SUPERSYMMETRY
Revised October 1999 by Howard E. Haber (Univ. of California,
Santa Cruz) Part I, and by M. Schmitt (Harvard Univ.) Part II

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SUPERSYMMETRY, PART I (THEORY)
(by H.E. Haber)

I.1. Introduction: Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa. It also provides a framework for the unification of particle physics and gravity [1–3], which is governed by the Planck scale, $M_P \approx 10^{19}$ GeV (defined to be the energy scale where the gravitational interactions of elementary particles become comparable to their gauge interactions). If supersymmetry were an exact symmetry of nature, then particles and their superpartners (which differ in spin by half a unit) would be degenerate in mass. Thus, supersymmetry cannot be an exact symmetry of nature, and must be broken. In theories of “low-energy” supersymmetry,
the effective scale of supersymmetry breaking is tied to the electroweak scale [4–6], which is characterized by the Standard Model Higgs vacuum expectation value $v = 246$ GeV. It is thus possible that supersymmetry will ultimately explain the origin of the large hierarchy of energy scales from the $W$ and $Z$ masses to the Planck scale.

At present, there are no unambiguous experimental results that require the existence of low-energy supersymmetry. However, if experimentation at future colliders uncovers evidence for supersymmetry, this would have a profound effect on the study of TeV-scale physics and the development of a more fundamental theory of mass and symmetry-breaking phenomena in particle physics.

I.2. Structure of the MSSM: The minimal supersymmetric extension of the Standard Model (MSSM) consists of taking the Standard Model and adding the corresponding supersymmetric partners [2,7]. In addition, the MSSM contains two hypercharge $Y = \pm 1$ Higgs doublets, which is the minimal structure for the Higgs sector of an anomaly-free supersymmetric extension of the Standard Model. The supersymmetric structure of the theory also requires (at least) two Higgs doublets to generate mass for both “up”-type and “down”-type quarks (and charged leptons) [8,9]. All renormalizable supersymmetric interactions consistent with (global) $B–L$ conservation ($B =$ baryon number and $L =$ lepton number) are included. Finally, the most general soft-supersymmetry-breaking terms are added [10].

If supersymmetry is associated with the origin of the scale of electroweak interactions, then the mass parameters introduced by the soft-supersymmetry-breaking terms must in general be of order 1 TeV or below [11] (although models have been proposed in which some supersymmetric particle masses can be larger, in the range of 1–10 TeV [12]). Some lower bounds on these parameters exist due to the absence of supersymmetric-particle production at current accelerators [13]. Additional constraints arise from limits on the contributions of virtual supersymmetric particle exchange to a variety of Standard Model processes [14,15]. In particular, the Standard Model fit (without supersymmetry)
to precision electroweak data is quite good [16]. If all supersymmetric particle masses are significantly heavier than $m_Z$ (in practice, masses greater than 300 GeV are sufficient [17]), then the effects of the supersymmetric particles decouple in loop-corrections to electroweak observables [18]. In this case the Standard Model global fit to precision data and the corresponding MSSM fit yield similar results. On the other hand, regions of parameter space with light supersymmetric particle masses can generate significant one-loop corrections, resulting in a poorer overall fit to the data [19]. Thus, the precision electroweak data provide some constraints on the magnitude of the soft-supersymmetry-breaking terms.

As a consequence of $B-L$ invariance, the MSSM possesses a multiplicative $R$-parity invariance, where $R = (-1)^{3(B-L)+2S}$ for a particle of spin $S$ [20]. Note that this formula implies that all the ordinary Standard Model particles have even $R$-parity, whereas the corresponding supersymmetric partners have odd $R$-parity. The conservation of $R$-parity in scattering and decay processes has a crucial impact on supersymmetric phenomenology. For example, starting from an initial state involving ordinary ($R$-even) particles, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay quickly into lighter states. However, $R$-parity invariance also implies that the lightest supersymmetric particle (LSP) is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle.

In order to be consistent with cosmological constraints, a stable LSP is almost certainly electrically and color neutral [21]. Consequently, the LSP in a $R$-parity-conserving theory is weakly-interacting in ordinary matter, i.e. it behaves like a stable heavy neutrino and will escape detectors without being directly observed. Thus, the canonical signature for conventional $R$-parity-conserving supersymmetric theories is missing (transverse) energy, due to the escape of the LSP. Moreover, the LSP is a prime candidate for “cold dark matter” [22], a
potentially important component of the non-baryonic dark matter that is required in many models of cosmology and galaxy formation [23].

In the MSSM, supersymmetry breaking is accomplished by including the most general renormalizable soft-supersymmetry-breaking terms consistent with the SU(3)×SU(2)×U(1) gauge symmetry and R-parity invariance. These terms parameterize our ignorance of the fundamental mechanism of supersymmetry breaking. If supersymmetry breaking occurs spontaneously, then a massless Goldstone fermion called the goldstino \( \tilde{G} \) must exist. The goldstino would then be the LSP and could play an important role in supersymmetric phenomenology [24]. However, the goldstino is a physical degree of freedom only in models of spontaneously broken global supersymmetry. If the supersymmetry is a local symmetry, then the theory must incorporate gravity; the resulting theory is called supergravity. In models of spontaneously broken supergravity, the goldstino is “absorbed” by the gravitino \( \tilde{g}_{3/2} \), the spin-3/2 partner of the graviton [25]. By this super-Higgs mechanism, the goldstino is removed from the physical spectrum and the gravitino acquires a mass \( m_{3/2} \).

