**B⁰–B̅⁰ MIXING**

Written March 2000 by O. Schneider (Univ. of Lausanne)

**Formalism in quantum mechanics**

There are two neutral B⁰–B̅⁰ meson systems, B_d–B̅_d and B_s–B̅_s (generically denoted B_q–B̅_q, q = s, d), which exhibit the phenomenon of particle-antiparticle mixing [1]. Such a system is produced in one of its two possible states of well-defined flavor: |B⁰⟩ (b̅q) or |B̅⁰⟩ (bq̅). Due to flavor-changing interactions, this initial state evolves into a time-dependent quantum superposition of the two flavor states, a(t)|B⁰⟩ + b(t)|B̅⁰⟩, satisfying the equation

\[ i \frac{\partial}{\partial t} \left( \begin{array}{c} a(t) \\ b(t) \end{array} \right) = \left( M - i \frac{\Gamma}{2} \right) \left( \begin{array}{c} a(t) \\ b(t) \end{array} \right), \]

where M and \( \Gamma \), known as the mass and decay matrices, describe the dispersive and absorptive parts of B⁰–B̅⁰ mixing. These matrices are hermitian, and CPT invariance requires \( M_{11} = M_{22} = M \) and \( \Gamma_{11} = \Gamma_{22} = \Gamma \), where M and \( \Gamma \) are the mass and decay width of the B⁰ and B̅⁰ flavor states.

The two eigenstates of the effective hamiltonian matrix \( (M - i \frac{\Gamma}{2}) \) are given by

\[ |B_\pm⟩ = p|B⁰⟩ ± q|B̅⁰⟩, \]  

and correspond to the eigenvalues

\[ \lambda_\pm = \left( M - i \frac{\Gamma}{2} \right) ± \frac{q}{p} \left( M_{12} - i \frac{\Gamma_{12}}{2} \right), \]

where

\[ \frac{q}{p} = \sqrt{ \frac{M_{12}^* - i \frac{\Gamma_{12}}{2}}{M_{12} - i \frac{\Gamma_{12}}{2}} }. \]

We choose a convention where Re\( (q/p) > 0 \) and \( CP|B⁰⟩ = |B̅⁰⟩ \).

An alternative notation is

\[ |B_\pm⟩ = \frac{(1 + \epsilon)|B⁰⟩ ± (1 - \epsilon)|B̅⁰⟩}{\sqrt{2(1 + |\epsilon|^2)}} \quad \text{with} \quad \frac{1 - \epsilon}{1 + \epsilon} = \frac{q}{p}. \]

The time dependence of these eigenstates of well-defined masses \( M_\pm = \text{Re}(\lambda_\pm) \) and widths \( \Gamma_\pm = -2 \text{Im}(\lambda_\pm) \) is given by
the phases $e^{-i\lambda^\pm t} = e^{-iM^\pm t}e^{-\frac{1}{2}\Gamma^\pm t}$: the evolution of a pure $|B^0\rangle$ or $|\bar{B}^0\rangle$ state at $t = 0$ is thus given by

$$|B^0(t)\rangle = g_+(t) |B^0\rangle + \frac{q}{p} g_-(t) |\bar{B}^0\rangle,$$

(6)

$$|\bar{B}^0(t)\rangle = g_+(t) |\bar{B}^0\rangle + \frac{p}{q} g_-(t) |B^0\rangle,$$

(7)

where

$$g_\pm(t) = \frac{1}{2} \left( e^{-i\lambda^+_t} \pm e^{-i\lambda^-_t} \right).$$

(8)

This means that the flavor states oscillate into each other with time-dependent probabilities proportional to

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) \pm \cos(\Delta m t) \right],$$

(9)

where

$$\Delta m = |M_+ - M_-|, \quad \Delta \Gamma = |\Gamma_+ - \Gamma_-|.$$

(10)

