QUARK MASSES

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A. Introduction:

This note discusses some of the theoretical issues relevant to the determination of quark masses, which are fundamental parameters of the Standard Model of particle physics. Unlike the leptons, quarks are confined inside hadrons and are not observed as physical particles. Quark masses, therefore, cannot be measured directly, but must be determined indirectly through their influence on hadronic properties. Although one often speaks loosely of quark masses as one would of the mass of the electron or muon, any quantitative statement about the value of a quark mass must make careful reference to the particular theoretical framework that is used to define it. It is important to keep this *scheme dependence* in mind when using the quark mass values tabulated in the data Listings.

Historically, the first determinations of quark masses were performed using quark models. The resulting masses only make sense in the limited context of a particular quark model, and cannot be related to the quark mass parameters of the Standard Model. In order to discuss quark masses at a fundamental level, definitions based on quantum field theory must be used, and the purpose of this note is to discuss these definitions and the corresponding determinations of the values of the masses.

B. Mass parameters and the QCD Lagrangian:

The QCD [1] Lagrangian for N_F quark flavors is

$$\mathcal{L} = \sum_{k=1}^{N_F} \overline{q}_k \left(i \not{D} - m_k \right) q_k - \tfrac{1}{4} G_{\mu\nu} G^{\mu\nu} \ , \tag{1}$$

where $\not{D} = \left(\partial_\mu - i g A_\mu \right) \gamma^\mu$ is the gauge covariant derivative, A_μ is the gluon field, $G_{\mu\nu}$ is the gluon field strength, m_k is the mass parameter of the $k^{\rm th}$ quark, and q_k is the quark Dirac field. After renormalization, the QCD Lagrangian Eq. (1) gives finite values for physical quantities, such as scattering amplitudes. Renormalization is a procedure that invokes a subtraction scheme to render the amplitudes finite, and requires the introduction of a dimensionful scale parameter μ. The mass parameters in the QCD Lagrangian Eq. (1) depend on the renormalization scheme used to define the theory, and also on the scale parameter μ. The most commonly used renormalization scheme for QCD perturbation theory is the $\overline{\mathrm{MS}}$ scheme.

The QCD Lagrangian has a chiral symmetry in the limit that the quark masses vanish. This symmetry is spontaneously broken by dynamical chiral symmetry breaking, and explicitly broken by the quark masses. The nonperturbative scale of dynamical chiral symmetry breaking, Λ_χ, is around 1 GeV [2]. It is conventional to call quarks heavy if $m > \Lambda_\chi$, so that explicit chiral symmetry breaking dominates (c, b, and t quarks are heavy), and light if $m < \Lambda_\chi$, so that spontaneous chiral symmetry breaking dominates (u, d, and s quarks are light). The determination of light- and heavy-quark masses is considered separately in sections ***D*** and ***E*** below.

At high energies or short distances, nonperturbative effects, such as chiral symmetry breaking, become small, and one can, in principle, determine quark masses by analyzing mass-dependent effects using QCD perturbation theory. Such computations are conventionally performed using the $\overline{\mathrm{MS}}$ scheme at a scale $\mu \gg \Lambda_\chi$, and give the $\overline{\mathrm{MS}}$ "running" mass $\overline{m}(\mu)$. We use the $\overline{\mathrm{MS}}$ scheme when reporting quark masses; one can readily convert these values into other schemes using perturbation theory.

The μ dependence of $\overline{m}(\mu)$ at short distances can be calculated using the renormalization group equation,

$$\mu^2 \frac{\mathrm{d}\overline{m}\,(\mu)}{\mathrm{d}\mu^2} = -\gamma(\overline{\alpha}_s\,(\mu))\ \overline{m}\,(\mu)\,, \tag{2}$$

where γ is the anomalous dimension which is now known to four-loop order in perturbation theory [3,4]. $\overline{\alpha}_s$ is the coupling constant in the $\overline{\mathrm{MS}}$ scheme. Defining the expansion coefficients γ_r by

$$\gamma\,(\overline{\alpha}_s) \equiv \sum_{r=1}^{\infty} \gamma_r \left(\frac{\overline{\alpha}_s}{4\pi}\right)^r ,$$