It is very difficult (perhaps impossible) to construct a model of spontaneously-broken low-energy supersymmetry where the supersymmetry breaking arises solely as a consequence of the interactions of the particles of the MSSM. A more viable scheme posits a theory consisting of at least two distinct sectors: a “hidden” sector consisting of particles that are completely neutral with respect to the Standard Model gauge group, and a “visible” sector consisting of the particles of the MSSM. There are no renormalizable tree-level interactions between particles of the visible and hidden sectors. Supersymmetry breaking is assumed to occur in the hidden sector, and then transmitted to the MSSM by some mechanism. Two theoretical scenarios have been examined in detail: gravity-mediated and gauge-mediated supersymmetry breaking.

Supergravity models provide a natural mechanism for transmitting the supersymmetry breaking of the hidden sector to
the particle spectrum of the MSSM. In models of gravity-mediated supersymmetry breaking, gravity is the messenger of supersymmetry breaking [26,27]. More precisely, supersymmetry breaking is mediated by effects of gravitational strength (suppressed by an inverse power of the Planck mass). In this scenario, the gravitino mass is of order the electroweak-symmetry-breaking scale, while its couplings are roughly gravitational in strength [1,28]. Such a gravitino would play no role in supersymmetric phenomenology at colliders.

In gauge-mediated supersymmetry breaking, supersymmetry breaking is transmitted to the MSSM via gauge forces. A typical structure of such models involves a hidden sector where supersymmetry is broken, a “messenger sector” consisting of particles (messengers) with SU(3)×SU(2)×U(1) quantum numbers, and the visible sector consisting of the fields of the MSSM [29,30]. The direct coupling of the messengers to the hidden sector generates a supersymmetry breaking spectrum in the messenger sector. Finally, supersymmetry breaking is transmitted to the MSSM via the virtual exchange of the messengers. If this approach is extended to incorporate gravitational phenomena, then supergravity effects will also contribute to supersymmetry breaking. However, in models of gauge-mediated supersymmetry breaking, one usually chooses the model parameters in such a way that the virtual exchange of the messengers dominates the effects of the direct gravitational interactions between the hidden and visible sectors. In this scenario, the gravitino mass is typically in the eV to keV range, and is therefore the LSP. The helicity $\pm \frac{1}{2}$ components of $g_{3/2}$ behave approximately like the goldstino; its coupling to the particles of the MSSM is significantly stronger than a coupling of gravitational strength.

**I.3. Parameters of the MSSM:** The parameters of the MSSM are conveniently described by considering separately the supersymmetry-conserving sector and the supersymmetry-breaking sector. A careful discussion of the conventions used in defining the MSSM parameters can be found in Ref. 31. For simplicity, consider the case of one generation of quarks, leptons, and their scalar superpartners. The parameters of
the supersymmetry-conserving sector consist of: (i) gauge couplings: \( g_s, g, \) and \( g' \), corresponding to the Standard Model gauge group \( SU(3) \times SU(2) \times U(1) \) respectively; (ii) a supersymmetry-conserving Higgs mass parameter \( \mu \); and (iii) Higgs-fermion Yukawa coupling constants: \( \lambda_u, \lambda_d, \) and \( \lambda_e \) (corresponding to the coupling of one generation of quarks, leptons, and their superpartners to the Higgs bosons and higgsinos).

The supersymmetry-breaking sector contains the following set of parameters: (i) gaugino Majorana masses \( M_3, M_2 \) and \( M_1 \) associated with the \( SU(3), SU(2), \) and \( U(1) \) subgroups of the Standard Model; (ii) five scalar squared-mass parameters for the squarks and sleptons, \( M^2_{Q}, M^2_{U'}, M^2_{D'}, M^2_{L}, \) and \( M^2_{E} \) [corresponding to the five electroweak gauge multiplets, \( i.e., \) superpartners of \( (u,d)_L, u^c_L, d^c_L, (\nu, e^-)_L, \) and \( e^c_L, ] \); (iii) Higgs-squark-squark and Higgs-slepton-slepton trilinear interaction terms, with coefficients \( A_u, A_d, \) and \( A_e \) (these are the so-called “\( A \)-parameters”); and (iv) three scalar Higgs squared-mass parameters—two of which contribute to the diagonal Higgs squared-masses, given by \( m^2_1 + |\mu|^2 \) and \( m^2_2 + |\mu|^2 \), and one off-diagonal Higgs squared-mass term, \( m^2_{12} \equiv B\mu \) (which defines the “\( B \)-parameter”). These three squared-mass parameters can be re-expressed in terms of the two Higgs vacuum expectation values, \( v_d \) and \( v_u \), and one physical Higgs mass. Here, \( v_d \) (\( v_u \)) is the vacuum expectation value of the Higgs field which couples exclusively to down-type (up-type) quarks and leptons. (Another notation often employed in the literature is \( v_1 \equiv v_d \) and \( v_2 \equiv v_u \).) Note that \( v_d^2 + v_u^2 = (246 \text{ GeV})^2 \) is fixed by the \( W \) mass, while the ratio

\[
\tan \beta = v_u / v_d
\]

is a free parameter of the model.

