12. CP VIOLATION

Revised April 2000 by L. Wolfenstein (Carnegie-Mellon Univ.).

The symmetries C (particle-antiparticle interchange) and P (space inversion) hold for strong and electromagnetic interactions. After the discovery of large C and P violation in the weak interactions, it appeared that the product CP was a good symmetry. In 1964 CP violation was observed in $K^0$ decays at a level given by the parameter $\epsilon \approx 2.3 \times 10^{-3}$. Larger CP-violation effects are anticipated in $B^0$ decays.

12.1. CP violation in Kaon decay

CP violation has been observed in the semi-leptonic decays $K^0_L \rightarrow \pi^+ \ell^+ \nu$ and in the nonleptonic decay $K^0_L \rightarrow 2\pi$. The experimental numbers that have been measured are

$$\delta = \frac{\Gamma(K^0_L \rightarrow \pi^- \ell^+ \nu) - \Gamma(K^0_S \rightarrow \pi^+ \ell^- \nu)}{\Gamma(K^0_L \rightarrow \pi^- \ell^+ \nu) + \Gamma(K^0_S \rightarrow \pi^+ \ell^- \nu)}$$

$$\eta_+ = A(K^0_L \rightarrow \pi^+ \pi^-)/A(K^0_S \rightarrow \pi^+ \pi^-)$$

$$\nu_{00} = A(K^0_L \rightarrow \pi^0 \nu)/A(K^0_S \rightarrow \pi^0 \nu)$$

where $A$ is the amplitude and $\nu$ is the CP-violation parameter in the decay $K_S \rightarrow \pi^+ \pi^- \pi^0$ [5], although limits on $\nu_{00}$ are still poor. The relation in Eq. (12.6a) is exact in the superweak theory so this is sometimes called the superweak phase. An important point for the analysis is that $|\cos(\phi - \phi')|$ is small. The consequence is that only two real quantities need be measured, the magnitude of $\epsilon$ and the value of $(\epsilon'/\epsilon)$ including its sign. The measured quantity $|\nu_{00}/\nu_{++}-\nu_{--}|$, which is very close to unity, is given by a good approximation by

$$|\nu_{00}/\nu_{++}-\nu_{--}| \approx 1 - 6 \Re(\epsilon'/\epsilon) \approx 1 - 6\epsilon'/\epsilon .$$

From the experimental measurements, one finds

$$\epsilon = (2.271 \pm 0.017) \times 10^{-3},$$

$$\Re(\epsilon'/\epsilon) = (2.1 \pm 0.5) \times 10^{-3},$$

$$|\phi_{+-} - \phi_0| = 43.5 \pm 0.5^\circ ,$$

$$|\phi_{00} - \phi_{+-}| = 0.1 \pm 0.8 ,$$

$$|\delta| = (3.3 \pm 0.14) \times 10^{-3} .$$

Direct CP violation, as indicated by $\epsilon'/\epsilon$, is expected in the Standard Model; most calculations [6] give a somewhat smaller value, but they have a large uncertainty. The value of $\epsilon'$ agrees with Eq. (12.5d). The values of $\phi_{+-}$ and $\phi_{00} - \phi_{+-}$ are used to set limits on CP violation. [See Tests of Conservation Laws.]

In the Standard Model, CP violation arises as a result of a single phase entering the CKM matrix (Sec. 11). As a result in what is now the standard phase convention, two elements have large phases, $V_{ub} \sim e^{-i\gamma}$, $V_{td} \sim e^{-i\delta}$. Because these elements have small magnitudes and involve the third generation, CP violation in the $K^0$ system is small. On the other hand, large effects are expected in the $B^0$ system, which is a major motivation for $B$ factories.

12.2. CP violation in B decay

CP violation in the $B^0$ system can be observed by comparing $B^0$ and $\bar{B}^0$ decays [7]. For a final CP eigenstate $a$, the decay rate has a time dependence given by

$$\Gamma_a \sim \epsilon^{-\gamma} \left( 1 + |\lambda_a|^2 \right) \approx 1 - |\lambda_a|^2 \cos(\Delta M t)$$

