K^0

$I(J^P) = \frac{1}{2}(0^-)$

K^0 MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
497.648±0.022 OUR FIT					
497.648±0.022 OUR AVERAGE					
497.625±0.001±0.031	655k	LAI	02	NA48	K^0_L beam
497.661±0.033	3713	BARKOV	87B	CMD	$e^+e^- \rightarrow K^0_L K^0_S$
497.742±0.085	780	BARKOV	85B	CMD	$e^+e^- \rightarrow K^0_L K^0_S$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
497.44 ±0.50		FITCH	67	OSPK	
498.9 ±0.5	4500	BALTAY	66	HBC	K^0 from $\overline{p}p$
497.44 ±0.33	2223	KIM	65B	HBC	K^0 from $\overline{p}p$
498.1 ±0.4		CHRISTENS...	64	OSPK	

$m_{K^0} - m_{K^\pm}$

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
3.972±0.027 OUR FIT Error includes scale factor of 1.2.						
• • • We do not use the following data for averages, fits, limits, etc. • • •						
3.95 ±0.21	417	HILL	68B	DBC	+	$K^+d \rightarrow K^0 pp$
3.90 ±0.25	9	BURNSTEIN	65	HBC	−	
3.71 ±0.35	7	KIM	65B	HBC	−	$K^- p \rightarrow n\overline{K}^0$
5.4 ±1.1		CRAWFORD	59	HBC	+	
3.9 ±0.6		ROSENFELD	59	HBC	−	

K^0 MEAN SQUARE CHARGE RADIUS

VALUE (fm^2)	DOCUMENT ID		TECN	COMMENT
−0.076±0.018 OUR AVERAGE Error includes scale factor of 1.1.				
−0.090±0.021	LAI	03C	NA48	$K^0_L \rightarrow \pi^+\pi^- e^+ e^-$
−0.054±0.026	MOLZON	78		K_S regen. by electrons
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−0.087±0.046	BLATNIK	79		VMD + dispersion relations
−0.050±0.130	FOETH	69B		K_S regen. by electrons

T-VIOLATION PARAMETER IN K^0-$\overline{K}^0$ MIXING

The asymmetry $A_T = \frac{\Gamma(\overline{K}^0 \rightarrow K^0) - \Gamma(K^0 \rightarrow \overline{K}^0)}{\Gamma(\overline{K}^0 \rightarrow K^0) + \Gamma(K^0 \rightarrow \overline{K}^0)}$ must vanish if T invariance holds.

ASYMMETRY A_T IN K^0-$\overline{K}^0$ MIXING

VALUE (units 10^{-3})	EVTS	DOCUMENT ID		TECN
6.6±1.3±1.0	640k	[1] ANGELOPO...	98E	CPLR

[1] ANGELOPOULOS 98E measures the asymmetry $A_T = [\Gamma(\overline{K}^0_{t=0} \to e^+\pi^-\nu_{t=\tau}) - \Gamma(K^0_{t=0} \to e^-\pi^+\overline{\nu}_{t=\tau})]/[\Gamma(\overline{K}^0_{t=0} \to e^+\pi^-\nu_{t=\tau}) + \Gamma(K^0_{t=0} \to e^-\pi^+\overline{\nu}_{t=\tau})]$ as a function of the neutral-kaon eigentime τ. The initial strangeness of the neutral kaon is tagged by the charge of the accompanying charged kaon in the reactions $p\overline{p} \to K^-\pi^+K^0$ and $p\overline{p} \to K^+\pi^-\overline{K}^0$. The strangeness at the time of the decay is tagged by the lepton charge. The reported result is the average value of A_T over the interval $1\tau_S < \tau < 20\tau_S$. From this value of A_T ANGELOPOULOS 01B, assuming CPT invariance in the $e\pi\nu$ decay amplitude, determine the T-violating as ΔS=ΔS conserving parameter (for its definition, see Review below) $4\mathrm{Re}(\epsilon) = (6.2 \pm 1.4 \pm 1.0) \times 10^{-3}$.

CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY

Revised 2003 by P. Bloch (CERN).