the first four coefficients are given by

$$\gamma_1 = 4,$$

$$\gamma_2 = \frac{202}{3} - \frac{20N_L}{9},$$

$$\gamma_3 = 1249 + \left(-\frac{2216}{27} - \frac{160}{3}\zeta(3)\right) N_L - \frac{140}{81} N_L^2,$$

$$\begin{aligned}\gamma_4 = {} & \frac{4603055}{162} + \frac{135680}{27}\zeta(3) - 8800\zeta(5) \\ & + \left(-\frac{91723}{27} - \frac{34192}{9}\zeta(3) + 880\zeta(4) + \frac{18400}{9}\zeta(5)\right) N_L \\ & + \left(\frac{5242}{243} + \frac{800}{9}\zeta(3) - \frac{160}{3}\zeta(4)\right) N_L^2 \\ & + \left(-\frac{332}{243} + \frac{64}{27}\zeta(3)\right) N_L^3,\end{aligned}$$

where N_L is the number of active light quark flavors at the scale μ, *i.e.*, flavors with masses $< \mu$, and ζ is the Riemann zeta function ($\zeta(3) \simeq 1.2020569$, $\zeta(4) \simeq 1.0823232$, and $\zeta(5) \simeq 1.0369278$).

C. Lattice Gauge Theory:

The use of the lattice simulations for *ab initio* determinations of the fundamental parameters of QCD, including the coupling constant and quark masses (except for the top-quark mass), is a very active area of research, with the current emphasis being on the reduction and control of the systematic uncertainties. We now briefly review some of the features of lattice QCD. In this approach, space-time is approximated by a finite, discrete *lattice* of points, and multi-local correlation functions are computed by the numerical evaluation of the corresponding functional integrals. To determine quark masses, one computes a convenient and appropriate set of physical quantities (frequently chosen to be a set of hadronic masses) using lattice QCD for a variety of input values of the quark masses. The true (physical) values of the quark masses are those which correctly reproduce the set of physical quantities being used for calibration.

The values of the quark masses obtained directly in lattice simulations are bare quark masses, with the lattice spacing a as the ultraviolet cut-off. In order for the lattice results to be useful in phenomenology, it is, therefore, necessary to relate the bare quark masses in a lattice formulation of QCD to renormalized masses in some standard renormalization scheme such as $\overline{\rm MS}$. Provided that both the ultraviolet cut-off a^{-1} and the renormalization scale are much greater than $\Lambda_{\rm QCD}$, the bare and renormalized masses can be related in perturbation theory (this is frequently facilitated by the use of chiral Ward identities). However, the coefficients in lattice perturbation theory are often found to be large, and our ignorance of higher-order terms is generally a significant source of systematic uncertainty (although techniques exist which help to resum some of the large higher-order effects). Increasingly, non-perturbative renormalization is used to calculate the relation between the bare and renormalized masses, circumventing the need for lattice perturbation theory.

The precision with which quark masses can be determined in lattice simulations is limited by the available computing resources. There are a number of sources of systematic uncertainty, and there has been considerable progress in recent years in reducing a number of these. Currently, the difficulty of performing a standard error analysis for lattice simulations is due predominantly to two sources of systematic uncertainty:

Quenching: Until recently most of the simulations have been performed in the "quenched" approximation, in which quark vacuum polarization effects are neglected. It is not possible, in general, to quantify the effects of quenching, although there is a folklore that they are of the order of $10-15\%$. Such an estimate is based on a comparison of results from quenched simulations, with experimental measurements for those quantities where this is possible, and with some (partially) unquenched calculations.

Extrapolation towards the Chiral Limit: Increasingly unquenched simulations are being performed, most often with two flavors of sea quarks. The difficulty, however, is that the masses of the u and d quarks (both valence and sea) used in these simulations are much larger than their physical values. The lattice results have, therefore, to be extrapolated as functions of m_u and m_d. Ideally such an extrapolation would be guided by the predictions of chiral perturbation theory, and there are some indications that this may be possible before too long. In general, however, it is likely that the values of m_u and m_d currently used in simulations are too large for the predictions of chiral perturbation theory to be useful. The results quoted below were obtained assuming there will be no major surprises when m_u and m_d are reduced.

