RARE KAON DECAYS

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A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–11]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focussed on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+ \ell^+ \ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0 \gamma\gamma$, which also scales a CP-conserving background to CP violation in $K_L \to \pi^0 \ell^+ \ell^-$ and $K_L \to \gamma \ell^+ \ell^-$, which could possibly shed light on long distance contributions to $K_L \to \mu^+ \mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 4.7 \times 10^{-12} (148\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of over 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to parity-odd couplings and the latter is sensitive to

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e\mu$	2.8×10^{-11}	BNL-865	01/12	9×10^{-12} (BNL-865)
$K_L \to \mu e$	4.7×10^{-12}	BNL-871	98/13	
$K_L \to \pi^0 e\mu$	4.4×10^{-10}*	FNAL-799	01/14	5×10^{-11} (KTeV)

*Conference result.

parity-even couplings. Related searches in μ and τ process are discussed in our section "Tests of Conservation Laws".

Another forbidden decay currently being pursued is $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, *etc.*). The 90% CL upper limit on this process is presently 1.1×10^{-10} [15]. Data already collected by BNL-787 are expected to yield a further factor ~ 2 in sensitivity to this process.

C. Measurements of Standard Model parameters: Until 1997, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ were motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [16] and long-distance contributions were known to be negligible [2]. However, BNL-787 has attained the sensitivity at which the observation of an event can no longer be unambiguously attributed to non-SM physics. In 1997 BNL-787 observed a single candidate event and in 2000 released the results of further running in which no additional events were seen, yielding a branching ratio of $(1.5^{+3.4}_{-1.2}) \times 10^{-10}$ [15]. Further data already collected are expected to increase the sensitivity by approximately a factor 2, and an upgrade to the experiment to collect roughly an order of magnitude more sensitivity is in progress [17]. This reaction is now interesting from the point of view of constraining SM parameters. The branching ratio can be written in terms of the very well-measured rate of K_{e3} as [2]:

$$B(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 B(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [18]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [19], and X_{NL}^{ℓ} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which are contained in X_{NL}^{ℓ}, are relatively small and now known [10] to $\leq 10\%$. Evaluating the constants in Eq. (1) with $m_t = 175$ GeV, one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [10]

$$B(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4[\eta^2 + (\rho_o - \rho)^2] \qquad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3}X_{NL}^e + \frac{1}{3}X_{NL}^{\tau})/(A^2 V_{us}^4 X(m_t)) \approx 1.4$. Thus, $B(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \frac{1}{A^2}\sqrt{\frac{B(K^+ \to \pi^+ \nu\overline{\nu})}{1.0 \times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ. For $m_t = 175$ GeV it is given by [10]:

$$B_{SD}(K_L \to \mu^+\mu^-) \approx 1.7 \times 10^{-9} A^4(\rho_o' - \rho)^2 \qquad (3)$$

where ρ_o' depends on the charm quark mass and is around 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $B_{abs}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $B(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ [20]. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present, it is not possible to compute this long-distance component reliably and, therefore, it is not possible to constrain ρ from this mode in a model independent way [21]. Several models exist to estimate this long-distance component [22,23] that are sufficient to place rough bounds on new physics from the measured rate for $K_L \to \mu^+\mu^-$ [24]. The decay $K_L \to e^+e^-$ is completely dominated by long distance physics and is easier to estimate. The result, $B(K_L \to e^+e^-) \sim 9 \times 10^{-12}$ [21,23], is in good agreement with the recent measurement [25]. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$, and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ, currently under active study by the KTeV and NA48 experiments, will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is parameterized in terms of α^*_K, discussed in the form-factors section of the K^0_L Particle Properties Listing in our 2000 edition [26]).