The total number of degrees of freedom of the MSSM is quite large, primarily due to the parameters of the soft-supersymmetry-breaking sector. In particular, in the case of three generations of quarks, leptons, and their superpartners, \( M^2_{Q}, M^2_{U}, M^2_{D}, M^2_{L}, \) and \( M^2_{E} \) are hermitian 3 \( \times \) 3 matrices, and the \( A \)-parameters are complex 3 \( \times \) 3 matrices. In addition, \( M_1, M_2, M_3, B \) and \( \mu \) are in general complex. Finally, as in the Standard
Model, the Higgs-fermion Yukawa couplings, $\lambda_f$ ($f = u, d, e$), are complex $3 \times 3$ matrices which are related to the quark and lepton mass matrices via: $M_f = \lambda_f v_f/\sqrt{2}$, where $v_e \equiv v_d$ (with $v_u$ and $v_d$ as defined above). However, not all these parameters are physical. Some of the MSSM parameters can be eliminated by expressing interaction eigenstates in terms of the mass eigenstates, with an appropriate redefinition of the MSSM fields to remove unphysical degrees of freedom. The analysis of Ref. 32 shows that the MSSM possesses 124 truly independent parameters. Of these, 18 parameters correspond to Standard Model parameters (including the QCD vacuum angle $\theta_{QCD}$), one corresponds to a Higgs sector parameter (the analogue of the Standard Model Higgs mass), and 105 are genuinely new parameters of the model. The latter include: five real parameters and three CP-violating phases in the gaugino/higgsino sector, 21 squark and slepton masses, 36 new real mixing angles to define the squark and slepton mass eigenstates and 40 new CP-violating phases that can appear in squark and slepton interactions. The most general $R$-parity-conserving minimal supersymmetric extension of the Standard Model (without additional theoretical assumptions) will be denoted henceforth as MSSM-124 [33].

**I.4. The supersymmetric-particle sector:** Consider the sector of supersymmetric particles ($sparticles$) in the MSSM. The supersymmetric partners of the gauge and Higgs bosons are fermions, whose names are obtained by appending “ino” at the end of the corresponding Standard Model particle name. The gluino is the color octet Majorana fermion partner of the gluon with mass $M_g = |M_3|$. The supersymmetric partners of the electroweak gauge and Higgs bosons (the gauginos and higgsinos) can mix. As a result, the physical mass eigenstates are model-dependent linear combinations of these states, called charginos and neutralinos, which are obtained by diagonalizing the corresponding mass matrices. The chargino-mass matrix depends on $M_2$, $\mu$, $\tan \beta$ and $m_W$ [34].
The corresponding chargino-mass eigenstates are denoted by \( \tilde{\chi}_1^+ \) and \( \tilde{\chi}_2^+ \), with masses

\[
M_{\tilde{\chi}_1^+ \tilde{\chi}_2^+}^2 = \frac{1}{2} \left\{ |\mu|^2 + |M_2|^2 + 2m_W^2 \mp \left[ (|\mu|^2 + |M_2|^2 + 2m_W^2)^2 - 4|\mu|^2|M_2|^2 - 4m_W^4 \sin^2 2\beta + 8m_W^2 \sin 2\beta \text{Re}(\mu M_2) \right]^{1/2} \right\},
\]

where the states are ordered such that \( M_{\tilde{\chi}_1^+} < M_{\tilde{\chi}_2^+} \). If CP-violating effects are neglected (in which case, \( M_2 \) and \( \mu \) are real parameters), then one can choose a convention where \( \tan \beta \) and \( M_2 \) are positive. (Note that the relative sign of \( M_2 \) and \( \mu \) is meaningful. The sign of \( \mu \) is convention-dependent; the reader is warned that both sign conventions appear in the literature.) The sign convention for \( \mu \) implicit in Eq. (2) is used by the LEP collaborations [13] in their plots of exclusion contours in the \( M_2 \) vs. \( \mu \) plane derived from the non-observation of \( e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \).

The neutralino mass matrix depends on \( M_1, M_2, \mu, \tan \beta, m_Z, \) and the weak mixing angle \( \theta_W \) [34]. The corresponding neutralino eigenstates are usually denoted by \( \tilde{\chi}_i^0 \) \( (i = 1, \ldots, 4) \), according to the convention that \( M_{\tilde{\chi}_1^0} \leq M_{\tilde{\chi}_2^0} \leq M_{\tilde{\chi}_3^0} \leq M_{\tilde{\chi}_4^0} \). If a chargino or neutralino eigenstate approximates a particular gaugino or higgsino state, it is convenient to employ the corresponding nomenclature. Specifically, if \( M_1 \) and \( M_2 \) are small compared to \( m_Z \) and \( |\mu| \), then the lightest neutralino \( \tilde{\chi}_1^0 \) would be nearly a pure photino, \( \tilde{\gamma} \), the supersymmetric partner of the photon. If \( M_1 \) and \( m_Z \) are small compared to \( M_2 \) and \( |\mu| \), then the lightest neutralino would be nearly a pure bino, \( \tilde{B} \), the supersymmetric partner of the weak hypercharge gauge boson. If \( M_2 \) and \( m_Z \) are small compared to \( M_1 \) and \( |\mu| \), then the lightest chargino pair and neutralino would constitute a triplet of roughly mass-degenerate pure winos, \( \tilde{W}^\pm \) and \( \tilde{W}_3^0 \), the supersymmetric partners of the weak SU(2) gauge bosons. Finally, if \( |\mu| \) and \( m_Z \) are small compared to \( M_1 \) and \( M_2 \), then the lightest neutralino would be nearly a pure higgsino. Each of the above cases leads to a strikingly different phenomenology.