The time evolution of a neutral kaon state state is described by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi \ , \qquad \Lambda \equiv M - \frac{i}{2}\Gamma \tag{1}$$

where M and Γ are Hermitian 2×2 matrices known as the mass and decay matrices. The corresponding eigenvalues are $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\gamma_{L,S}$. CPT invariance requires the diagonal elements of Λ to be equal. The CPT-violation complex parameter δ is defined as

$$\begin{aligned}\delta &= \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{2(\lambda_L - \lambda_S)} \\ &= \delta_{\parallel}\exp\Big(i\phi_{SW}\Big) + \delta_{\perp}\exp\Big(i(\phi_{SW} + \frac{\pi}{2})\Big)\end{aligned} \tag{2}$$

where we have introduced the projections $\delta_{\parallel}$ and $\delta_{\perp}$ respectively parallel and perpendicular to the superweak direction $\phi_{SW} = \tan^{-1}(2\Delta m/\Delta\gamma)$, where $\Delta m = m_L - m_S$ and $\Delta\gamma = \gamma_S - \gamma_L$, the positive mass and width differences between K_L and K_S. These projections are linked to the mass and width difference between K^0 and $\overline{K}^0$:

$$\delta_{\parallel} = \frac{1}{4}\frac{\gamma_{K^0} - \gamma_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ , \ \delta_{\perp} = \frac{1}{2}\frac{m_{K^0} - m_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ . \tag{3}$$

$\mathrm{Re}(\delta)$ can be directly measured by studying the time evolution of the strangeness content of initially pure K^0 and $\overline{K}^0$ states, for example through the asymmetry

$$A_{CPT} = \frac{P[\overline{K}^0 \to \overline{K}^0(t)] - P[K^0 \to K^0(t)]}{P[\overline{K}^0 \to \overline{K}^0(t)] + P[K^0 \to K^0(t)]} = 4\mathrm{Re}(\delta) \qquad (4)$$

where $P[a \to b(t)]$ is the probability that the pure initial state a is seen as state b at proper time t. This method has been used by tagging the initial strangeness with strong interactions and the final strangeness with the semileptonic decay (a more appropriate combination of semileptonic rates allows to be independent of any direct CPT violation in the decay itself) and yields today's best value of $\mathrm{Re}(\delta)$, compatible with zero with an error of $\sim 3 \times 10^{-4}$.

As an alternative it has been proposed to compare the semileptonic charge asymmetries for K_L and K_S

$$\delta_{L,S} = \frac{R(K_{L,S} \to \pi^- \ell^+ \nu) - R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})}{R(K_{L,S} \to \pi^- \ell^+ \nu) + R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})} \ ,$$

$$\delta_S - \delta_L = 4\mathrm{Re}(\delta) \ . \qquad (5)$$

δ_L has been accurately measured and δ_S should be measured in the near future with tagged K_S at ϕ factories. Note however that Eq. (5) assumes CPT invariance in the $\Delta S = -\Delta Q$ semileptonic decay amplitude.

$\delta_\perp$ can be obtained from the measurement of the $\pi\pi$ decays CP-violation parameters η_{+-} and η_{00}. Figure 1 shows the various contributions to $\eta_{\pi\pi}$ [1]. The T-violation parameter ϵ_T

$$\epsilon_T = i\frac{|\Lambda_{K^0\overline{K}^0}|^2 - |\Lambda_{\overline{K}^0 K^0}|^2}{\Delta\gamma(\lambda_L - \lambda_S)} \qquad (6)$$