where the top sign is for $B^0$ and the bottom for $\bar{B}^0$ and

$$\lambda_a = (q_B/\bar{q}_B) \ |A_a|/A_a .$$

The quantities $p_B$ and $q_B$ come from the analogue for $B^0$ of Eq. (12.2), and $A_a(\Delta M)$ is the decay amplitude to state $a$ for $B^0(\bar{B}^0)$. However, for $B^0$, the eigenstates are expected to have a negligible lifetime difference and are only distinguished by the mass difference $\Delta M$; also as a consequence $|q_B/p_B| \approx 1$ so that $\epsilon_B$ is purely imaginary.
If only one quark weak transition contributes to the decay, \( |\mathcal{A}_a/\mathcal{A}_b| = 1 \) so that \( |\mathcal{A}_b| = 1 \) and the \( \cos(\Delta M t) \) term vanishes. In this case, the difference between \( B^0 \) and \( \bar{B}^0 \) decays is given by the \( \sin(\Delta M t) \) term with the asymmetric coefficient

\[
a_a = \frac{\Gamma_a(t) - \Gamma_b(t)}{\left( \Gamma_a(t) + \Gamma_b(t) \right)} = \tan \sin \left( 2(\phi_M + \phi_D) \right),
\]

where \( 2\phi_M \) is the phase of the \( B^0 \bar{B}^0 \) mixing, \( \phi_D \) is the weak phase of the decay transition, and \( \tan \) is the \( CP \) eigenvalue of \( a \).

For \( B^0 \bar{B}^0 \rightarrow \psi K_S \) from the transition \( b \rightarrow c \bar{s} \), one finds in the Standard Model that the asymmetry is given directly in terms of a CKM phase with no hadronic uncertainty:

\[
a_{\psi K_S} = -\sin 2\beta .
\]

From the constraints on the CKM matrix (Sec. 11) \( \sin 2\beta \) is predicted to be between 0.3 and 0.9. A significantly different value could be a sign of new physics.

A second decay of interest is \( B^0 \rightarrow \pi^+ \pi^- \) from the transition \( b \rightarrow u \bar{d} \) with

\[
a_{\pi\pi} = \sin 2(\beta + \gamma) .
\]

While either of these asymmetries could be ascribed to \( B^0 \bar{B}^0 \) mixing \( (q_B \bar{q}_{\bar{B}} \text{ or } \bar{q}_B q_{\bar{B}}) \), the difference between the two asymmetries is evidence for direct \( CP \) violation. From Eq. (12.10) it is seen that this corresponds to a phase difference between \( A_{\psi K_S} \) and \( A_{\pi\pi} \). Thus this is analogous to \( \epsilon' \). In the standard phase convention, \( \cos 2\beta \) in Eqs. (12.12) and (12.13) arises from \( B^0 \bar{B}^0 \) mixing whereas the \( \gamma \) in Eq. (12.13) comes from \( V_{ub} V_{cb}^\ast \) in the transition \( b \rightarrow u \bar{d} \). The result in Eq. (12.13) may have a sizeable correction due to what is called a penguin diagram. This is a one-loop graph producing \( b \rightarrow d + \text{gluon} \) with a \( W \) and a quark, predominantly the \( t \) quark, in the loop. This leads to an amplitude proportional to \( V_{ud}^\ast V_{tb} \), which has a weak phase different from that of the original tree amplitude proportional to \( V_{ub} V_{cb}^\ast \). There are several methods to approximately determine this correction using additional measurements [8].

\( CP \) violation in the decay amplitude is also revealed by the \( \cos(\Delta M t) \) term in Eq. (12.9) or by a difference in rates of \( B^+ \) and \( B^- \) to charge-conjugate states. These effects, however, require two contributing amplitudes to the decay (such as a tree amplitude plus a penguin) and also require final-state interaction phases. Predicted effects are very uncertain and are generally small [9].

In the case of the \( B_s \) system, the mass difference \( \Delta M \) is much larger than for \( B^0 \) and has not yet been measured. As a result, it will be difficult to isolate the \( \sin(\Delta M t) \) term to measure asymmetries. Furthermore, in the Standard Model with the standard phase convention, \( \phi_M \) is very small so that decays due to \( b \rightarrow c \bar{s} \), yielding \( B_s \rightarrow \psi f_1^0 \), would have zero asymmetry. Decays due to \( b \rightarrow u \bar{d} \) yielding \( B_s \rightarrow \psi f_1^3 \), would have an asymmetry \( \sin 2\beta \) in the tree approximation. The width difference \( \Delta \Gamma \) is also expected to be much larger for \( B_s \) so that \( \Delta \Gamma/\Gamma \) might be as large as 0.15. In this case, there might be a possibility of detecting \( CP \) violation as in the case of \( \epsilon' \) by observing the \( B_s \) states with different lifetimes decaying into the same \( CP \) eigenstate [10].

For further details, see the notes on \( CP \) violation in the \( K^0_L, K^0_S \), and \( B^0 \) Particle Listings of this Review.

References: