

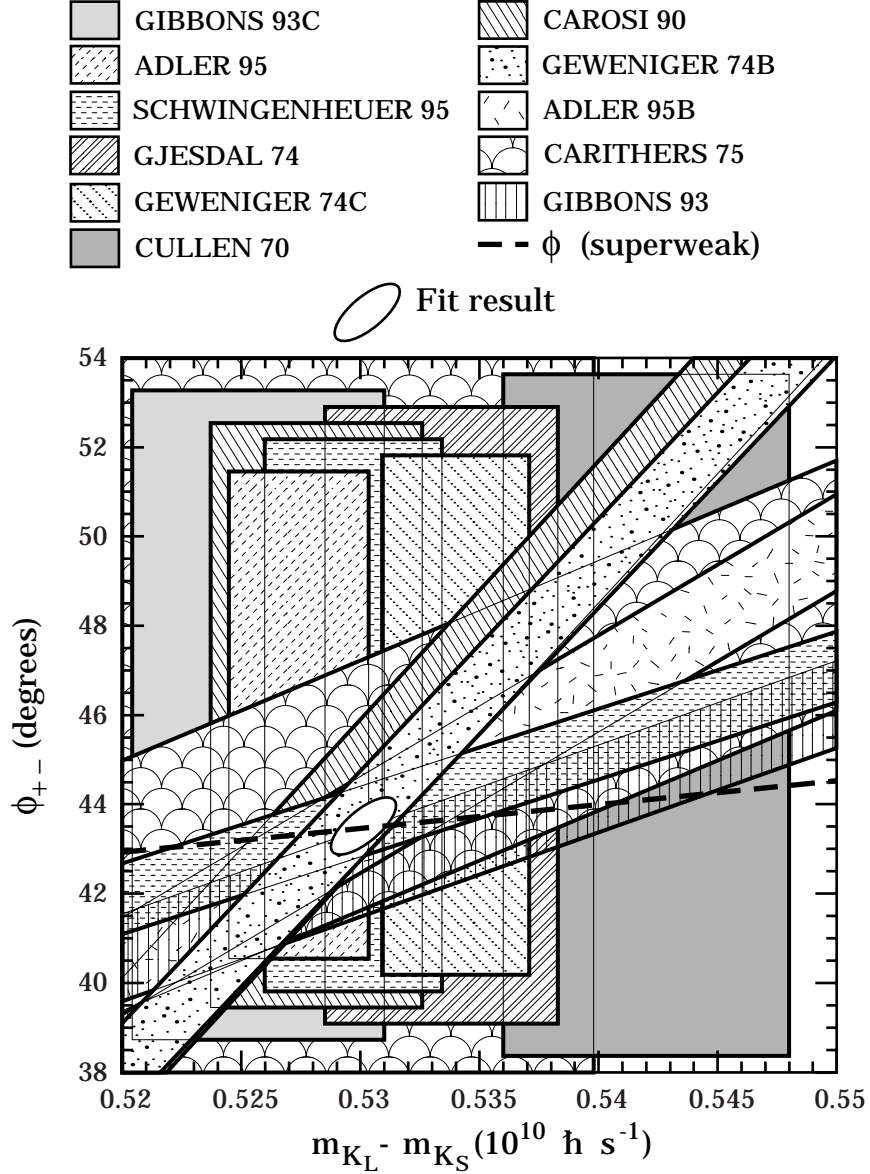
## FITS FOR $K_L^0$ $CP$ -VIOLATION PARAMETERS

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In recent years,  $K_L^0$   $CP$ -violation experiments have improved our knowledge of  $CP$ -violation parameters and their consistency with the expectations of  $CPT$  invariance and unitarity. For definitions of  $K_L^0$   $CP$ -violation parameters and a brief discussion of the theory, see the article “ $CP$  Violation” by L. Wolfenstein in Section 12 of this *Review*.

This note describes our two fits for the  $CP$ -violation parameters in  $K_L^0 \rightarrow \pi^+\pi^-$  and  $\pi^0\pi^0$  decay, one for the phases  $\phi_{+-}$  and  $\phi_{00}$ , and another for the amplitudes  $|\eta_{+-}|$  and  $|\eta_{00}|$ .