D. Searches for direct CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,27]. The Standard Model predicts a branching ratio $(3.0 \pm 1.3) \times 10^{-11}$; for $m_t = 175$ GeV it is given approximately by [10]:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4\eta^2 \; . \tag{4}$$

The current upper bound is $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 5.9 \times 10^{-7}$ [28] and KTeV (FNAL799II) is expected to place a bound of order 10^{-8} [29]. The 90% CL bound on $K^+ \to \pi^+\nu\bar{\nu}$ provides a nearly model independent bound $B(K_L \to \pi^0\nu\bar{\nu}) < 3 \times 10^{-9}$ [30]. A KEK experiment to reach the 3×10^{-10}/event level is in preparation [31]. The KOPIO [32] proposal aims to make a $\sim 20\%$ measurement of $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu})$ at the BNL AGS.

There has been much recent theoretical work on possible contributions to ϵ'/ϵ and rare K decays within a generic supersymmetric extension of the Standard Model with R parity conservation and minimal particle content [24,33]. These conclude that contributions to rare decays much larger than those of the Standard Model are possible without violating current phenomenological constraints.

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the product $A^4\eta^2$. It has a direct CP-violating component that for $m_t =$ 175 GeV is given by [10]:

$$\mathrm{B}_{\mathrm{dir}}(K_L \to \pi^0 e^+ e^-) \approx 6.7 \times 10^{-11} A^4 \eta^2 \ . \tag{5}$$

However, like $K_L \to \mu^+\mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects. It has an indirect CP-violating component given by:

$$\mathrm{B}_{\mathrm{ind}}(K_L \to \pi^0 e^+ e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+ e^-) \ , \tag{6}$$

that has been estimated to be less than 10^{-12} [34], but that will not be known precisely until a measurement of $K_S \to \pi^0 e^+ e^-$ is available [4,35]. The 90% CL upper limit, $B(K_S \to \pi^0 e^+ e^-) < 1.4 \times 10^{-7}$, recently obtained by NA48 [36] is about two orders of magnitude short of the expected level. NA48 proposes to reach $\sim 10^{-9}$/event sensitivity for this mode in their upcoming K_S run [37]. There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0 \gamma\gamma$.

To understand the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$ in $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory it is necessary to go beyond leading order. The measured rate and spectrum can be accommodated naturally, for example, by allowing only one of the free parameters that occur, a_V, to vary [38]. The published data on this decay from KTeV [39] and a fit to the distribution has given $a_V = -0.72 \pm 0.05 \pm 0.06$. This value suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ could be comparable to the direct CP-violating component [35,39]. However a new conference result from NA48, $a_V = -0.46 \pm 0.03 \pm 0.03 \pm 0.02$ [40] would suggest that this contribution is smaller. The related process,

$K_L \to \pi^0\gamma e^+e^-$, is potentially an additional background in some region of phase space [41]. This process has recently been observed with a branching ratio of $(2.34 \pm 0.35_{\rm stat} \pm 0.13_{\rm sys}) \times 10^{-8}$ [42] Finally, BNL-845 observed a potential background to $K_L \to \pi^0 e^+e^-$ from the decay $K_L \to \gamma\gamma e^+e^-$ [43]. This has recently been confirmed with a 500-fold larger sample by FNAL-799 [44], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 10^{-11} [45], comparable to the signal level. Because of this, the observation of $K_L \to \pi^0 e^+e^-$ will depend on background subtraction with good statistics.

The current 90% CL preliminary upper bound for the process $K_L \to \pi^0 e^+e^-$ is 5.1×10^{-10} [46]. For the closely related muonic process, the corresponding upper bound is $\mathrm{B}(K_L \to \pi^0\mu^+\mu^-) \leq 3.8 \times 10^{-10}$ [47]. KTeV expects to reach a sensitivity of roughly 10^{-11} for both reactions [29].

E. Other long distance dominated modes:

The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) are described by leading order chiral perturbation theory in terms of one parameter, ω^+ [48]. It now appears that this parameterization is not sufficient to account for both the rate and the detailed shape of the spectrum in $K^+ \to \pi^+e^+e^-$ [49] An analysis beyond leading order in chiral perturbation theory can accommodate both the rate and the spectrum [50], at the cost of introducing at least one new parameter.

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