In addition, one has to consider the uncertainties due to the fact that the lattice spacing is non-zero (lattice artifacts), and that the volume is not infinite. The former are studied by observing the stability of the results as a is varied, or by using "improved" formulations of lattice QCD. By varying the volume of the lattice one checks that finite-volume effects are indeed small.

D. Light quarks:

For light quarks, one can use the techniques of chiral perturbation theory to extract quark mass ratios. The mass term for light quarks is

$$\overline{\Psi} M \Psi = \overline{\Psi}_L M \Psi_R + \overline{\Psi}_R M \Psi_L , \tag{3}$$

where M is the light quark mass matrix M,

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} , \tag{4}$$

and $\Psi = (u, d, s)$. The mass term $\overline{\Psi} M \Psi$ is the only term in the QCD Lagrangian that mixes left- and right-handed quarks. In the limit $M \to 0$, there is an independent SU(3) × U(1) flavor symmetry for the left- and right-handed quarks. The vector U(1) symmetry is baryon number; the axial U(1) symmetry of the classical theory is broken in the quantum theory, due to the anomaly. The remaining $G_\chi = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral symmetry of the QCD Lagrangian is spontaneously broken to $\mathrm{SU}(3)_V$, which, in the limit $M \to 0$, leads to eight massless Goldstone bosons, the π's, K's, and η.

The symmetry G_χ is only an approximate symmetry, since it is explicitly broken by the quark mass matrix M. The Goldstone bosons acquire masses which can be computed in a systematic expansion in M, in terms of certain unknown nonperturbative parameters of the theory. For example, to first order in M, one finds that [5]

$$\begin{aligned} m^2_{\pi^0} =& B\left(m_u+m_d\right) \ , \\ m^2_{\pi^\pm} =& B\left(m_u+m_d\right)+\Delta_{\rm em} \ , \\ m^2_{K^0}=m^2_{\overline{K}^0} =& B\left(m_d+m_s\right) \ , \\ m^2_{K^\pm} =& B\left(m_u+m_s\right)+\Delta_{\rm em} \ , \\ m^2_\eta =& \frac{1}{3}B\left(m_u+m_d+4m_s\right) \ , \end{aligned} \tag{5}$$

with two unknown parameters B and $\Delta_{\rm em}$, the electromagnetic mass difference. From Eq. (5), one can determine the quark mass ratios [5]

$$\begin{aligned} \frac{m_u}{m_d} =& \frac{2m^2_{\pi^0}-m^2_{\pi^+}+m^2_{K^+}-m^2_{K^0}}{m^2_{K^0}-m^2_{K^+}+m^2_{\pi^+}}=0.56 \ , \\ \frac{m_s}{m_d} =& \frac{m^2_{K^0}+m^2_{K^+}-m^2_{\pi^+}}{m^2_{K^0}+m^2_{\pi^+}-m^2_{K^+}}=20.1 \ , \end{aligned} \tag{6}$$

to lowest order in chiral perturbation theory, with an error which will be estimated below. Since the mass ratios extracted using chiral perturbation theory use the symmetry transformation property of M under the chiral symmetry G_χ, it is important to use a renormalization scheme for QCD that does not change this transformation law. Any mass-independent subtraction scheme, such as $\overline{\rm MS}$, is suitable. The ratios of quark masses are scale-independent in such a scheme, and Eq. (6) can be taken to be the ratio of $\overline{\rm MS}$ masses. Chiral perturbation theory cannot determine the overall scale of the quark masses, since it uses only the symmetry properties of M, and any multiple of M has the same G_χ transformation law as M.