Time-integrated mixing probabilities are only well defined when considering decays to flavor-specific final states, i.e. final states $f$ such that the instantaneous decay amplitudes $A_f = \langle \bar{f} | H | B^0 \rangle$ and $\bar{A}_f = \langle f | H | \bar{B}^0 \rangle$, where $H$ is the weak interaction hamiltonian, are both zero. Due to mixing, a produced $B^0$ can decay to the final state $\bar{f}$ (mixed event) in addition to the final state $f$ (unmixed event). Restricting the sample to these two decay channels, the time-integrated mixing probability is given by

$$\chi_{\bar{f} \rightarrow f}^{B^0} = \frac{\int_0^\infty |\langle \bar{f} | H | B^0(t) \rangle|^2 dt}{\int_0^\infty |\langle \bar{f} | H | B^0(t) \rangle|^2 dt + \int_0^\infty |\langle f | H | B^0(t) \rangle|^2 dt}$$

$$= \frac{\xi_f^2 (x^2 + y^2)}{\xi_f^2 (x^2 + y^2) + 2 + x^2 - y^2},$$

(11)

where we have defined $\xi_f = \frac{q A_{\bar{f}}}{p A_f}$ and

$$x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}.$$

(12)

The mixing probability $\chi_{\bar{f} \rightarrow \bar{B}^0}^{B^0}$ for the case of a produced $\bar{B}^0$ is obtained by replacing $\xi_f$ with $1/\xi_f$ in Eq. (11). It is different from $\chi_{f \rightarrow \bar{B}^0}^{B^0}$ if $|\xi_f|^2 \neq 1$, a condition reflecting non-invariance under the $CP$ transformation. $CP$ violation in the
decay amplitudes is discussed elsewhere [2] and we assume \(|\mathcal{T}_f| = |A_f|\) from now on. The deviation of \(|q/p|^2\) from 1, namely the quantity

\[
1 - \left|\frac{q}{p}\right|^2 = \frac{4\text{Re}(\epsilon)}{1 + |\epsilon|^2} + O\left(\left(\frac{\text{Re}(\epsilon)}{1 + |\epsilon|^2}\right)^2\right),
\]  

(13)
decribes \(CP\) violation in \(B^0 - \bar{B}^0\) mixing. As can be seen from Eq. (4), this can occur only if \(M_{12} \neq 0\), \(\Gamma_{12} \neq 0\) and if the phase difference between \(M_{12}\) and \(\Gamma_{12}\) is different from 0 or \(\pi\).

In the absence of \(CP\) violation, \(|q/p|^2 = 1\), \(\text{Re}(\epsilon) = 0\), the mass eigenstates are also \(CP\) eigenstates,

\[
CP|B_{\pm}\rangle = \pm|B_{\pm}\rangle,
\]

(14)
the phases \(\varphi_{M_{12}} = \text{arg}(M_{12})\) and \(\varphi_{\Gamma_{12}} = \text{arg}(\Gamma_{12})\) satisfy

\[
\sin(\varphi_{M_{12}} - \varphi_{\Gamma_{12}}) = 0,
\]

(15)
the mass and decay width differences reduce to

\[
\Delta m = 2|M_{12}|, \quad \Delta \Gamma = 2|\Gamma_{12}|,
\]

(16)
and the time-integrated mixing probabilities \(\chi_{f}^{B^0 \to \bar{B}^0}\) and \(\chi_{f}^{\bar{B}^0 \to B^0}\) become both equal to

\[
\chi = \frac{x^2 + y^2}{2(x^2 + 1)}.
\]

(17)

**Standard Model predictions and phenomenology**

In the Standard Model, the transitions \(B_q^0 \to \bar{B}_q^0\) and \(\bar{B}_q^0 \to B_q^0\) are due to the weak interaction. They are described, at the lowest order, by the box diagrams involving two \(W\) bosons and two up-type quarks, as is the case for \(K^0 - \bar{K}^0\) mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral \(B\) meson systems, because the large \(B\) mass is away from the region of hadronic resonances. The calculation of the dispersive and absorptive
parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} S_0(m_i^2/m_W^2) (V_{tq}^* V_{tb})^2$$  \hspace{1cm} (18)$$

$$\Gamma_{12} = \frac{G_F^2 m_W^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi}$$

$$\times \left[ (V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right]$$

$$+ (V_{cq}^* V_{cb})^2 \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right)$$  \hspace{1cm} (19)$$

where $G_F$ is the Fermi constant, $m_W$ the $W$ mass, $m_i$ the mass of quark $i$, and where $m_{B_q} = M$, $f_{B_q}$ and $B_{B_q}$ are the $B^0_q$ mass, decay constant and bag parameter. The known function $S_0(x_t)$ can be approximated very well with $0.784 x_t^{0.76}$ [4] and $V_{ij}$ are the elements of the CKM matrix [5]. The QCD corrections $\eta_B$ and $\eta'_B$ are of order unity. The only non negligible contributions to $M_{12}$ are from top-top diagrams. The phases of $M_{12}$ and $\Gamma_{12}$ satisfy