The supersymmetric partners of the quarks and leptons are spin-zero bosons: the squarks, charged sleptons, and sneutrinos. For simplicity, only the one-generation case is illustrated below.
(using first-generation notation). For a given fermion $f$, there are two supersymmetric partners $\tilde{f}_L$ and $\tilde{f}_R$ which are scalar partners of the corresponding left and right-handed fermion. (There is no $\tilde{\nu}_R$ in the MSSM.) However, in general, $\tilde{f}_L$ and $\tilde{f}_R$ are not mass-eigenstates since there is $\tilde{f}_L$-$\tilde{f}_R$ mixing which is proportional in strength to the corresponding element of the scalar squared-mass matrix [35]

$$M^2_{LR} = \begin{cases} m_d(A_d - \mu \tan \beta), & \text{for "down"-type } f \\ m_u(A_u - \mu \cot \beta), & \text{for "up"-type } f, \end{cases}$$ (3)

where $m_d$ ($m_u$) is the mass of the appropriate “down” (“up”) type quark or lepton. The signs of the $A$-parameters are also convention-dependent; see Ref. 31. Due to the appearance of the fermion mass in Eq. (3), one expects $M_{LR}$ to be small compared to the diagonal squark and slepton masses, with the possible exception of the top-squark, since $m_t$ is large, and the bottom-squark and tau-slepton if $\tan \beta \gg 1$.

The (diagonal) $L$- and $R$-type squark and slepton squared-masses are given by

$$M^2_{\tilde{f}_L} = M^2_{\tilde{F}} + m_f^2 + (T_{3f} - e_f \sin^2 \theta_W) m_Z^2 \cos 2\beta,$$

$$M^2_{\tilde{f}_R} = M^2_{\tilde{R}} + m_f^2 + e_f \sin^2 \theta_W m_Z^2 \cos 2\beta,$$ (4)

where $M^2_{\tilde{F}} \equiv M^2_{\tilde{Q}}$ [$M^2_{\tilde{L}}$] for $\tilde{u}_L$ and $\tilde{d}_L$ [\tilde{\nu}_L and \tilde{e}_L], and $M^2_{\tilde{R}} \equiv M^2_{\tilde{U}}$, $M^2_{\tilde{D}}$, and $M^2_{\tilde{E}}$ for $\tilde{u}_R$, $\tilde{d}_R$, and $\tilde{e}_R$, respectively. In addition, $e_f = \frac{2}{3}$, $-\frac{1}{3}$, 0, -1 for $f = u$, $d$, $\nu$, and $e$, respectively, $T_{3f} = \frac{1}{2}$ $[-\frac{1}{2}]$ for up-type [down-type] squarks and sleptons, and $m_f$ is the corresponding quark or lepton mass. Squark and slepton mass eigenstates, generically called $\tilde{f}_1$ and $\tilde{f}_2$ (these are linear combinations of $\tilde{f}_L$ and $\tilde{f}_R$), are obtained by diagonalizing the corresponding $2 \times 2$ squared-mass matrices.

In the case of three generations, the general analysis is more complicated. The scalar squared-masses [$M^2_{\tilde{F}}$ and $M^2_{\tilde{R}}$ in Eq. (4)], the fermion masses $m_f$ and the $A$-parameters are now $3 \times 3$ matrices as noted in Section I.3. Thus, to obtain the squark and slepton mass eigenstates, one must diagonalize $6 \times 6$ mass matrices. As a result, intergenerational mixing is possible, although there are some constraints from the nonobservation of
FCNC’s [14,15]. In practice, because off-diagonal scalar mixing is appreciable only for the third generation, this additional complication can usually be neglected.

It should be noted that all mass formulae quoted in this section are tree-level results. One-loop corrections will modify all these results, and eventually must be included in any precision study of supersymmetric phenomenology [36].

I.5. The Higgs sector of the MSSM: Next, consider the Higgs sector of the MSSM [8,9,37]. Despite the large number of potential $CP$-violating phases among the MSSM-124 parameters, one can show that the tree-level MSSM Higgs sector is automatically $CP$-conserving. That is, unphysical phases can be absorbed into the definition of the Higgs fields such that $\tan \beta$ is a real parameter (conventionally chosen to be positive). Moreover, the physical neutral Higgs scalars are $CP$ eigenstates. There are five physical Higgs particles in this model: a charged Higgs boson pair ($H^\pm$), two $CP$-even neutral Higgs bosons (denoted by $H_1^0$ and $H_2^0$ where $m_{H_1^0} \leq m_{H_2^0}$) and one $CP$-odd neutral Higgs boson ($A^0$).

The properties of the Higgs sector are determined by the Higgs potential, which is made up of quadratic terms [whose squared-mass coefficients were mentioned above Eq. (1)] and quartic interaction terms. The strengths of the interaction terms are directly related to the gauge couplings by supersymmetry (and are not affected at tree-level by supersymmetry breaking). As a result, $\tan \beta$ [defined in Eq. (1)] and one Higgs mass determine the tree-level Higgs-sector parameters. These include the Higgs masses, an angle $\alpha$ [which measures the component of the original $Y = \pm 1$ Higgs doublet states in the physical $CP$-even neutral scalars], and the Higgs boson couplings.