***Fit to  $\phi_{+-}$ ,  $\phi_{00}$ ,  $\Delta\phi$ ,  $\Delta m$ , and  $\tau_S$  data:*** We perform a joint fit to the data on  $\phi_{+-}$ ,  $\phi_{00}$ , the phase difference  $\Delta\phi = \phi_{00} - \phi_{+-}$ , the  $K_L^0 - K_S^0$  mass difference  $\Delta m$ , and the  $K_S^0$  mean life  $\tau_S$ , including the effects of correlations. Measurements of  $\phi_{+-}$  and  $\phi_{00}$  are highly correlated with  $\Delta m$  and  $\tau_S$ . Some measurements of  $\tau_S$  are correlated with  $\Delta m$ . The correlations are given in the footnotes of the  $\phi_{+-}$  and  $\phi_{00}$  sections of the  $K_L^0$  Particle Listings and the  $\tau_S$  section of the  $K_S^0$  Particle listings. In editions of the *Review* prior to 1996, we adjusted the experimental values of  $\phi_{+-}$  and  $\phi_{00}$  to account for correlations with  $\Delta m$  and  $\tau_S$  but did not include the effects of these correlations when evaluating  $\Delta m$  and  $\tau_S$ . When a joint fit including these correlations is done, the  $\phi_{+-}$  measurements have a strong influence on the fitted value of  $\Delta m$ . This is because the CERN NA31 vacuum regeneration experiments (CAROSI 90 [1] and GEWENIGER 74B [2]), the Fermilab E773/E731 regenerator experiments (SCHWINGENHEUER 95 [3] and GIBBONS 93 [4]), and the CPLEAR  $K^0 - \bar{K}^0$  asymmetry experiment (ADLER 95B [5]) have very different dependences of  $\phi_{+-}$  on  $\Delta m$ , as can be seen from their diagonal bands in Fig. 1. The region where the  $\phi_{+-}$  bands from these experiments cross gives a powerful measurement of  $\Delta m$  which decreases the fitted  $\Delta m$  relative to our pre-1996 average  $\Delta m$  and earlier measurements such as CULLEN 70 [6], GEWENIGER 74C [7], and GJESDAL 74 [8]. This decrease



**Figure 1:**  $\phi_{+-}$  vs  $\Delta m$ .  $\Delta m$  measurements appear as vertical bands spanning  $\Delta m \pm 1\sigma$ , some of which are cut near the top to aid the eye. The  $\phi_{+-}$  measurements appear as diagonal bands spanning  $\phi_{+-} \pm \sigma_\phi$ . The dashed line shows  $\phi(\text{superweak})$ . The ellipse shows the  $1\sigma$  contour of the fit result. See Table 1 for data references.

brings the  $\Delta m$ -dependent  $\phi_{+-}$  measurements into good agreement with each other and with  $\phi(\text{superweak})$ , where

$$\phi(\text{superweak}) = \tan^{-1} \left( \frac{2\Delta m}{\Delta\Gamma} \right) = \tan^{-1} \left( \frac{2\Delta m \tau_S \tau_L}{\hbar(\tau_L - \tau_S)} \right). \quad (1)$$

**Table 1:** References and location of input data for Fig. 1 and Fig. 2. Unless otherwise indicated by a footnote, a check ( $\checkmark$ ) indicates that the data can be found in the  $\phi_{+-}$  or  $\Delta m$  sections of the  $K_L$  Particle Listings, or the  $\tau_S$  section of the  $K_S$  Particle Listings, according to the column headers.

Location of input data					
Fig. 1		Fig. 2		PDG Document ID	Ref.
$\phi_{+-}$	$\Delta m$	$\phi_{+-}$	$\tau_S$		
$\checkmark$	$\checkmark^*$	$\checkmark$	$\checkmark^*$	CAROSI 90	[1]
$\checkmark$		$\checkmark^\dagger$	$\checkmark$	GEWENIGER 74B	[2]
$\checkmark$		$\checkmark$		ADLER 95B	[5]
$\checkmark$	$\checkmark^\ddagger$	$\checkmark^\dagger$	$\checkmark$	CARITHERS 75	[10]
$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	SCHWINGENHEUER 95	[3]
$\checkmark$		$\checkmark$	$\checkmark$	GIBBONS 93	[4]
	$\checkmark$			GIBBONS 93C	[11]
	$\checkmark$			ADLER 95	[12]
	$\checkmark$			GJESDAL 74	[8]
	$\checkmark$			GEWENIGER 74C	[7]
	$\checkmark$			CULLEN 70	[6]
			$\checkmark$	ARONSON 76	[13]
			$\checkmark$	GROSSMAN 87	[14]
			$\checkmark$	SKJEGGESTAD 72	[15]
			$\checkmark$	BERTANZA 97	[9]