$$\varphi_{M_{12}} - \varphi_{\Gamma_{12}} = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right)$$  \hspace{1cm} (20)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the $K^0 - \bar{K}^0$ system, the “heavy” state with mass $M_{\text{heavy}} = \max(M_+, M_-)$ has a smaller decay width than that of the “light” state with mass $M_{\text{light}} = \min(M_+, M_-)$. We thus redefine

$$\Delta m = M_{\text{heavy}} - M_{\text{light}}, \quad \Delta \Gamma = \Gamma_{\text{light}} - \Gamma_{\text{heavy}},$$  \hspace{1cm} (21)$$

where $\Delta m$ is positive by definition and $\Delta \Gamma$ is expected to be positive in the Standard Model.

Furthermore, since $\Gamma_{12}$ is, like $M_{12}$, dominated by the top-top diagrams, the quantity

$$\left| \frac{\Gamma_{12}}{M_{12}} \right| \approx \frac{3\pi}{2} \frac{m_b^2}{m_W^2} \frac{1}{S_0(m_i^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_i^2}\right)$$  \hspace{1cm} (22)$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left| \frac{q}{p} \right|^2 = 1 + \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin(\varphi_{M_{12}} - \varphi_{\Gamma_{12}}) + \mathcal{O}\left(\frac{\left| \Gamma_{12}/M_{12} \right|^2}{1}\right).$$  \hspace{1cm} (23)$$
Therefore, considering both Eqs. (20) and (22), the $CP$-violating parameter

$$1 - \left| \frac{q}{p} \right|^2 \simeq \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right)$$

(24)

is expected to be tiny: $\sim \mathcal{O}(10^{-3})$ for the $B_d - \overline{B}_d$ system and $\lesssim \mathcal{O}(10^{-4})$ for the $B_s - \overline{B}_s$ system [6].

In the approximation of negligible $CP$ violation in the mixing, the ratio $\Delta \Gamma/\Delta m$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eq. (22); it is hence independent of CKM matrix elements, i.e. the same for the $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ systems. It can be calculated with lattice QCD techniques; typical results are $\sim 5 \times 10^{-3}$ with quoted uncertainties of 30% at least. Given the current experimental knowledge (discussed below) on the mixing parameter $x$,

$$\begin{cases} x_d = 0.73 \pm 0.03 & (B_d - \overline{B}_d \text{ system}) \\ x_s \gtrsim 20 & \text{at 95\% CL (} B_s - \overline{B}_s \text{ system}) \end{cases}$$

(25)

the Standard Model thus predicts that $\Delta \Gamma/\Gamma$ is very small for the $B_d - \overline{B}_d$ system (below 1%), but may be quite large for the $B_s - \overline{B}_s$ system (up to $\sim 20\%$). This width difference is caused by the existence of final states to which both the $B^0_q$ and $\overline{B}^0_q$ mesons can decay. Such decays involve $b \rightarrow c\bar{s}q$ quark-level transitions, which are Cabibbo-suppressed if $q = d$ and Cabibbo-allowed if $q = s$. If the final states common to $B^0_s$ and $\overline{B}^0_s$ are predominantly $CP$-even as discussed in Ref. 7, then the $B_s - \overline{B}_s$ mass eigenstate with the largest decay width corresponds to the $CP$-even eigenstate. Taking Eq. (21) into account, one thus expects $\Gamma_{\text{light}} = \Gamma_+$ and

$$\Delta m_s = M_- - M_+ > 0, \quad \Delta \Gamma_s = \Gamma_+ - \Gamma_- > 0.$$ 

(26)

**Experimental issues and methods for oscillation analyses**

Time-integrated measurements of $B^0 - \overline{B}^0$ mixing were published for the first time in 1987 by UA1 [8] and ARGUS [9], and since then by many different experiments. These are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced $b\overline{b}$ pairs. At high
energy colliders, such analyses cannot easily separate the $B_d$ and $B_s$ contributions, therefore experiments at $\Upsilon(4S)$ machines are best suited to measure $\chi_d$.