When one-loop radiative corrections are incorporated, additional parameters of the supersymmetric model enter via virtual loops. The impact of these corrections can be significant [38]. For example, at tree-level, MSSM-124 predicts $m_{H_1^0} \leq m_Z |\cos 2\beta| \leq m_Z$ [8,9]. If this prediction were unmodified, it would imply that $H_1^0$ must be discovered at the LEP collider (running at its maximum energy and luminosity); otherwise MSSM-124 would be ruled out. However, when
radiative corrections are included, the light Higgs-mass upper bound may be significantly increased. The qualitative behavior of the radiative corrections can be most easily seen in the large top-squark mass limit, where in addition, both the splitting of the two diagonal entries [Eq. (4)] and the two off-diagonal entries [Eq. (3)] of the top-squark squared-mass matrix are small in comparison to the average of the two top-squark squared-masses, $M_S^2 \equiv \frac{1}{2}(M_{t_1}^2 + M_{t_2}^2)$. In this case (assuming $m_{A^0} > m_Z$), the upper bound on the lightest CP-even Higgs mass at one-loop is approximately given by

$$m_{H^0_1}^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left\{ \ln \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right\},$$

where $X_t \equiv A_t - \mu \cot \beta$ is the top-squark mixing factor [see Eq. (3)]. A more complete treatment of the radiative corrections [39] shows that Eq. (5) somewhat overestimates the true upper bound of $m_{H^0_1}$. These more refined computations, which incorporate renormalization group improvement and the leading two-loop contributions, yield $m_{H^0_1} \lesssim 130$ GeV (with an accuracy of a few GeV) for $m_t = 175$ GeV and $M_S \lesssim 1$ TeV [39].

In addition, one-loop radiative corrections can also introduce CP-violating effects in the Higgs sector, which depend on some of the CP-violating phases among the MSSM-124 parameters [40]. Although these effects are more model-dependent, they can have a non-trivial impact on the Higgs searches at LEP and future colliders.

**I.6. Reducing the MSSM parameter freedom:** Even in the absence of a fundamental theory of supersymmetry breaking, one is hard-pressed to regard MSSM-124 as a fundamental theory. For example, no fundamental explanation is provided for the origin of electroweak symmetry breaking. Moreover, MSSM-124 is not a phenomenologically viable theory over most of its parameter space. Among the phenomenologically deficiencies are: (i) no conservation of the separate lepton numbers $L_e$, $L_\mu$, and $L_\tau$; (ii) unsuppressed FCNC’s; and (iii) new sources of CP-violation that are inconsistent with the experimental bounds. As a result, almost the entire MSSM-124 parameter
space is ruled out! This theory is viable only at very special “exceptional” points of the full parameter space.

MSSM-124 is also theoretically deficient since it provides no explanation for the origin of the supersymmetry-breaking parameters (and in particular, why these parameters should conform to the exceptional points of the parameter space mentioned above). Moreover, the MSSM contains many new sources of $CP$ violation. For example, some combination of the complex phases of the gaugino-mass parameters, the $A$-parameters, and $\mu$ must be less than of order $10^{-2} - 10^{-3}$ (for a supersymmetry-breaking scale of 100 GeV) to avoid generating electric dipole moments for the neutron, electron, and atoms in conflict with observed data [41,42].

There are two general approaches for reducing the parameter freedom of MSSM-124. In the low-energy approach, an attempt is made to elucidate the nature of the exceptional points in the MSSM-124 parameter space that are phenomenologically viable. Consider the following two possible choices. First, one can assume that $M^2_{Q}, M^2_{U}, M^2_{D}, M^2_{L}, M^2_{E}$ and the matrix $A$-parameters are generation-independent (horizontal universality [5,32,43]). Alternatively, one can simply require that all the aforementioned matrices are flavor diagonal in a basis where the quark and lepton mass matrices are diagonal (flavor alignment [44]). In either case, $L_e, L_\mu,$ and $L_\tau$ are separately conserved, while tree-level FCNC’s are automatically absent. In both cases, the number of free parameters characterizing the MSSM is substantially less than 124. Both scenarios are phenomenologically viable, although there is no strong theoretical basis for either scenario.

In the high-energy approach, one treats the parameters of the MSSM as running parameters and imposes a particular structure on the soft-supersymmetry-breaking terms at a common high-energy scale [such as the Planck scale ($M_P$)]. Using the renormalization group equations, one can then derive the low-energy MSSM parameters. The initial conditions (at the appropriate high-energy scale) for the renormalization group equations depend on the mechanism by which supersymmetry breaking is communicated to the effective low energy
theory. Examples of this scenario are provided by models of
gravity-mediated and gauge-mediated supersymmetry breaking
(see Section I.2). One bonus of such an approach is that one of
the diagonal Higgs squared-mass parameters is typically driven
negative by renormalization group evolution. Thus, electroweak
symmetry breaking is generated radiatively, and the resulting
electroweak symmetry-breaking scale is intimately tied to the
scale of low-energy supersymmetry breaking.

One prediction of the high-energy approach that arises in
most grand unified supergravity models and gauge-mediated
supersymmetry-breaking models is the unification of gaugino
mass parameters at some high-energy scale $M_X$, i.e.,

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}. \quad (6)$$

Consequently, the effective low-energy gaugino mass parameters
(at the electroweak scale) are related:

$$M_3 = (g_s^2/g^2)M_2, \quad M_1 = (5g_1^2/3g^2)M_2 \simeq 0.5M_2. \quad (7)$$

In this case, the chargino and neutralino masses and mixing
angles depend only on three unknown parameters: the gluino
mass, $\mu$, and $\tan \beta$. If in addition $|\mu| \gg M_1, m_Z$, then the
lightest neutralino is nearly a pure bino, an assumption often
made in supersymmetric particle searches at colliders.

