FITS FOR K_L^0 CP-VIOLATION PARAMETERS

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In recent years, K_L^0 CP-violation experiments have improved our knowledge of CP-violation parameters and their consistency with the expectations of CPT invariance and unitarity. For definitions of K_L^0 CP-violation parameters and a brief discussion of the theory, see the article "CP Violation" by L. Wolfenstein in the *Reviews, Tables, and Plots* section of this *Review*.

This note describes our two types of fits for the CP-violation parameters in $K_L^0 \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ decay, one for the phases ϕ_{+-} and ϕ_{00} jointly with Δm and τ_S, and the other for the amplitudes $|\eta_{+-}|$ and $|\eta_{00}|$ jointly with the $K_L^0 \rightarrow \pi\pi$ branching fractions. In this edition, phase fits are done without and with the assumption of CPT invariance, the latter giving a significant improvement in the precision of Δm and τ_S.

Fit to ϕ_{+-}, ϕ_{00}, $\Delta\phi$, Δm, and τ_S data: This is a joint fit to the data on ϕ_{+-}, ϕ_{00}, the phase difference $\Delta\phi = \phi_{00} - \phi_{+-}$, the $K_L^0 - K_S^0$ mass difference Δm, and the K_S^0 mean life τ_S, including the effects of correlations. Measurements of ϕ_{+-} and ϕ_{00} are highly correlated with Δm and τ_S. Some measurements of τ_S are correlated with Δm. The correlations are given in the footnotes of the ϕ_{+-} and ϕ_{00} sections of the K_L^0 Particle Listings and the τ_S section of the K_S^0 Particle listings. In editions of the Review prior to 1996, we adjusted the experimental values of ϕ_{+-} and ϕ_{00} to account for correlations with Δm and τ_S but did not include the effects of these correlations when evaluating Δm and τ_S. In 1996, we introduced a joint fit including these correlations. In the joint fit, the ϕ_{+-} measurements have a strong influence on the fitted value of Δm. This is because the CERN NA31 vacuum regeneration experiments (CAROSI 90 [1] and GEWENIGER 74B [2]), the Fermilab E773/E731 regenerator experiments (SCHWINGENHEUER 95 [3] and GIBBONS 93 [4]), and the CPLEAR $K^0 - \overline{K}^0$ asymmetry experiment (APOSTOLAKIS 99C [5]) have very different dependences of ϕ_{+-} on Δm, as can be seen from their diagonal bands e, d, b, c, and a, respectively, in Fig. 1.