However, better sensitivity is obtained from time-dependent analyses aimed at the direct measurement of the oscillation frequencies $\Delta m_d$ and $\Delta m_s$, from the proper time distributions of $B_d$ or $B_s$ candidates identified through their decay in (mostly) flavor-specific modes and suitably tagged as mixed or unmixed. This is particularly true for the $B_s - \bar{B}_s$ system where the large value of $x_s$ implies maximal mixing, i.e. $\chi_s \simeq 1/2$. In such analyses, performed at high-energy colliders, the neutral $B$ mesons are either partially reconstructed from a charm meson, or selected from a lepton with high transverse momentum with respect to the $b$ jet, or selected from a reconstructed displaced vertex. The proper time $t = \frac{m_B}{p}L$ is measured from the distance $L$ between the production vertex and the $B$ decay vertex, as measured with a silicon vertex detector, and from an estimate of the $B$ momentum $p$.

The statistical significance $S$ of an oscillation signal can be approximated as [10]

$$S \approx \sqrt{N/2} f_{\text{sig}} (1 - 2\eta) e^{-(\Delta m \sigma_t)^2/2},$$

(27)

where $N$ and $f_{\text{sig}}$ are the number of candidates and the fraction of signal in the selected sample, $\eta$ is the mistag probability, and $\sigma_t$ is the proper time resolution. The quantity $S$ decreases very quickly as $\Delta m$ increases; this dependence is controlled by $\sigma_t$, which is therefore a critical parameter for $\Delta m_s$ analyses. The proper time resolution $\sigma_t \sim \frac{m_B}{\langle p \rangle} \sigma_L \oplus t \frac{\sigma_p}{p}$ includes a constant contribution due to the decay length resolution $\sigma_L$ (typically 0.1–0.3 ps), and a term due to the relative momentum resolution $\frac{\sigma_p}{p}$ (typically 10–20% for partially reconstructed decays), which increases with proper time.

In order to tag a $B$ candidate as mixed or unmixed, it is necessary to determine its flavor state both at production (initial state) and at decay (final state). The initial and final state mistag probabilities, $\eta_i$ and $\eta_f$, degrade $S$ by a total factor $(1 - 2\eta) = (1 - 2\eta_i)(1 - 2\eta_f)$. In inclusive lepton analyses,
the final state is tagged by the charge of the lepton from $b \to \ell^-$ decays; the biggest contribution to $\eta_f$ is then due to $\bar{b} \to \tau \to \ell^-$ decays. Alternatively, the charge of a reconstructed charm meson ($D^{*-}$ from $B^0_d$ or $D^-_s$ from $B^0_s$), or that of a kaon thought to come from a $b \to c \to s$ decay [11], can be used. For fully inclusive analyses based on topological vertexing, final state tagging techniques include jet charge [12] and charge dipole methods [11].

The initial state tags are somewhat less dependent on the procedure used to select $B$ candidates. They can be divided in two groups: the ones that tag the initial charge of the $b$ quark contained in the $B$ candidate itself (same-side tag), and the ones that tag the initial charge of the other $b$ quark produced in the event (opposite-side tag). On the same side, the charge of a track from the primary vertex is correlated with the production state of the $B$ if that track is a decay product of a $B^{**}$ state or the first particle in the fragmentation chain [13,14]. Jet charge techniques work on both sides. Finally, the charge of a lepton from $b \to \ell^-$ or of a kaon from $b \to c \to s$ can be used as opposite side tags, keeping in mind that their performance depends on integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the $Z \to b\bar{b}$ decays and provided another very interesting and effective initial state tag based on the polar angle of the $B$ candidate [11]. Initial state tags have also been combined to reach $\eta_i \sim 26\%$ at LEP [14,15] or even $16\%$ at SLD [11] with full efficiency. The equivalent figure at CDF is currently $\sim 40\%$ [16].