Recently, attention has been given to a class of supergravity
models in which Eq. (7) does not hold. In models where no
tree-level gaugino masses are generated, one finds a model-
independent contribution to the gaugino mass whose origin can
be traced to the super-conformal (super-Weyl) anomaly which
is common to all supergravity models [45]. This approach has
been called anomaly-mediated supersymmetry breaking. Eq. (7)
is then replaced (in the one-loop approximation) by:

$$M_i \simeq \frac{b_i g_i^2}{16\pi^2} m_{3/2}, \quad (8)$$

where $m_{3/2}$ is the gravitino mass (assumed to be of order 1 TeV),
and $b_i$ are the coefficients of the MSSM gauge beta-functions cor-
responding to the corresponding U(1), SU(2) and SU(3) gauge
groups: $(b_1, b_2, b_3) = (\frac{33}{2}, 1, -3)$. Eq. (8) yields $M_1 \simeq 2.8M_2$. 
and $M_3 \simeq -8.3 M_2$, which implies that the lightest chargino pair and neutralino make up a nearly-mass degenerate triplet of winos. The corresponding supersymmetric phenomenology differs significantly from the standard phenomenology based on Eq. (7), and is explored in detail in Ref. [46]. Anomaly-mediated supersymmetry breaking also generates (approximate) flavor-diagonal squark and slepton mass matrices. However, in the MSSM this cannot be the sole source of supersymmetry-breaking in the slepton sector (which yields negative squared-mass contributions for the sleptons).

I.7. The constrained MSSMs: mSUGRA, GMSB, and SGUTs: One way to guarantee the absence of significant FCNC’s mediated by virtual supersymmetric-particle exchange is to posit that the diagonal soft-supersymmetry-breaking scalar squared-masses are universal at some energy scale. In models of gauge-mediated supersymmetry breaking, scalar squared-masses are expected to be flavor independent since gauge forces are flavor-blind. In the minimal supergravity (mSUGRA) framework [1–3], the soft-supersymmetry-breaking parameters at the Planck scale take a particularly simple form in which the scalar squared-masses and the $A$-parameters are flavor diagonal and universal [26]:

\[
M_2^Q(\mathcal{M}_P) = M_2^U(\mathcal{M}_P) = M_2^D(\mathcal{M}_P) = m_0^2 \mathbf{1},
\]

\[
M_2^L(\mathcal{M}_P) = M_2^E(\mathcal{M}_P) = m_0^2 \mathbf{1},
\]

\[
m_2^U(\mathcal{M}_P) = m_2^D(\mathcal{M}_P) = m_0^2,
\]

\[
A_U(\mathcal{M}_P) = A_D(\mathcal{M}_P) = A_L(\mathcal{M}_P) = A_0 \mathbf{1}, \tag{9}
\]

where $\mathbf{1}$ is a $3 \times 3$ identity matrix in generation space. Renormalization group evolution is then used to derive the values of the supersymmetric parameters at the low-energy (electroweak) scale. For example, to compute squark and slepton masses, one must use the low-energy values for $M_2^F$ and $M_2^R$ in Eq. (4). Through the renormalization group running with boundary conditions specified in Eq. (7) and Eq. (9), one can show that the low-energy values of $M_2^F$ and $M_2^R$ depend primarily on $m_0^2$ and $m_{1/2}^2$. A number of useful approximate analytic expressions.
for superpartner masses in terms of the mSUGRA parameters can be found in Ref. 47.

Clearly, in the mSUGRA approach, the MSSM-124 parameter freedom has been sharply reduced. For example, typical mSUGRA models give low-energy values for the scalar mass parameters that satisfy $M_L \approx M_E < M_Q \approx M_U \approx M_D$ with the squark mass parameters somewhere between a factor of 1–3 larger than the slepton mass parameters (e.g., see Ref. 47). More precisely, the low-energy values of the squark mass parameters of the first two generations are roughly degenerate, while $M_{Q_3}$ and $M_{U_3}$ are typically reduced by a factor of 1–3 from the values of the first and second generation squark mass parameters because of renormalization effects due to the heavy top quark mass.

As a result, one typically finds that four flavors of squarks (with two squark eigenstates per flavor) and $\tilde{b}_R$ are nearly mass-degenerate. The $\tilde{b}_L$ mass and the diagonal $\tilde{t}_L$ and $\tilde{t}_R$ masses are reduced compared to the common squark mass of the first two generations. (If $\tan \beta \gg 1$, then the pattern of third generation squark masses is somewhat altered; e.g., see Ref. 48.) In addition, there are six flavors of nearly mass-degenerate sleptons (with two slepton eigenstates per flavor for the charged sleptons and one per flavor for the sneutrinos); the sleptons are expected to be somewhat lighter than the mass-degenerate squarks. Finally, third generation squark masses and tau-slepton masses are sensitive to the strength of the respective $\tilde{f}_L$–$\tilde{f}_R$ mixing as discussed below Eq. (3).

Due to the implicit $m_{1/2}$ dependence in the low-energy values of $M_{Q_1}^2, M_U^2$, and $M_D^2$, there is a tendency for the gluino in mSUGRA models to be lighter than the first and second generation squarks. Moreover, the LSP is typically the lightest neutralino, $\chi_1^0$, which is dominated by its bino component. However, there are some regions of mSUGRA parameter space where the above conclusions do not hold. For example, one can reject those mSUGRA parameter regimes in which the LSP is a chargino.