The second-order quark-mass term [9]

$$\left(M^\dagger\right)^{-1}\det M^\dagger \tag{7}$$

(which can be generated by instantons) transforms in the same way under G_χ as M. Chiral perturbation theory cannot distinguish between M and $\left(M^\dagger\right)^{-1} \det M^\dagger$; one can make the replacement $M \to M(\lambda) = M + \lambda M \left(M^\dagger M\right)^{-1} \det M^\dagger$ in the chiral Lagrangian,

$$\begin{aligned} M(\lambda) &= \mathrm{diag}\left(m_u(\lambda)\,,\ \ m_d(\lambda)\,,\ \ m_s(\lambda)\right) \\ &= \mathrm{diag}\left(m_u + \lambda m_d m_s\,,\ \ m_d + \lambda m_u m_s\,,\ \ m_s + \lambda m_u m_d\right), \end{aligned} \tag{8}$$

and leave all observables unchanged.

The combination

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1 \tag{9}$$

where

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}, \qquad \hat{m} = \frac{1}{2}\left(m_u + m_d\right),$$

is insensitive to the transformation in Eq. (8). Eq. (9) gives an ellipse in the $m_u/m_d - m_s/m_d$ plane. The ellipse is well-determined by chiral perturbation theory, but the exact location on the ellipse, and the absolute normalization of the quark masses, has larger uncertainties. Q is determined to be in the range 21–25 from $\eta \to 3\pi$ decay and the electromagnetic contribution to the K^+–K^0 and π^+–π^0 mass differences [10].

Chiral perturbation theory is a systematic expansion in powers of the light quark masses. The typical expansion parameter is $m_K^2/\Lambda_\chi^2 \sim 0.25$ if one uses SU(3) chiral symmetry, and $m_\pi^2/\Lambda_\chi^2 \sim 0.02$ if one uses SU(2) chiral symmetry. Electromagnetic effects at the few percent level also break SU(2) and SU(3) symmetry. The mass formulæ Eq. (5) were derived using SU(3) chiral symmetry, and are expected to have a 25% uncertainty due to second-order corrections.

It is particularly important to determine the quark mass ratio m_u/m_d, since there is no strong CP problem if $m_u = 0$. The chiral symmetry G_χ of the QCD Lagrangian is not enhanced even if $m_u = 0$. [The possible additional axial u-quark number symmetry is anomalous. The only additional symmetry when $m_u = 0$ is CP.] As a result, $m_u = 0$ is not a special value for chiral perturbation theory. One can try and extend the chiral perturbation expansion Eq. (5) to second order in the quark masses M, to get a more accurate determination of the quark mass ratios. However, as we have seen, due to the ambiguity Eq. (8) at second order, one cannot accurately determine m_u/m_d, only the combination Eq. (9).

The absolute normalization of the quark masses can be determined by using methods that go beyond chiral perturbation theory, such as spectral function sum rules for hadronic correlation functions or lattice simulations. In the former approach, one computes a hadron spectral function using QCD perturbation theory, and compares the result with the experimental data. The comparison must necessarily take place at large q^2, where QCD perturbation theory is valid. Quark mass effects are of order m/q, so that the spectral functions are not very sensitive to m at large q^2. The extraction of the absolute value of quark masses is very sensitive to theoretical and experimental uncertainties. The strange quark mass has been extracted from hadronic tau decays using this procedure, since the relevant scale m_τ is large enough for perturbation theory to be valid [11].

Lattice simulations allow for detailed studies of the behavior of hadronic masses and matrix elements as functions of the quark masses. Moreover, the quark masses do not have to take their physical values, but can be varied freely, and chiral perturbation theory applies also for unphysical masses, provided that they are sufficiently light. From such recent studies of pseudoscalar masses and decay constants, the relevant higher-order couplings in the chiral Lagrangian have been estimated, strongly suggesting that $m_u \neq 0$ [6–8]. In order to make this evidence conclusive, the lattice systematic errors must be reduced; in particular, the range of light quark masses should be increased, and the validity of chiral perturbation theory for this range established.

There have been numerous quenched-lattice determinations of the light quark masses, using a variety of formulations of lattice QCD (see, for example, the recent set of results in Refs. [12–22]). Given the different systematic errors in these determinations (*e.g.*, the different lattice formulations of QCD, the use of perturbative and non-perturbative renormalization), the level of agreement is satisfying. There have also been a number of unquenched studies with two flavors of sea quarks, Refs. [16,23,24,25] and results from the APE and MILC Collaborations cited in the review article Ref. 26.