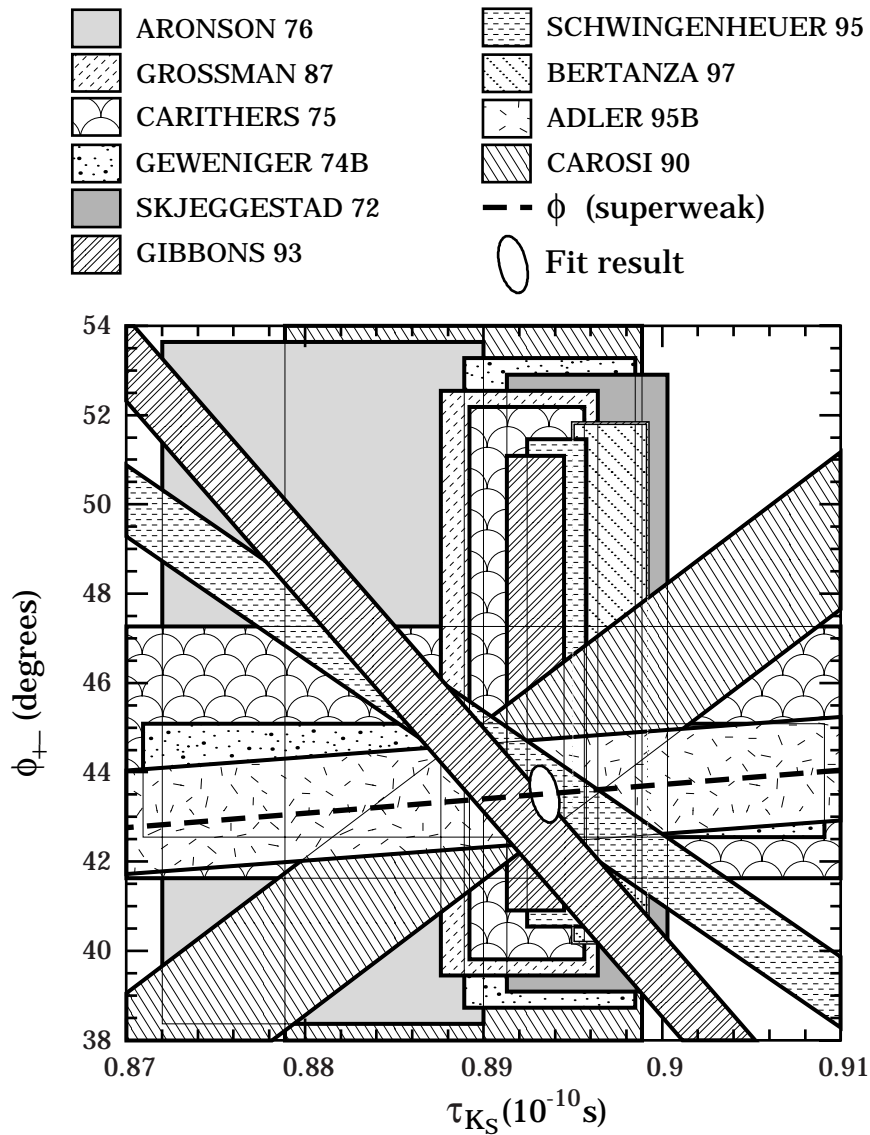
\* from  $\phi_{00}(\Delta m, \tau_S)$  in  $\phi_{00}$  Particle Listings.

$\dagger$  from  $\phi_{+-}(\Delta m)$  in  $\phi_{+-}$  Particle Listings.

$\ddagger$  from  $\tau_S(\Delta m)$  in  $\tau_S$  Particle Listings.