One can count the number of independent parameters in the mSUGRA framework. In addition to 18 Standard Model
parameters (excluding the Higgs mass), one must specify $m_0$, $m_{1/2}$, $A_0$, and Planck-scale values for $\mu$ and $B$-parameters (denoted by $\mu_0$ and $B_0$). In principle, $A_0$, $B_0$ and $\mu_0$ can be complex, although in the mSUGRA approach, these parameters are taken (arbitrarily) to be real. As previously noted, renormalization group evolution is used to compute the low-energy values of the mSUGRA parameters, which then fixes all the parameters of the low-energy MSSM. In particular, the two Higgs vacuum expectation values (or equivalently, $m_Z$ and $\tan \beta$) can be expressed as a function of the Planck-scale supergravity parameters. The simplest procedure is to remove $\mu_0$ and $B_0$ in favor of $m_Z$ and $\tan \beta$ (the sign of $\mu_0$ is not fixed in this process). In this case, the MSSM spectrum and its interaction strengths are determined by five parameters: $m_0$, $A_0$, $m_{1/2}$, $\tan \beta$, and the sign of $\mu_0$, in addition to the 18 parameters of the Standard Model. However, the mSUGRA approach is probably too simplistic. Theoretical considerations suggest that the universality of Planck-scale soft-supersymmetry-breaking parameters is not generic [49].

In the minimal gauge-mediated supersymmetry-breaking (GMSB) approach, there is one effective mass scale, $\Lambda$, that determines all low-energy scalar and gaugino mass parameters through loop-effects (while the resulting $A$-parameters are suppressed). In order that the resulting superpartner masses be of order 1 TeV or less, one must have $\Lambda \sim 100$ TeV. The origin of the $\mu$ and $B$-parameters is quite model dependent and lies somewhat outside the ansatz of gauge-mediated supersymmetry breaking. The simplest models of this type are even more restrictive than mSUGRA, with two fewer degrees of freedom. However, minimal GMSB is not a fully realized model. The sector of supersymmetry-breaking dynamics can be very complex, and no complete model of gauge-mediated supersymmetry yet exists that is both simple and compelling.

It was noted in Section I.2 that the gravitino is the LSP in GMSB models. Thus, in such models, the next-to-lightest supersymmetric particle (NLSP) plays a crucial role in the phenomenology of supersymmetric particle production and decay. Note that unlike the LSP, the NLSP can be charged. In GMSB
models, the most likely candidates for the NLSP are $\tilde{\chi}^0_1$ and $\tilde{\tau}^\pm_R$. The NLSP will decay into its superpartner plus a gravitino (e.g., $\tilde{\chi}^0_1 \rightarrow \gamma \tilde{g}_{3/2}$, $\tilde{\chi}^0_1 \rightarrow Z \tilde{g}_{3/2}$ or $\tilde{\tau}^\pm_R \rightarrow \tau^\pm \tilde{g}_{3/2}$), with lifetimes and branching ratios that depend on the model parameters.

Different choices for the identity of the NLSP and its decay rate lead to a variety of distinctive supersymmetric phenomenologies [30,50]. For example, a long-lived $\tilde{\chi}^0_1$-NLSP that decays outside collider detectors leads to supersymmetric decay chains with missing energy in association with leptons and/or hadronic jets (this case is indistinguishable from the canonical phenomenology of the $\tilde{\chi}^0_1$-LSP). On the other hand, if $\tilde{\chi}^0_1 \rightarrow \gamma \tilde{g}_{3/2}$ is the dominant decay mode, and the decay occurs inside the detector, then nearly all supersymmetric particle decay chains would contain a photon. In contrast, the case of a $\tilde{\tau}^\pm_R$-NLSP would lead either to a new long-lived charged particle (i.e., the $\tilde{\tau}^\pm_R$) or to supersymmetric particle decay chains with $\tau$-leptons.

Finally, grand unification can impose additional constraints on the MSSM parameters. Perhaps one of the most compelling hints for low-energy supersymmetry is the unification of SU(3)$\times$SU(2)$\times$U(1) gauge couplings predicted by models of supersymmetric grand unified theories (SGUTs) [5,51] (with the supersymmetry-breaking scale of order 1 TeV or below). Gauge coupling unification, which takes place at an energy scale of order $10^{16}$ GeV, is quite robust (i.e., the unification depends weakly on the details of the theory at the unification scale). In particular, given the low-energy values of the electroweak couplings $g(m_Z)$ and $g'(m_Z)$, one can predict $\alpha_s(m_Z)$ by using the MSSM renormalization group equations to extrapolate to higher energies and imposing the unification condition on the three gauge couplings at some high-energy scale, $M_X$. This procedure (which fixes $M_X$) can be successful (i.e., three running couplings will meet at a single point) only for a unique value of $\alpha_s(m_Z)$. The extrapolation depends somewhat on the low-energy supersymmetric spectrum (so-called low-energy “threshold effects”) and on the SGUT spectrum (high-energy threshold effects), which can somewhat alter the evolution of couplings. Ref. [52]
summarizes the comparison of present data with the expectations of SGUTs, and shows that the measured value of $\alpha_s(m_Z)$ is in good agreement with the predictions of supersymmetric grand unification for a reasonable choice of supersymmetric threshold corrections.