In current lattice simulations, it is the combination $(m_u + m_d)/2$ which can be determined. In the evaluation of m_s, one gets a result which is about 20–25% larger if the ϕ meson is used as input rather than the K meson. This is evidence that the errors due to quenching are significant. It is reassuring that this difference is eliminated or reduced significantly in the cited unquenched studies.

The quark masses for light quarks discussed so far are often referred to as current quark masses. Nonrelativistic quark models use constituent quark masses, which are of order 350 MeV for the u and d quarks. Constituent quark masses model the effects of dynamical chiral symmetry breaking, and are not related to the quark mass parameters m_k of the QCD Lagrangian Eq. (1). Constituent masses are only defined in the context of a particular hadronic model.

E. Heavy quarks:

The masses and decay rates of hadrons containing a single heavy quark, such as the B and D mesons, can be determined using the heavy quark effective theory (HQET) [35]. The theoretical calculations involve radiative corrections computed in perturbation theory with an expansion in $\alpha_s(m_Q)$, and non-perturbative corrections with an expansion in powers of $\Lambda_{\rm QCD}/m_Q$. Due to the asymptotic nature of the QCD perturbation series, the two kinds of corrections are intimately related; renormalon effects in the perturbative expansion are an example of this, which are associated with non-perturbative corrections.

Systems containing two heavy quarks, such as the Υ or J/ψ, are treated using NRQCD [36]. The typical momentum and energy transfers in these systems are $\alpha_s m_Q$, and $\alpha_s^2 m_Q$, respectively, so these bound states are sensitive to scales much smaller than m_Q. However, smeared observables, such as the cross-section for $e^+e^- \to \overline{b}b$, averaged over some range of s that includes several bound state energy levels, are better behaved and only sensitive to scales near m_Q. For this reason, most determinations of the b quark mass using perturbative calculations compare smeared observables with experiment [37,38,39].

Lattice simulations of heavy-quark systems have been performed using effective theories, including HQET and NRQCD, as well as directly in QCD. The systematic uncertainties in the two cases are different, so both approaches contribute to the final results. Simulating the effective theory requires lattice spacings to be fine enough to resolve the size of the hadron, whereas simulating QCD requires much finer lattice spacings, of order the inverse quark mass. For this reason, and because available computing resources limit the lattice spacings which can be used ($a^{-1} \simeq 2-3\,\mathrm{GeV}$), simulations for the b quark using the QCD action are currently done at quark mass values near the c quark, and then extrapolated to the physical b-quark mass. On the other hand, in effective theories, when evaluating non-leading terms in $1/m_b$, one encounters power divergences in $1/a$ which have to be subtracted.

For an observable particle such as the electron, the position of the pole in the propagator is the definition of the particle mass. In QCD, this definition of the quark mass is known as the pole mass. It is known that the on-shell quark propagator has no infrared divergences in perturbation theory [27,28], so this provides a perturbative definition of the quark mass. The pole mass cannot be used to arbitrarily high accuracy because of nonperturbative infrared effects in QCD. The full quark propagator has no pole because the quarks are confined, so that the pole mass cannot be defined outside of perturbation theory. The relation between the pole mass m_Q and the $\overline{\mathrm{MS}}$ mass $\overline{m}_Q$ is known to three loops [29,30,31]

$$
\begin{aligned}
m_Q = \overline{m}_Q(\overline{m}_Q)\Bigg\{1 &+ \frac{4\overline{\alpha}_s(\overline{m}_Q)}{3\pi} \\
&+ \left[-1.0414\sum_k\left(1-\frac{4}{3}\frac{\overline{m}_{Q_k}}{\overline{m}_Q}\right)+13.4434\right]\left[\frac{\overline{\alpha}_s(\overline{m}_Q)}{\pi}\right]^2 \\
&+ \left[0.6527N_L^2 - 26.655N_L + 190.595\right]\left[\frac{\overline{\alpha}_s(\overline{m}_Q)}{\pi}\right]^3\Bigg\}, \quad (10)
\end{aligned}
$$