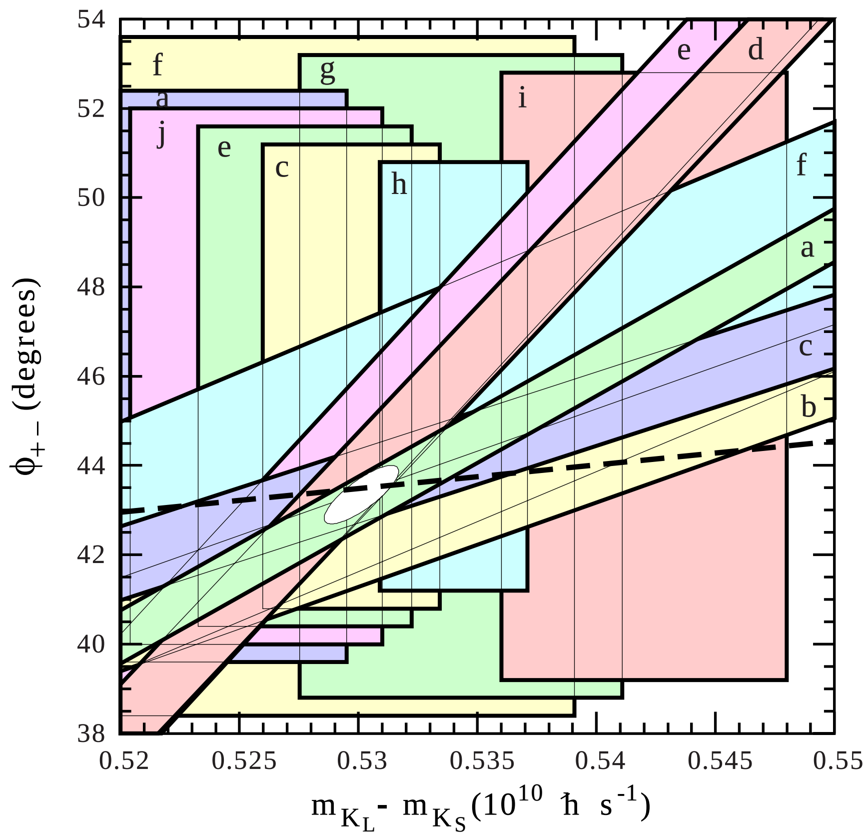

Figure 1: ϕ_{+-} vs Δm for experiments which do not assume CPT invariance. Δm measurements appear as vertical bands spanning $\Delta m \pm 1\sigma$, cut near the top and bottom to aid the eye. The ϕ_{+-} measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the 1σ contour of the fit result. Data are labeled by letters and cited in Table 1.

The region where the ϕ_{+-} bands from these experiments cross gives a powerful measurement of Δm which decreases the fitted Δm value relative to our pre-1996 average Δm and earlier measurements such as CULLEN 70 [6] and GEWENIGER 74C [7], i and h respectively in Fig. 1. This decrease brings the Δm-dependent ϕ_{+-} measurements into good agreement with each other and with ϕ(superweak), where

$$\phi(\text{superweak}) = \tan^{-1}\left(\frac{2\Delta m}{\Delta\Gamma}\right) = \tan^{-1}\left(\frac{2\Delta m \tau_S \tau_L}{\hbar(\tau_L - \tau_S)}\right) \ , \quad (1)$$

which is shown as a dashed line in Fig. 1 and Fig. 2. In this edition, we have taken care in these figures to exclude

Table 1: References for data in the figures and fits. The letters in the first four columns label the bands in Fig. 1 and Fig. 2. Columns 1 and 3 label the diagonal ϕ_{+-} bands while columns 2 and 4 label the vertical Δm and τ_S bands. A check ($\surd$) in a column means that the data are excluded from the figures because they assume CPT. The data are given in the ϕ_{+-} and Δm sections of the K_L Particle Listings, and the τ_S section of the K_S Particle Listings, unless otherwise footnoted.

Location of input data					
Fig. 1		Fig. 2			
ϕ_{+-}	Δm	ϕ_{+-}	τ_S	PDG Document ID	Ref.
a	a	a		APOSTOLAKIS 99C	[5]
b		b	$\surd$	GIBBONS 93	[4]
c	c	c	$\surd$	SCHWINGENHEUER 95	[3]
d		d	d	GEWENIGER 74B	[2]
e	e*	e	e*	CAROSI 90	[1]
f	f†	f	f	CARITHERS 75	[8]
	g			ANGELOPOULOS 01	[9]
	h			GEWENIGER 74C	[7]
	i			CULLEN 70	[6]
	j			GIBBONS 93C	[10]
	$\surd$			ANGELOPOULOS 98D	[11]
	$\surd$			GJESDAL 74	[12]
			k	BERTANZA 97	[13]
			l	GROSSMAN 87	[14]
			m	SKJEGGESTAD 72	[15]

* from $\phi_{00}(\Delta m, \tau_S)$ in ϕ_{00} Particle Listings.

† from $\tau_S(\Delta m)$ in τ_S Particle Listings.

experiments which assume CPT invariance. This was not the case in the 2000 edition of the *Review* and earlier editions.