In the absence of experimental evidence for a width difference, and since $\Delta \Gamma/\Delta m$ is predicted to be very small, oscillation analyses typically neglect $\Delta \Gamma$ and describe the data with the physics functions $\Gamma e^{-\Gamma t}(1 \pm \cos \Delta mt)/2$. As can be seen from Eq. (9), a non zero value of $\Delta \Gamma$ would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Whereas measurements of $\Delta m_d$ are usually extracted from the data using a maximum likelihood fit, no significant $B_s - \bar{B}_s$ oscillations have been seen so far, and all $B_s$
analyses set lower limits on $\Delta m_s$. The original technique used to set such limits was to study the likelihood as a function of $\Delta m_s$. However, these limits turned out to be difficult to combine. A method was therefore developed [10], in which a $B_s$ oscillation amplitude $\mathcal{A}$ is measured at each fixed value of $\Delta m_s$, using a maximum likelihood fit based on the functions $\Gamma_s e^{-\Gamma_s t} (1 \pm \mathcal{A} \cos \Delta m_s t) / 2$. To a very good approximation, the statistical uncertainty on $\mathcal{A}$ is Gaussian and equal to $1/$. Measurements of $\mathcal{A}$ performed at a given value of $\Delta m_s$ can be averaged easily. If $\Delta m_s = \Delta m_s^{\text{true}}$, one expects $\mathcal{A} = 1$ within the total uncertainty $\sigma_A$; however, if $\Delta m_s$ is far from its true value, a measurement consistent with $\mathcal{A} = 0$ is expected. A value of $\Delta m_s$ can be excluded at 95% CL if $\mathcal{A} + 1.645 \sigma_A \leq 1$. If $\Delta m_s^{\text{true}}$ is very large, one expects $\mathcal{A} = 0$, and all values of $\Delta m_s$ such that $1.645 \sigma_A (\Delta m_s) < 1$ are expected to be excluded at 95% CL. Because of the proper time resolution, the quantity $\sigma_A (\Delta m_s)$ is an increasing function of $\Delta m_s$ and one therefore expects to be able to exclude individual $\Delta m_s$ values up to $\Delta m_s^{\text{sens}}$, where $\Delta m_s^{\text{sens}}$, called here the sensitivity of the analysis, is defined by $1.645 \sigma_A (\Delta m_s^{\text{sens}}) = 1$.

$B_d$ mixing studies

Many $B_d \overline{B_d}$ oscillations analyses have been performed by the ALEPH [17,12], CDF [13,18], DELPHI [19], L3 [20], OPAL [21] and SLD [11] collaborations. Although a variety of different techniques have been used, the $\Delta m_d$ results have remarkably similar precision. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or $b$-hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the $b$-hadron lifetimes and fractions published in this Review. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of $b$ hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all published measurements [17,13,19,20,21] and accounting
for all identified correlations as described in Ref. 22 yields \( \Delta m_d = 0.478 \pm 0.012\text{(stat)} \pm 0.013\text{(syst)} \) \( \text{ps}^{-1} \).

On the other hand, ARGUS and CLEO have published time-integrated measurements based on semileptonic decays \([23,24]\), which average to \( \chi_d^{T(4S)} = 0.156 \pm 0.024 \). The width difference \( \Delta \Gamma_d \) could in principle be extracted from the measured value of \( \Gamma_d \), and the above averages for \( \Delta m_d \) and \( \chi_d \) (see Eqs. (12) and (17)). The results are however compatible with \( \Delta \Gamma_d = 0 \), and their precision is still insufficient to provide an interesting constraint. Neglecting \( \Gamma_d \) and using the measured \( B_d \) lifetime, the \( \Delta m_d \) and \( \chi_d \) results are combined to yield the world average

\[
\Delta m_d = 0.472 \pm 0.017 \text{ \( \text{ps}^{-1} \)} \quad (28)
\]

or, equivalently,

\[
\chi_d = 0.174 \pm 0.009 \text{ .} \quad (29)
\]

Evidence for \( CP \) violation in \( B_d \) mixing has been searched for, both with semileptonic and inclusive \( B_d \) decays, in samples where the initial flavor state is tagged. In the semileptonic case, where the final state tag is also available, the following asymmetry

\[
\frac{N(B_d^0(t) \to \ell^+\nu_{\ell}X) - N(B_d^0(t) \to \ell^-\bar{\nu}_{\ell}X)}{N(B_d^0(t) \to \ell^+\nu_{\ell}X) + N(B_d^0(t) \to \ell^-\bar{\nu}_{\ell}X)} = a_{CP} \simeq 1 - \frac{|q/p|^2}{d} \approx \frac{4\text{Re}(\epsilon_d)}{1 + |\epsilon_d|^2} \quad (30)
\]