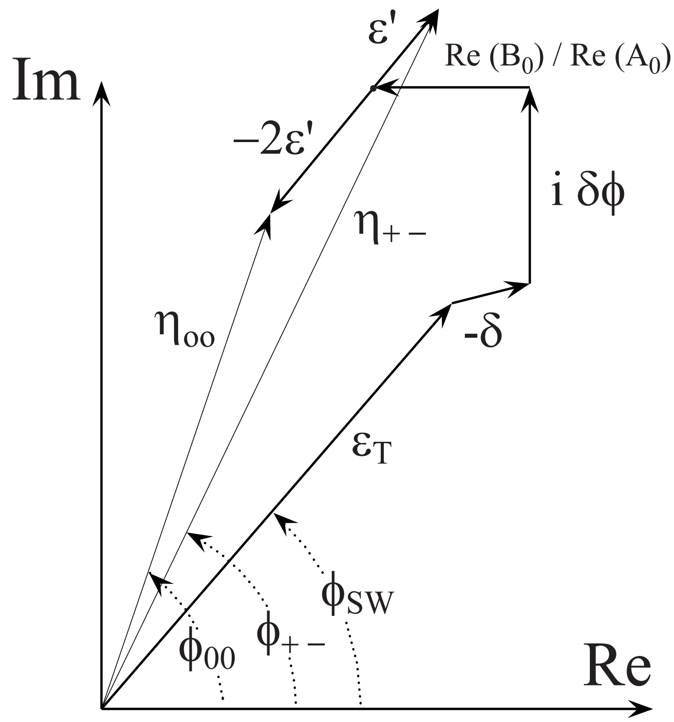

Figure 1: *CP*- and *CPT*-violation parameters in 2π decay.

has been defined in such a way that it is exactly aligned along the superweak direction [‡]. A_I (resp. B_I) is the CPT-conserving (resp. violating) decay amplitude for the $\pi\pi$ Isospin I state, ε' is the direct CP/CPT-violation parameter $[\varepsilon' = 1/3(\eta_{+-} - \eta_{00})]$ and $\delta\phi = \frac{1}{2}\left[\varphi_\Gamma - \arg(A_0^*\overline{A}_0)\right]$ is the phase difference between the $I = 0$ component of the decay amplitude and the matrix element $\Gamma_{K^0\overline{K}^0}$. From Fig. 1 one obtains

$$\delta_\perp = |\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}) - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\sin(\phi_{SW}) + \delta\phi \ \ \cos(\phi_{SW}) \ . \tag{7}$$

The present accuracy on the term $|\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00})$ is 2.6×10^{-5}. $\delta\phi$ gets contributions from CP violation in semileptonic and 3π decays [2,3] and can only be neglected at the present time if one assumes that η_{000} is not significantly larger than η_{+-0}. Furthermore, B_0 is not directly measured, so additional assumptions (for example, CPT conservation in the decay which implies $B_0 = 0$) or a combination with other measurements are necessary to obtain $\delta_\perp$.

If one assumes unitarity, one can measure $\mathrm{Im}(\delta)$ using the Bell-Steinberger relation which relates K_S and K_L decay amplitudes into all final states f:

$$\mathrm{Re}(\epsilon_T) - i\mathrm{Im}(\delta) = \frac{1}{2(i\Delta m + \frac{1}{2}(\gamma_L + \gamma_S))} \times \sum A_{f_L} A^*_{f_S} \ . \quad (8)$$

Since the $\pi\pi$ amplitudes dominate, the result relies also strongly on the $\phi_{\pi\pi}$ phase measurements. The advantage is that B_0 does not enter. Using all available data, one obtains a value of $\mathrm{Im}(\delta)$ compatible with zero with a precision of 5×10^{-5}. The precision here is also limited by the poor measurement of η_{000}.

The results on $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ can be combined to obtain $\delta_\parallel$ and $\delta_\perp$ and therefore the K^0–$\overline{K}^0$ mass and width difference shown in Fig. 2. The current accuracy is a few 10^{-18} GeV for both.

If one assumes that CPT is conserved in the decays ($\gamma_{K^0} = \gamma_{\overline{K}^0}$, $\delta_\parallel = 0$, $B_I = 0$), the phase of δ is known, and the $\delta_\perp$ and Bell-Steinberger methods are identical. Assuming in addition $\eta_{+-0} = \eta_{000}$, one in this case obtains a limit for $|m_{K^0} - m_{\overline{K}^0}|$ of 4.4×10^{-19} GeV (90%CL).