Table 2 column 2, "Fit w/o CPT," gives the resulting fitted parameters, while Table 3 gives the correlation matrix for this fit. The $\chi^2 = 1$ contour for the fit result is shown as a white ellipse in Fig. 1 and Fig. 2. The fit is seen to be consistent with the ϕ(superweak) dashed line.

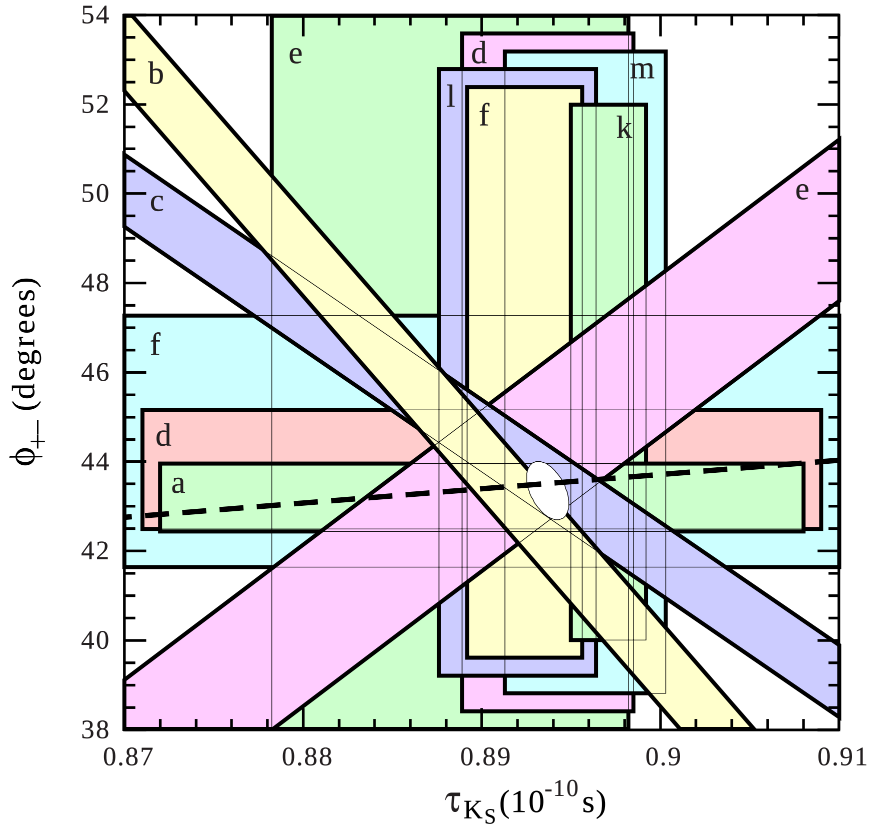

Figure 2: ϕ_{+-} vs τ_S. τ_S measurements appear as vertical bands spanning $\tau_S \pm 1\sigma$, some of which are cut near the top and bottom to aid the eye. The ϕ_{+-} measurements appear as diagonal or horizontal bands spanning $\phi_{+-} \pm \sigma_\phi$. The dashed line shows ϕ(superweak). The ellipse shows the fit result's 1σ contour. Data are labeled by letters and cited in Table 1.

For experiments which have dependencies on unseen fit parameters, that is, parameters other than those shown on the x or y axis of the figure, their band positions are evaluated using the fit results and their band widths include the fitted uncertainty in the unseen parameters.

If CPT invariance is assumed, four experimental results, those indicated by a check (√) in column 2 or 4 of Table 1, are added to the fit. In addition, we require that $\phi_{+-} = \phi_{00} = \phi$(superweak). The result is shown in Table 2 column 3, "Fit w/ CPT," and the correlation matrix is shown in Table 4. The resulting ϕ_{+-} and ϕ_{00} are just ϕ(superweak) from Eq. (1),

Table 2: Fit results for ϕ_{+-}, ϕ_{00}, $\phi_{00} - \phi_{+-}$, Δm, and τ_S without and with CPT assumption.

