### 31. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

#### Note:
A square-root sign is to be understood over every coefficient, e.g., for $-8/15$ read $-\sqrt{8/15}$.

#### 1/2x1/2
\[
Y^0 = \sqrt{\frac{3}{4\pi}} \cos \theta \\
Y^1 = \sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}
\]

#### 1x1/2
\[
Y^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{8} \cos^2 \theta - \frac{1}{2} \right) \\
Y^1 = \sqrt{\frac{15}{8\pi}} \sin \theta e^{i\phi}
\]

#### 2x1
\[
Y^m = (-1)^m Y^m_{-m} \\
d_{m0,m'0} = \sqrt{\frac{4\pi}{2m+1}} Y^m_{-m} Y^{m'}_{-m'}
\]

#### 3/2x3/2
\[
d_{1/2,1/2}^{1/2,1/2} = \cos \theta \\
d_{1/2,-1/2}^{1/2,-1/2} = \sin \theta
\]

#### 3/2x1/2
\[
d_{3/2,1/2}^{3/2,1/2} = \frac{1 + \cos \theta}{2} \\
d_{3/2,-1/2}^{3/2,-1/2} = \frac{1 - \cos \theta}{2}
\]

#### 3/2x2/2
\[
d_{3/2,3/2} = \frac{1 + \cos \theta}{2} \\
d_{3/2,1/2}^{3/2,1/2} = -\sqrt{\frac{1 + \cos \theta}{2}} \sin \frac{\theta}{2}
\]

**Figure 31.1:** The sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.