Footnotes and References

[‡] Many authors have a different definition of the T-violation parameter, $\epsilon = (\Lambda_{\overline{K}^0 K^0} - \Lambda_{K^0 \overline{K}^0})/(2(\lambda_L - \lambda_S))$. ϵ is not

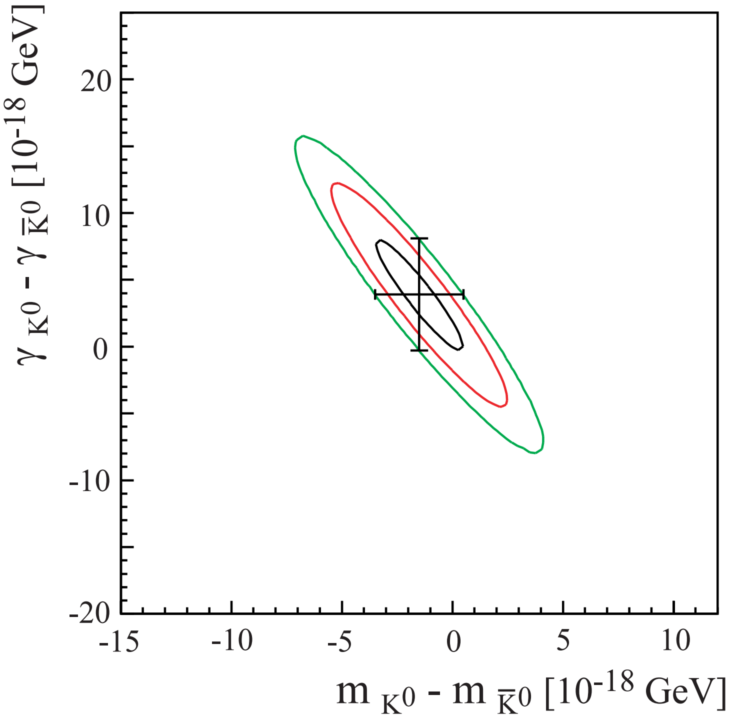

Figure 2: K^0–$\overline{K}^0$ mass vs width difference.

exactly aligned with the superweak direction. The two definitions can be related through $\epsilon = \epsilon_T + i\delta\phi$.

1. See for instance, C.D. Buchanan *et al.*, Phys. Rev. **D45**, 4088 (1992). See also the Second Daphne Handbook, Ed. L.Maiani *et al.*, INFN Frascati (1995).
2. V.V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).
3. L. Lavoura, Mod. Phys. Lett. **A7**, 1367 (1992).

CPT-VIOLATION PARAMETERS

In K^0-$\overline{K}^0$ mixing, if *CP*-violating interactions include a T conserving part then

$$|K_S\rangle = [|K_1\rangle + (\epsilon+\delta)|K_2\rangle]/\sqrt{1+|\epsilon+\delta|^2}$$
$$|K_L\rangle = [|K_2\rangle + (\epsilon-\delta)|K_1\rangle]/\sqrt{1+|\epsilon-\delta|^2}$$

where

$$|K_1\rangle = [|K^0\rangle + |\overline{K}^0\rangle]/\sqrt{2}$$

$$|K_2\rangle = [|K^0\rangle - |\overline{K}^0\rangle]/\sqrt{2}$$

and

$$|\overline{K}^0\rangle = CP|K^0\rangle.$$

The parameter δ specifies the *CPT*-violating part.

Estimates of δ are given below assuming the validity of the $\Delta S = \Delta Q$ rule. See also THOMSON 95 for a test of *CPT*-symmetry conservation in K^0 decays using the Bell-Steinberger relation.

REAL PART OF δ

A nonzero value violates *CPT* invariance.

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
2.9± 2.6±0.6	1.3M	[2] ANGELOPO...	98F	CPLR	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
2.4± 2.8		[3] APOSTOLA...	99B	RVUE	
180 ±200	6481	[4] DEMIDOV	95		$K_{\ell 3}$ reanalysis

[2] If $\Delta S = \Delta Q$ is not assumed, ANGELOPOULOS 98F finds Re$\delta = (3.0 \pm 3.3 \pm 0.6) \times 10^{-4}$.
[3] APOSTOLAKIS 99B assumes only unitarity and combines CPLEAR and other results.
[4] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

IMAGINARY PART OF δ

A nonzero value violates *CPT* invariance.