Quantity(units)	Fit w/o CPT	Fit w/ CPT
$\phi_{+-}(^\circ)$	43.4 ± 0.7	43.51 ± 0.06
$\Delta m(10^{10}\hbar\ \mathrm{s}^{-1})$	0.5301 ± 0.0016	0.5303 ± 0.0009
$\tau_S(10^{-10}\mathrm{s})$	0.8937 ± 0.0012	0.8935 ± 0.0008
$\phi_{00}(^\circ)$	43.2 ± 1.0	43.51 ± 0.06
$\Delta\phi(^\circ)$	-0.1 ± 0.8	−−−−−−−
χ^2	13.6	14.5
No. Deg. Freedom	16	21

Table 3: Correlation matrix for the results of the fit without the CPT assumption

	ϕ_{+-}	Δm	τ_S	ϕ_{00}	$\Delta\phi$
ϕ_{+-}	1.00	0.80	−0.45	0.62	−0.02
Δm	0.80	1.00	−0.33	0.54	0.04
τ_S	−0.45	−0.33	1.00	−0.25	0.05
ϕ_{00}	0.62	0.54	−0.25	1.00	0.78
$\Delta\phi$	−0.02	0.04	0.05	0.78	1.00

Table 4: Correlation matrix for the results of the fit with the CPT assumption

	ϕ_{+-}	Δm	τ_S	ϕ_{00}
ϕ_{+-}	1.00	0.90	0.49	1.00
Δm	0.90	1.00	0.06	0.90
τ_S	0.49	0.06	1.00	0.49
ϕ_{00}	1.00	0.90	0.49	1.00

evaluated for Δm and τ_S from this fit. The Δm and τ_S precision are improved significantly by the CPT assumption.

Fit for ϵ'/ϵ, $|\eta_{+-}|$, $|\eta_{00}|$, and B$(K_L \to \pi\pi)$

We list measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$ and ϵ'/ϵ. Independent information on $|\eta_{+-}|$ and $|\eta_{00}|$ can be obtained from measurements of the K_L^0 and K_S^0 lifetimes (τ_L, τ_S) and branching ratios (B) to $\pi\pi$, using the relations

$$|\eta_{+-}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau_L} \; \frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2} \; , \qquad (2a)$$

$$|\eta_{00}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\tau_L} \; \frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}\right]^{1/2} \; . \qquad (2b)$$

For historical reasons the branching ratio fits and the CP-violation fits are done separately, but we want to include the influence of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ measurements on $\mathrm{B}(K_L^0 \to \pi^+\pi^-)$ and $\mathrm{B}(K_L^0 \to \pi^0\pi^0)$ and vice versa. We approximate a global fit to all of these measurements by first performing two independent fits: 1) BRFIT, a fit to the K_L^0 branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{+-}/\eta_{00}|$, and ϵ'/ϵ measurements. The results from fit 1, along with the K_S^0 values from this edition are used to compute values of $|\eta_{+-}|$ and $|\eta_{00}|$ which are included as measurements in the $|\eta_{00}|$ and $|\eta_{+-}|$ sections with a document ID of BRFIT 02. Thus the fit values of $|\eta_{+-}|$ and $|\eta_{00}|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\eta|$ measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 02 values) are used along with the K_L^0 and K_S^0 mean lives and the $K_S^0 \to \pi\pi$ branching fractions to compute the K_L^0 branching ratios $\Gamma(K_L^0 \to \pi^+\pi^-)/\Gamma(\text{total})$ and $\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)$. These branching ratio values are included as measurements in the branching ratio section with a document ID of ETAFIT 02. Thus the K_L^0 branching ratio fit values in this edition include the results of direct measurements of $|\eta_{+-}|$, $|\eta_{00}|$, $|\eta_{00}/\eta_{+-}|$, and ϵ'/ϵ. A more detailed discussion of these fits is given in the 1990 edition of this *Review* [16].

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