has been measured, either in time-integrated analyses at CLEO \([24]\) and CDF \([25]\), or in more recent and sensitive time-dependent analyses at LEP \([26,27,28]\). In the inclusive case, also investigated at LEP \([29,27,30]\), no final state tag is used, and the asymmetry \([31]\)

\[
\frac{N(B_d^0(t) \to \text{all}) - N(B_d^0(t) \to \text{all})}{N(B_d^0(t) \to \text{all}) + N(B_d^0(t) \to \text{all})} = a_{CP} \left[ \frac{x_d}{2} \sin(\Delta m_d t) - \sin^2 \left( \frac{\Delta m_d t}{2} \right) \right] \quad (31)
\]

must be measured as a function of the proper time to extract information on \( CP \) violation. In all cases asymmetries compatible with zero have been found, with a precision limited by
the available statistics. A simple average of all published and preliminary results \([24–30]\) neglecting small possible statistical correlations and assuming half of the systematics to be correlated, is \(a_{CP} = -0.017 \pm 0.016\), a result which does not yet constrain the Standard Model.

The \(\Delta m_d\) result of Eq. (28) provides an estimate of \(|M_{12}|\) and can be used, together with Eqs. (16) and (18), to extract the modulus of the CKM matrix element \(V_{td}\) within the Standard Model \([32]\). The main experimental uncertainties on the resulting estimate of \(|V_{td}|\) come from \(m_t\) and \(\Delta m_d\); however, these are at present completely dominated by the 15–20% uncertainty usually quoted on the hadronic matrix element \(f_{B_d}\sqrt{B_{B_d}} \sim 200\) MeV obtained from lattice QCD calculations \([33]\).

**B_\(s\)** mixing studies

\(B_s\)–\(\bar{B}_s\) oscillation has been the subject of many recent studies from ALEPH \([14]\), CDF \([34]\), DELPHI \([35,15]\), OPAL \([36]\) and SLD \([37]\). No oscillation signal has been found so far. The most sensitive analyses appear to be the ones based on inclusive lepton samples, and on samples where a lepton and a \(D_s\) meson have been reconstructed in the same jet. All results are limited by the available statistics. These are combined to yield the amplitudes \(A\) shown in Fig. 1 as a function of \(\Delta m_s\) \([22]\).

As before, the individual results have been adjusted to common physics inputs, and all known correlations have been accounted for; furthermore, the sensitivities of the inclusive analyses, which depend directly through Eq. (27) on the assumed fraction \(f_s\) of \(B_s\) mesons in an unbiased sample of weakly-decaying \(b\) hadrons, have been rescaled to a common value of \(f_s = 0.100 \pm 0.012\) \([22]\). The combined sensitivity for 95% CL exclusion of \(\Delta m_s\) values is found to be 14.5 ps\(^{-1}\). All values of \(\Delta m_s\) below 14.3 ps\(^{-1}\) are excluded at 95% CL, and no deviation from \(A = 0\) is seen in Fig. 1 that would indicate the observation of a signal.

Some \(\Delta m_s\) analyses are still preliminary \([15,37]\). Using only published results, the combined \(\Delta m_s\) result is

\[
\Delta m_s > 10.6 \text{ ps}^{-1} \quad \text{at 95\% CL},
\]

\(32\)
Figure 1: Combined measurements of the $B_s$ oscillation amplitude as a function of $\Delta m_s$ [22], including all preliminary results available at the end of 1999. The measurements are dominated by statistical uncertainties. Neighboring points are statistically correlated.

with a sensitivity of 12.1 ps$^{-1}$.

The information on $|V_{ts}|$ obtained, in the framework of the Standard Model, from the combined limit is hampered by the hadronic uncertainty, as in the $B_d$ case. However, many uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2,$$

where $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$, of order unity, is currently estimated from lattice QCD with a 5–6% uncertainty [33].
results on $\Delta m_d$, $\Delta m_s$, $|V_{ub}/V_{cb}|$ and $\epsilon_K$, together with theoretical inputs and unitarity conditions [32]. Given the information available from $|V_{ub}/V_{cb}|$ and $\epsilon_K$ measurements, the constraint from our knowledge on the ratio $\Delta m_d/\Delta m_s$ is presently more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the $\Delta m_d$ measurements alone, due to the reduced hadronic uncertainty in Eq. (33). We note also that the Standard Model would not easily accommodate values of $\Delta m_s$ above $\sim 25$ ps$^{-1}$.