where $\overline{\alpha}_s(\mu)$ is the strong interaction coupling constants in the $\overline{\rm MS}$ scheme, and the sum over k extends over the N_L flavors Q_k lighter than Q. The complete mass dependence of the α_s^2 term can be found in Ref. 29; the mass dependence of the α_s^3 term is not known. For the b quark, Eq. (10) reads

$$m_b = \overline{m}_b\,(\overline{m}_b)\,[1 + 0.09 + 0.05 + 0.03]\,, \qquad (11)$$

where the contributions from the different orders in α_s are shown explicitly. The two- and three-loop corrections are comparable in size, and have the same sign as the one-loop term. This is a signal of the asymptotic nature of the perturbation series [there is a renormalon in the pole mass]. Such a badly behaved perturbation expansion can be avoided by directly extracting the $\overline{\rm MS}$ mass from data without extracting the pole mass as an intermediate step.

F. Numerical values and caveats:

The quark masses in the Particle Data Group's Listings have been obtained by using a wide variety of methods. Each method involves its own set of approximations and errors. In most cases, the errors are a best guess at the size of neglected higher-order corrections or other uncertainties. The expansion parameters for some of the approximations are not very small (for example, they are $m_K^2/\Lambda_\chi^2 \sim 0.25$ for the chiral expansion, and $\Lambda_{\rm QCD}/m_b \sim 0.1$ for the heavy-quark expansion), so an unexpectedly large coefficient in a neglected higher-order term could significantly alter the results. It is also important to note that the quark mass values can be significantly different in the different schemes.

The heavy quark masses obtained using HQET, QCD sum rules, or lattice gauge theory are consistent with each other if they are all converted into the same scheme. When using the data listings, it is important to remember that the numerical value for a quark mass is meaningless without specifying the particular scheme in which it was obtained.

We have specified all masses in the $\overline{\rm MS}$ scheme. For light quarks, the renormalization scale has been chosen to be $\mu = 2\,{\rm GeV}$, and for heavy quarks, the quark mass itself (*i.e.*, we quote $\overline{m}(\mu = \overline{m})$). If necessary, we have converted the values in the original papers using the two-loop formulæ. The light quark masses at 1 GeV are significantly different from those at 2 GeV, $\overline{m}(1\,\text{GeV})/\overline{m}(2\,\text{GeV}) = 1.35$.

From the spread of results, and taking into account the treatment of systematic errors in each of the lattice simulations, we quote as the current best results for the quark masses renormalized in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV:

$$\frac{1}{2}\left(\overline{m}_u + \overline{m}_d\right)\Big|_{\mu=2\ \text{GeV}} = (4.2 \pm 1.0)\,\text{MeV} \quad \text{[Lattice only]},$$

and

$$\overline{m}_s\Big|_{\mu=2\ \text{GeV}} = (105 \pm 25)\,\text{MeV} \quad \text{[Lattice only]}.$$

It should be noted that recent results from simulations with two flavors of sea quarks suggest that the light-quark masses may be in the lower parts of the ranges quoted above (for example Refs. [16,25] find that $m_s \sim 90\,\text{MeV}$, with an error of about 7 MeV, and $(m_u + m_d)/2 \sim 3.5\,\text{MeV}$, with an error of perhaps 0.3 MeV). As such studies become more widespread, and use a variety of approaches to study and reduce systematic uncertainties, we can confidently expect that the errors quoted above for the best results will decrease significantly.

Continuum determinations of the absolute values of light quark masses have significant systematic uncertainties. The values are consistent with the lattice extractions above. The u- and d-quark masses are in the range

$$1.5\,\text{MeV} \le \overline{m}_u\Big|_{\mu=2\ \text{GeV}} \le 5\,\text{MeV} \quad \text{[Excluding lattice]},$$

$$5\,\text{MeV} \le m_d\Big|_{\mu=2\ \text{GeV}} \le 9\,\text{MeV} \quad \text{[Excluding lattice]}.$$