Additional SGUT predictions arise through the unification of the Higgs-fermion Yukawa couplings ($\lambda_f$). There is some evidence that $\lambda_b = \lambda_{\tau}$ leads to good low-energy phenomenology [53], and an intriguing possibility that $\lambda_b = \lambda_{\tau} = \lambda_t$ may be phenomenologically viable [54,48] in the parameter regime where $\tan \beta \simeq m_t/m_b$. Finally, grand unification imposes constraints on the soft-supersymmetry-breaking parameters. For example, gaugino-mass unification leads to the relations given in Eq. (7). Diagonal squark and slepton soft-supersymmetry-breaking scalar masses may also be unified, which is analogous to the unification of Higgs-fermion Yukawa couplings.

In the absence of a fundamental theory of supersymmetry breaking, further progress will require a detailed knowledge of the supersymmetric-particle spectrum in order to determine the nature of the high-energy parameters. Of course, any of the theoretical assumptions described in this section could be wrong and must eventually be tested experimentally.

**I.8. Beyond the MSSM:** Non-minimal models of low-energy supersymmetry can also be constructed. One approach is to add new structure beyond the Standard Model at the TeV scale or below. The supersymmetric extension of such a theory would be a non-minimal extension of the MSSM. Possible new structures include: (i) the supersymmetric generalization of the see-saw model of neutrino masses [55,56]; (ii) an enlarged electroweak gauge group beyond SU(2)$\times$U(1) [57]; (iii) the addition of new, possibly exotic, matter multiplets [e.g., a vector-like color triplet with electric charge $\frac{1}{3}e$; such states sometimes occur as low-energy remnants in E$_6$ grand unification models]; and/or (iv) the addition of low-energy SU(3)$\times$SU(2)$\times$U(1) singlets [58]. A possible theoretical motivation for such new structure arises from the study of phenomenologically viable string theory ground states [59].
A second approach is to retain the minimal particle content of the MSSM but remove the assumption of \( R \)-parity invariance. The most general \( R \)-parity-violating (RPV) theory involving the MSSM spectrum introduces many new parameters to both the supersymmetry-conserving and the supersymmetry-breaking sectors. Each new interaction term violates either \( B \) or \( L \) conservation. For example, consider new scalar-fermion Yukawa couplings derived from the following interactions:

\[
(\lambda_L)_{pmn} \tilde{L}_p \tilde{L}_m \tilde{E}_n^c + (\lambda_L')_{pmn} \tilde{L}_p \tilde{Q}_m \tilde{D}_n^c + (\lambda_B)_{pmn} \tilde{U}_p \tilde{D}_m^c \tilde{D}_n^c ,
\]

where \( p, m, \) and \( n \) are generation indices, and gauge group indices are suppressed. In the notation above, \( \tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \) and \( \tilde{E}^c \) respectively represent \((u,d)_L, u_L^c, d_L^c, (\nu, e^-)_L, \) and \( e_L^c \) and the corresponding superpartners. The Yukawa interactions are obtained from Eq. (10) by taking all possible combinations involving two fermions and one scalar superpartner. Note that the term in Eq. (10) proportional to \( \lambda_B \) violates \( B \), while the other two terms violate \( L \).

Phenomenological constraints on various low-energy \( B \)- and \( L \)-violating processes yield limits on each of the coefficients \((\lambda_L)_{pmn}, (\lambda_L')_{pmn} \) and \((\lambda_B)_{pmn} \) taken one at a time [60]. If more than one coefficient is simultaneously non-zero, then the limits are in general more complicated. All possible RPV terms cannot be simultaneously present and unsuppressed; otherwise the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose \( B \)- or \( L \)-invariance (either one alone would suffice). Otherwise, one must accept the requirement that certain RPV coefficients must be extremely suppressed.

If \( R \)-parity is not conserved, supersymmetric phenomenology exhibits features that are quite distinct from that of the MSSM. The LSP is no longer stable, which implies that not all supersymmetric decay chains must yield missing-energy events at colliders. Both \( \Delta L = 1 \) and \( \Delta L = 2 \) phenomena are allowed (if \( L \) is violated), leading to neutrino masses and mixing [61], neutrinoless double beta decay [62], sneutrino-antisneutrino mixing [56,63,64], and \( s \)-channel resonant production of the sneutrino in \( e^+e^- \) collisions [65]. Since the distinction between
the Higgs and matter multiplets is lost, \( R \)-parity violation permits the mixing of sleptons and Higgs bosons, the mixing of neutrinos and neutralinos, and the mixing of charged leptons and charginos, leading to more complicated mass matrices and mass eigenstates than in the MSSM. Note that if \( \chi_L \neq 0 \), then squarks can behave as leptoquarks since the following processes are allowed: \( e^+ \tau_m \rightarrow \bar{d}_n \rightarrow e^+ \pi_m, \pi d_m \) and \( e^+ d_m \rightarrow \bar{u}_n \rightarrow e^+ d_m \). (As above, \( m \) and \( n \) are generation labels, so that \( d_2 = s, d_3 = b \), etc.)

The theory and phenomenology of alternative low-energy supersymmetric models and its consequences for collider physics have recently begun to attract significant attention. In particular, experimental and theoretical constraints place some non-trivial restrictions on \( R \)-parity-violating alternatives to the MSSM (see, e.g., Refs. [60,66] for further details).

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