The  $(\phi_{+-}, \tau_S)$  correlations influence the  $\tau_S$  fit result in a similar manner, as can be seen in Fig. 2. The influence of the  $\phi_{+-}$  experiments is not as great on  $\tau_S$  as it is on  $\Delta m$  because the indirect measurements of  $\tau_S$  derived from the diagonal crossing bands in Fig. 2 are not as precise as the direct measurements of  $\tau_S$  from E773 (SCHWINGENHEUER 95 [3]), E731 (GIBBONS 93 [4]), and NA31 (BERTANZA 97 [9]).

In Fig. 1 [Fig. 2] the slope of the diagonal  $\phi_{+-}$  bands shows the  $\Delta m$  [ $\tau_S$ ] dependence; the unseen  $\tau_S$  [ $\Delta m$ ] dependent term



**Figure 2:**  $\phi_{+-}$  vs  $\tau_S$ .  $\tau_S$  measurements appear as vertical bands spanning  $\tau_S \pm 1\sigma$ , some of which are cut near the top to aid the eye. The  $\phi_{+-}$  measurements appear as diagonal bands spanning  $\phi_{+-} \pm \sigma_\phi$ . The dashed line shows  $\phi$ (superweak). The ellipse shows the fit result's  $1\sigma$  contour. See Table 1 for data references.

is evaluated using the fitted  $\tau_S$  [ $\Delta m$ ]. The vertical half-width  $\sigma_\phi$  of each band is the  $\phi_{+-}$  error for fixed  $\Delta m$  [ $\tau_S$ ] and includes the systematic error due to the error in the fitted  $\tau_S$  [ $\Delta m$ ].

Table 2 gives the resulting fit values for the parameters and Table 3 gives the correlation matrix. The resulting  $\phi_{+-}$  is in

good agreement with  $\phi(\text{superweak}) = 43.50 \pm 0.08^\circ$  obtained from Eq. (1) using  $\Delta m$  and  $\tau_S$  from Table 2.

**Table 2:** Results of the fit for  $\phi_{+-}$ ,  $\phi_{00}$ ,  $\phi_{00} - \phi_{+-}$ ,  $\Delta m$ , and  $\tau_S$ . The fit has  $\chi^2 = 15.4$  for 18 degrees of freedom (22 measurements  $-5$  parameters  $+1$  constraint).

Quantity	Fit Result
$\phi_{+-}$	$43.5 \pm 0.6^\circ$
$\Delta m$	$(0.5301 \pm 0.0014) \times 10^{10} \hbar \text{ s}^{-1}$
$\tau_S$	$(0.8934 \pm 0.0008) \times 10^{-10} \text{ s}$
$\phi_{00}$	$43.4 \pm 1.0^\circ$
$\Delta\phi$	$-0.1 \pm 0.8^\circ$

**Table 3:** Correlation matrix for the fitted parameters.

	$\phi_{+-}$	$\Delta m$	$\tau_S$	$\phi_{00}$	$\Delta\phi$
$\phi_{+-}$	1.00	0.72	-0.35	0.60	-0.02
$\Delta m$	0.72	1.00	-0.22	0.48	0.04
$\tau_S$	-0.35	-0.22	1.00	-0.18	0.04
$\phi_{00}$	0.60	0.48	-0.18	1.00	0.79
$\Delta\phi$	-0.02	0.04	0.04	0.79	1.00

The  $\chi^2$  is 15.4 for 18 degrees of freedom, indicating good agreement of the input data. Nevertheless, there has been criticism that Fermilab E773 (SCHWINGENHEUER 95 [3]) and E731 (GIBBONS 93 [4]) measure  $\phi_{+-} - \phi_f$  and calculate the regeneration phase  $\phi_f$  from the power law momentum dependence of the regeneration amplitude using analyticity and dispersion relations. In the E731 result, a systematic error of  $\pm 0.5$  degrees for departures from a pure power-law is included. For the E773 result, they modeled a variety of effects that do distort the amplitude from a pure power law and ascribed a  $\pm 0.35^\circ$  systematic error from uncertainties in these effects. Even so, the E731 result remains valid within its quoted errors. KLEINKNECHT 94 [16] and KLEINKNECHT 95 [17] argue

that these systematic errors should be around 3°, primarily because of the absence of data on the momentum dependence of the regeneration amplitude above 160 GeV/ $c$ . BRIERE 95 [18] and BRIERE 95C [19] reply that the current understanding of regeneration is sufficient to allow a precise and reliable correction for the region above 160 GeV/ $c$ . The question is one of judgement about the reliability of the assumptions used. In the absence of any contradictory evidence, we choose to accept the judgement of the E731/E773 experimenters in setting their systematic errors.