The s-quark mass in more recent determinations tends to be smaller than in older extractions. The newer calculations use both better experimental data and perturbative calculations, which tend to reduce m_s. The continuum extractions give

$$80\,\text{MeV} \le \overline{m}_s\Big|_{\mu=2\ \text{GeV}} \le 155\,\text{MeV} \quad \text{[Excluding lattice]}.$$

Using the continuum determinations of the c-quark mass, we quote

$$1\,\mathrm{GeV} \leq \overline{m}_c\,(\overline{m}_c) \leq 1.4\,\mathrm{GeV} \quad \text{[Excluding lattice]}$$

as a best value. Recent determinations include at least two-loop corrections, and give values consistent with this range. The value $\overline{m}_c\,(\overline{m}_c)$ is sensitive to higher-order perturbative corrections, since α_s starts to get large below the charm quark scale.

There are rather few lattice determinations of m_c, as the charm quark is too light for comfortable use of HQET, and yet heavy enough that one must be careful about lattice artifacts. All the results are from quenched simulations, and most are still preliminary. For the best result, we take

$$\overline{m}_c\,(\overline{m}_c) = (1.26 \pm 0.13 \pm 0.20)\,\mathrm{GeV} \quad \text{[Lattice only]},$$

which is consistent with continuum extractions. The second error of 15% is our estimate of possible quenching effects.

There has been much recent work on the b-quark mass. As a best value from continuum extractions, we quote

$$4\,\mathrm{GeV} \leq \overline{m}_b\,(\overline{m}_b) \leq 4.5\,\mathrm{GeV} \quad \text{[Excluding lattice]},$$

which is consistent with continuum extractions. The dominant uncertainties in the b-quark mass are the non-perturbative corrections in the B and Υ systems.

As the current best lattice result for $\overline{m}_b$ we take:

$$\overline{m}_b\,(\overline{m}_b) = (4.26 \pm 0.15 \pm 0.15)\,\mathrm{GeV} \quad \text{[Lattice only]}.$$

The second error is our estimate of possible quenching effects (15% on $M_B - \overline{m}_b$).

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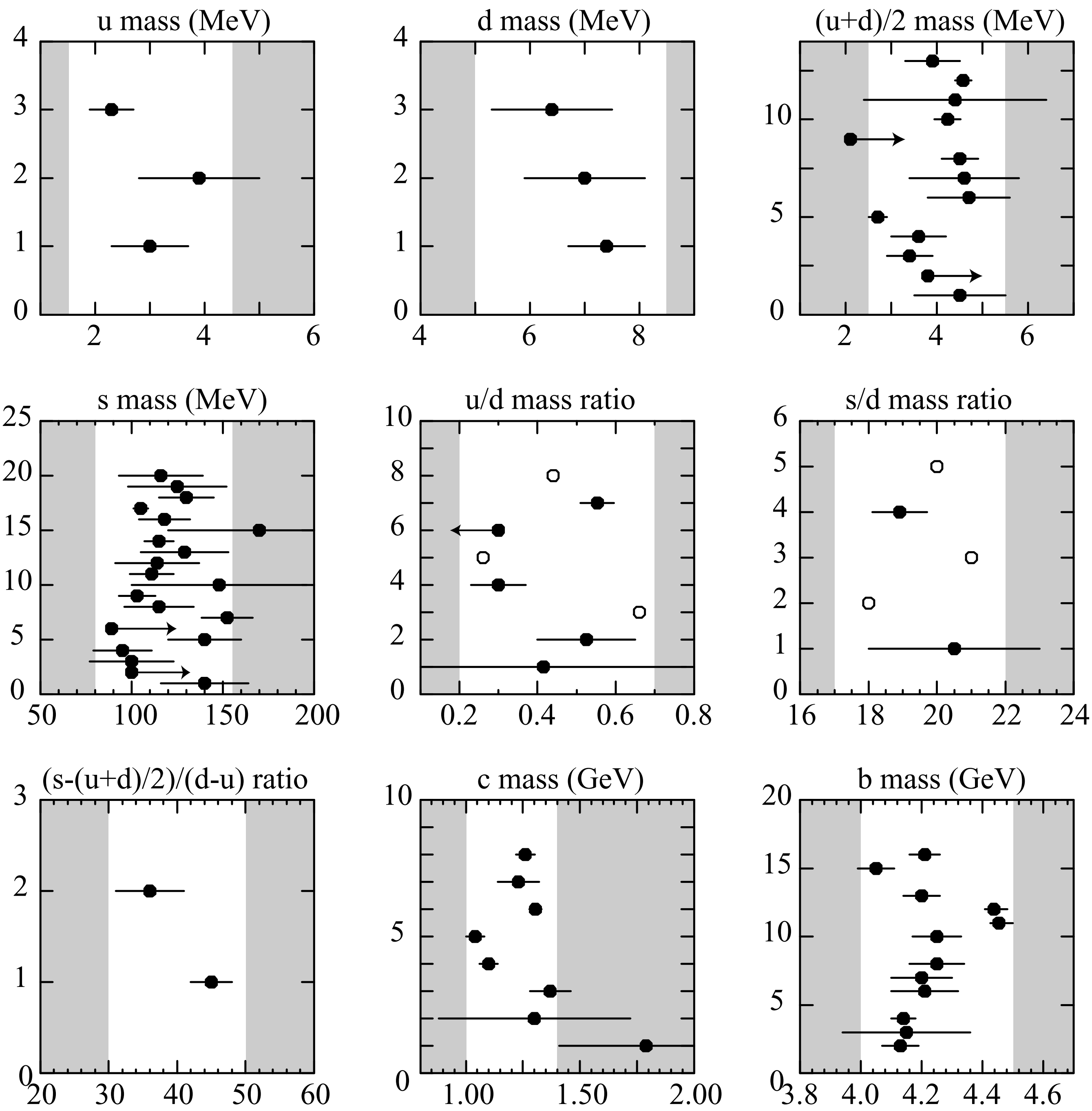

