### 31. Clebsch-Gordan Coefficients, Spherical Harmonics, and D Functions

Note: A square-root sign is to be understood over every coefficient, e.g., for $-8/15$ read $-\sqrt{8}/10$.

#### 1/2 x 1/2

- $Y^0_{1/2} = \sqrt{\frac{3}{4\pi}} \cos \theta$
- $Y^1_{1/2} = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$
- $Y^2_{1/2} = \sqrt{\frac{5(3/2)^2}{4\pi}} (\cos^2 \theta - 1/2)$

#### 2 x 1

- $Y^0_{3/2} = \sqrt{\frac{12}{20\pi}} \sin \theta (\cos \theta + 1/2)$
- $Y^1_{3/2} = \sqrt{\frac{12}{20\pi}} \sin \theta (\cos \theta - 1/2)$
- $Y^2_{3/2} = \sqrt{\frac{12}{20\pi}} \sin \theta (\cos \theta + 3/2)$

#### 3 x 1

- $Y^0_{1} = \sqrt{\frac{1}{2\pi}} \cos \theta$
- $Y^1_{1} = -\sqrt{\frac{1}{2\pi}} \sin \theta e^{i\phi}$
- $Y^2_{1} = \sqrt{\frac{6}{2\pi}} \sin^2 \theta e^{i\phi}$

#### 2 x 3/2

- $Y^0_{3/2} = \sqrt{\frac{3}{8\pi}} \sin \theta (\cos \theta + 1/2)$
- $Y^1_{3/2} = \sqrt{\frac{3}{8\pi}} \sin \theta (\cos \theta - 1/2)$
- $Y^2_{3/2} = \sqrt{\frac{3}{8\pi}} \sin \theta (\cos \theta + 3/2)$

#### 3 x 3/2

- $Y^0_{1} = \sqrt{\frac{1}{2\pi}} \cos \theta$
- $Y^1_{1} = -\sqrt{\frac{1}{2\pi}} \sin \theta e^{i\phi}$
- $Y^2_{1} = \sqrt{\frac{6}{2\pi}} \sin^2 \theta e^{i\phi}$

#### Notation:

- $J_J \ldots$
- $d_{m,J}$
- $m, m'$
- $\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle$

#### Figure 31.1