Information on $\Delta \Gamma_s$ can be obtained by studying the proper time distribution of untagged data samples enriched in $B_s$ mesons [38]. In the case of an inclusive $B_s$ selection [39] or a semileptonic $B_s$ decay selection [40,41], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants $\Gamma_s \pm \Delta \Gamma_s/2$. In principle, this provides sensitivity to both $\Gamma_s$ and $(\Delta \Gamma_s/\Gamma_s)^2$. Ignoring $\Delta \Gamma_s$ and fitting for a single exponential leads to an estimate of $\Gamma_s$ with a relative bias proportional to $(\Delta \Gamma_s/\Gamma_s)^2$. An alternative approach, which is directly sensitive to first order in $\Delta \Gamma_s/\Gamma_s$, is to determine the lifetime of $B_s$ candidates decaying to $CP$ eigenstates; measurements already exist for $B^0_s \rightarrow J/\psi\phi$ [42] and $B^0_s \rightarrow D^{(*)}_s D^{(*)}_s$ [43], which are mostly $CP$-even states [7]. An estimate of $\Delta \Gamma_s/\Gamma_s$ has also been obtained directly from a measurement of the $B^0_s \rightarrow D^{(*)}_s D^{(*)}_s$ branching ratio [43], under the assumption that these decays practically account for all the $CP$-even final states.

Present data is not precise enough to efficiently constrain both $\Gamma_s$ and $\Delta \Gamma_s/\Gamma_s$; since the $B_s$ and $B_d$ lifetimes are predicted to be equal within less than a percent [44], an expectation compatible with the current experimental data [45], the constraint $\Gamma_s = \Gamma_d$ can also be used to extract $\Delta \Gamma_s/\Gamma_s$. Applying the combination procedure described in Ref. 22 on the published $B_s$ lifetime results [40,42,46] yields

$$\Delta \Gamma_s/\Gamma_s < 0.65 \quad \text{at 95\% CL} \quad (34)$$

without external constraint, or

$$\Delta \Gamma_s/\Gamma_s < 0.33 \quad \text{at 95\% CL} \quad (35)$$
when constraining $1/\Gamma_s$ to the measured $B_d$ lifetime. These results are not yet precise enough to test Standard Model predictions.

**Average $b$-hadron mixing and $b$-hadron production fractions**

Let $f_u$, $f_d$, $f_s$ and $f_{\text{baryon}}$ be the $B_u$, $B_d$, $B_s$ and $b$-baryon fractions composing an unbiased sample of weakly-decaying $b$ hadrons produced in high energy colliders. LEP experiments have measured $f_s \times \text{BR}(B_s^0 \to D_s^- \ell^+ \nu_\ell X)$ [47], $\text{BR}(b \to A_b^0) \times \text{BR}(A_b^0 \to \Lambda_c^+ \ell^- \bar{\nu}_\ell X)$ [48] and $\text{BR}(b \to \Xi_b^-) \times \text{BR}(\Xi_b^- \to \Xi^- \ell^- \bar{\nu}_\ell X)$ [49] from partially reconstructed final states including a lepton, $f_{\text{baryon}}$ from protons identified in $b$ events [50], and the production rate of charged $b$ hadrons [51]. The various $b$ hadron fractions have also been measured at CDF from electron-charm final states [52]. All the published results have been combined following the procedure and assumptions described in Ref. 22, to yield $f_u = f_d = (38.4 \pm 1.8)\%$, $f_s = (11.7 \pm 3.0)\%$ and $f_{\text{baryon}} = (11.5 \pm 2.0)\%$ under the constraints

$$f_u = f_d \quad \text{and} \quad f_u + f_d + f_s + f_{\text{baryon}} = 1. \quad (36)$$

Time-integrated mixing analyses performed with lepton pairs from $b\bar{b}$ events produced at high energy colliders measure the quantity

$$\chi = f_d' \chi_d + f_s' \chi_s, \quad (37)$$

where $f_d'$ and $f_s'$ are the fractions of $B_d$ and $B_s$ hadrons in a sample of semileptonic $b$-hadron decays. Assuming that all $b$ hadrons have the same semileptonic decay width implies $f_q' = f_q/\Gamma_q \tau_b$ ($q = s, d$), where $\tau_b$ is the average $b$-hadron lifetime. Hence $\chi$ measurements can be used to improve our knowledge on the fractions $f_u$, $f_d$, $f_s$ and $f_{\text{baryon}}$.