Figure 1: The values of each quark mass parameter taken from the Data Listings. Points from papers reporting no error bars are open circles. Arrows indicate limits reported. The grey regions indicate values excluded by our evaluations; some regions were determined in part though examination of Fig. 2.

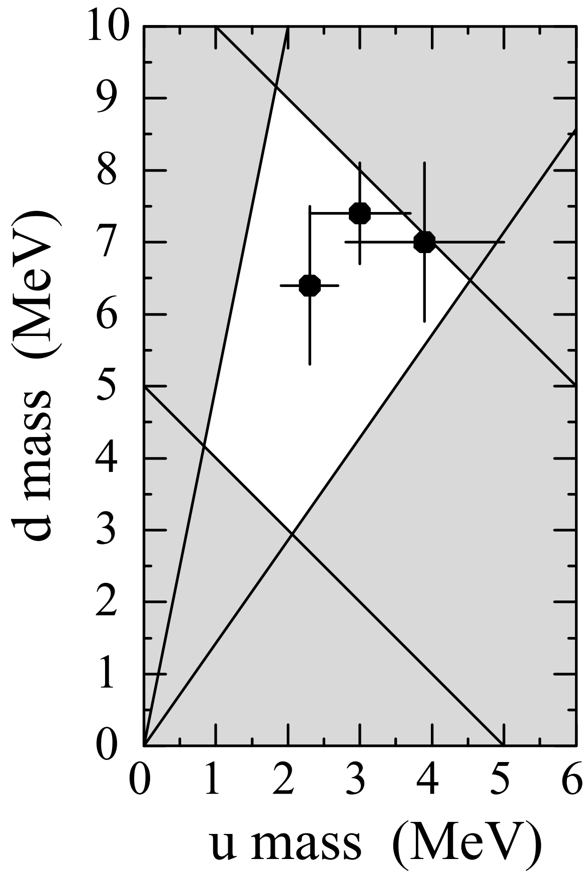

Figure 2: The allowed region (shown in white) for up quark and down quark masses. This region was determined in part from papers reporting values for m_u and m_d (data points shown), and in part from analysis of the allowed ranges of other mass parameters (see Fig. 1). The parameter $(m_u + m_d)/2$ yields the two downward-sloping lines, while m_u/m_d yields the two rising lines originating at (0,0).