A similar analysis has been done by the CPLEAR Collaboration [20]. The small differences between their results and ours are due primarily to different treatments of  $\tau_S$ . Their fit constrains  $\tau_S$  to the PDG 1994 value, while our fit includes the more recent SCHWINGENHEUER 95 [3] and BERTANZA 97 [9]  $\tau_S$  measurements.

### Fit for $\epsilon'/\epsilon$ , $|\eta_{+-}|$ , $|\eta_{00}|$ , and $B(K_L \rightarrow \pi\pi)$

We list measurements of  $|\eta_{+-}|$ ,  $|\eta_{00}|$ ,  $|\eta_{00}/\eta_{+-}|$  and  $\epsilon'/\epsilon$ . Independent information on  $|\eta_{+-}|$  and  $|\eta_{00}|$  can be obtained from measurements of the  $K_L^0$  and  $K_S^0$  lifetimes ( $\tau_L$ ,  $\tau_S$ ) and branching ratios (B) to  $\pi\pi$ , using the relations

$$|\eta_{+-}| = \left[ \frac{B(K_L^0 \rightarrow \pi^+\pi^-)}{\tau_L} \frac{\tau_S}{B(K_S^0 \rightarrow \pi^+\pi^-)} \right]^{1/2}, \quad (2a)$$

$$|\eta_{00}| = \left[ \frac{B(K_L^0 \rightarrow \pi^0\pi^0)}{\tau_L} \frac{\tau_S}{B(K_S^0 \rightarrow \pi^0\pi^0)} \right]^{1/2}. \quad (2b)$$

For historical reasons the branching ratio fits and the  $CP$ -violation fits are done separately, but we want to include the influence of  $|\eta_{+-}|$ ,  $|\eta_{00}|$ ,  $|\eta_{00}/\eta_{+-}|$ , and  $\epsilon'/\epsilon$  measurements on  $B(K_L^0 \rightarrow \pi^+\pi^-)$  and  $B(K_L^0 \rightarrow \pi^0\pi^0)$  and vice versa. We approximate a global fit to all of these measurements by first performing two independent fits: 1) BRFIT, a fit to the  $K_L^0$  branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the  $|\eta_{+-}|$ ,  $|\eta_{00}|$ ,  $|\eta_{+-}/\eta_{00}|$ , and  $\epsilon'/\epsilon$  measurements. The results from fit 1, along with the  $K_S^0$  values from this edition are used to compute values of  $|\eta_{+-}|$  and  $|\eta_{00}|$  which are included as

measurements in the  $|\eta_{00}|$  and  $|\eta_{+-}|$  sections with a document ID of BRFIT 98. Thus the fit values of  $|\eta_{+-}|$  and  $|\eta_{00}|$  given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct  $|\eta|$  measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 98 values) are used along with the  $K_L^0$  and  $K_S^0$  mean lives and the  $K_S^0 \rightarrow \pi\pi$  branching fractions to compute the  $K_L^0$  branching ratios  $\Gamma(K_L^0 \rightarrow \pi^+\pi^-)/\Gamma(\text{total})$  and  $\Gamma(K_L^0 \rightarrow \pi^0\pi^0)/\Gamma(K_L^0 \rightarrow \pi^+\pi^-)$ . These branching ratio values are included as measurements in the branching ratio section with a document ID of ETAFIT 98. Thus the  $K_L^0$  branching ratio fit values in this edition include the results of direct measurements of  $|\eta_{+-}|$ ,  $|\eta_{00}|$ ,  $|\eta_{00}/\eta_{+-}|$ , and  $\epsilon'/\epsilon$ . A more detailed discussion of these fits is given in the 1990 edition of this *Review* [21].

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