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
0.024± 0.050		[5] APOSTOLA...	99B	RVUE	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
− 0.9 ± 2.9 ±1.0	1.3M	[6] ANGELOPO...	98F	CPLR	
21 ±37	6481	[7] DEMIDOV	95		$K_{\ell 3}$ reanalysis

[5] APOSTOLAKIS 99B assumes only unitarity and combines CPLEAR and other results.
[6] If $\Delta S = \Delta Q$ is not assumed, ANGELOPOULOS 98F finds Im$\delta = (-15 \pm 23 \pm 3) \times 10^{-3}$.
[7] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

$|m_{K^0} - m_{\overline{K}^0}| \,/\, m_{\text{average}}$

A test of *CPT* invariance. "Our Evaluation" is described in the "Tests of Conservation Laws" section. It assumes *CPT* invariance in the decay and neglects some contributions from decay channels other than $\pi\pi$.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*
< 10^{-18} (CL = 90%) OUR EVALUATION				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(-\ 3 \pm 4) \times 10^{-18}$		[8] ANGELOPO...	99B	RVUE

[8] ANGELOPOULOS 99B assumes only unitarity and combines CPLEAR and other results.

$(\Gamma_{K^0} - \Gamma_{\overline{K}^0})/m_{\text{average}}$

A test of *CPT* invariance.

VALUE	*DOCUMENT ID*	*TECN*
$(7.8 \pm 8.4) \times 10^{-18}$	[9] ANGELOPO... 99B	RVUE

[9] ANGELOPOULOS 99B assumes only unitarity and combines CPLEAR with other results. Correlated with $(m_{K^0} - m_{\overline{K}^0})$ / m_{average} with a correlation coefficient of -0.95.

K^0 REFERENCES

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ANGELOPO...	99B	PL B471 332	A. Angelopoulos *et al.*	(CPLEAR Collab.)
APOSTOLA...	99B	PL B456 297	A. Apostolakis *et al.*	(CPLEAR Collab.)
ANGELOPO...	98E	PL B444 43	A. Angelopoulos *et al.*	(CPLEAR Collab.)
ANGELOPO...	98F	PL B444 52	A. Angelopoulos *et al.*	(CPLEAR Collab.)
Also	01B	EPJ C22 55	A. Angelopoulos *et al.*	(CPLEAR Collab.)
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From YAF 58 1041.				
THOMSON	95	PR D51 1412	G.B. Thomson, Y. Zou	(RUTG)
BARKOV	87B	SJNP 46 630	L.M. Barkov *et al.*	(NOVO)
		Translated from YAF 46 1088.		
BARKOV	85B	JETPL 42 138	L.M. Barkov *et al.*	(NOVO)
		Translated from ZETFP 42 113.		
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MOLZON	78	PRL 41 1213	W.R. Molzon *et al.*	(EFI+)
NIEBERGALL	74	PL 49B 103	F. Niebergall *et al.*	(CERN, ORSAY, VIEN)
HART	73	NP B66 317	J.C. Hart *et al.*	(CAVE, RHEL)
FOETH	69B	PL 30B 276	H. Foeth *et al.*	(AACH, CERN, TORI)
HILL	68B	PR 168 1534	D.G. Hill *et al.*	(BNL, CMU)
FITCH	67	PR 164 1711	V.L. Fitch *et al.*	(PRIN)
BALTAY	66	PR 142 932	C. Baltay *et al.*	(YALE, BNL)
BURNSTEIN	65	PR 138B 895	R.A. Burnstein, H.A. Rubin	(UMD)
KIM	65B	PR 140B 1334	J.K. Kim, L. Kirsch, D. Miller	(COLU)
CHRISTENS...	64	PRL 13 138	J.H. Christenson *et al.*	(PRIN)
CRAWFORD	59	PRL 2 112	F.S. Crawford *et al.*	(LRL)
ROSENFELD	59	PRL 2 110	A.H. Rosenfeld, F.T. Solmitz, R.D. Tripp	(LRL)