Combining the above estimates of these fractions with the average $\chi = 0.118 \pm 0.005$ (published in this Review), $\chi_d$ from Eq. (29) and $\chi_s = \frac{1}{2}$ yields, under the constraints of Eq. (36),

$$f_u = f_d = (38.9 \pm 1.3)\%, \quad (38)$$

$$f_s = (10.7 \pm 1.4)\%, \quad (39)$$

$$f_{\text{baryon}} = (11.6 \pm 2.0)\%, \quad (40)$$
showing that mixing information substantially reduces the uncertainty on $f_s$. These results and the averages quoted in Eqs. (28) and (29) for $\Delta m_d$ and $\Delta m_d$ have been obtained in a consistent way by the $B$ oscillations working group [22], taking into account the fact that many individual measurements of $\Delta m_d$ depend on the assumed values for the $b$-hadron fractions.

Summary and prospects

$B^0-\bar{B}^0$ mixing has been a field of intense study in the last few years. The mass difference in the $B_d-\bar{B}_d$ system is very well measured (with an accuracy of $\sim 3.5\%$) but, despite an impressive theoretical effort, the hadronic uncertainty still limits the precision of the extracted estimate of $|V_{td}|$. The mass difference in the $B_s-\bar{B}_s$ system is much larger and still unmeasured. However, the current experimental lower limit on $\Delta m_s$ already provides, together with $\Delta m_d$, a significant constraint on the CKM matrix within the Standard Model.

No strong experimental evidence exists yet for the rather large decay width difference expected in the $B_s-\bar{B}_s$ system. It is interesting to recall that the ratio $\Delta \Gamma_s/\Delta m_s$ does not depend on CKM matrix elements in the Standard Model (see Eq. (22)), and that a measurement of either $\Delta m_s$ or $\Delta \Gamma_s$ could be turned into a Standard Model prediction of the other one.

The LEP and SLD experiments have still not finalized all their $B_s$ oscillation analyses, but a measurement of $\Delta m_s$ from data collected at the $Z$ pole becomes unlikely. In the near future, the most promising prospects for $B_s$ mixing are from Run II at the Tevatron, where both $\Delta m_s$ and $\Delta \Gamma_s$ are expected to be measured; CDF will be able to observe $B_s$ oscillations for values of $\Delta m_s$ up to $\sim 40$ ps$^{-1}$ [53], well above the current Standard Model prediction.

$CP$ violation in $B$ mixing, which has not been seen yet, as well as the phases involved in $B$ mixing, will be further investigated with the large statistics that will become available both at the $B$ factories and at the Tevatron.

$B$ mixing may not have delivered all its secrets yet, because it is one of the phenomena where new physics might very well reveal itself (for example new particles involved in the box
diagrams). Theoretical calculations in lattice QCD are becoming more reliable and further progress in reducing hadronic uncertainties is expected. In the long term, a stringent check of the consistency, within the Standard Model, of the \( B_d \) and \( B_s \) mixing measurements with all other measured observables in \( B \) physics (including \( CP \) asymmetries in \( B \) decays) will be possible, allowing to place limits on new physics or, better, discover new physics.

References

   see also the review on \( B^0-\bar{B}^0 \) mixing by H. Quinn in C. Caso et al., Eur. Phys. J. C3, 1 (1998).
2. See the review on \( CP \) violation in \( B \) decays by H. Quinn and A. Sanda in this publication.


22. ALEPH, CDF, DELPHI, L3, OPAL, and SLD Collab., “Combined results on $b$-hadron production rates, lifetimes, oscillations, and semileptonic decays,” LEPHFS note 99-02, to be subm. as CERN–EP preprint; the combined results on $B$ mixing and $b$ hadron fractions included in the above paper or published in this Review have been obtained by the $B$ oscillations working group; see http://www.cern.ch/LEPBOSC/ for more information.


30. **ALEPH** Collab., conf. note 98-032, contrib. 4-396 to Int. Europhysics Conf. on High Energy Physics, Tampere, 1999.


32. See the review on the CKM quark-mixing matrix by F.J. Gilman, K. Kleinknecht and B. Renk in this publication.


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45. See the review on production and decay of b-hadrons by L. Gibbons and K. Honscheid in this publication.


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