DETERMINATION OF $|V_{cb}|$

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I. Introduction

In the framework of the Standard Model, the quark sector is characterized by a rich pattern of flavor-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see CKM review [1]). This report focuses on the quark mixing parameter $|V_{cb}|$.

Two different methods have been used to extract this parameter from data: the **exclusive** measurement, where $|V_{cb}|$ is extracted by studying $B \to D^\star \ell\nu$ or $B \to D\ell\nu$ decay processes; and the **inclusive** measurement, which uses the semileptonic width of b-hadron decays ($B \to X\ell\overline{\nu}$). Theoretical estimates play a crucial role in extracting $|V_{cb}|$, and an understanding of their uncertainties is very important.

II. $|V_{cb}|$ determination from exclusive channels

The exclusive $|V_{cb}|$ determination is obtained studying $B \to D^\star\ell\nu$ or $B \to D\ell\nu$ decays, using Heavy Quark Effective Theory (HQET), an exact theory in the limit of infinite quark masses. Currently, the $B \to D\ell\nu$ transition provides a less precise value, and is used as a check.

The decay $B \to D^\star\ell\nu$ in HQET: HQET predicts that the differential partial decay width for this process, $d\Gamma/dw$, is related to $|V_{cb}|$ through:

$$\frac{d\Gamma}{dw}(B \to D^\star\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\mathcal{K}(w)\mathcal{F}(w)^2, \tag{1}$$

where w is the inner product of the B and $D^\star$ meson 4-velocities, $\mathcal{K}(w)$ is a known phase-space factor, and the form factor $\mathcal{F}(w)$ is generally expressed as the product of a normalization constant, $\mathcal{F}(1)$, and a function, $g(w)$, constrained by experimental studies of the helicity amplitudes characterizing this decay [2] and dispersion relations [3].

There are several different corrections to the infinite mass value $\mathcal{F}(1) = 1$ [4]:

$$\mathcal{F}(1) = \eta_{\rm QED}\eta_A\left[1 + \delta_{1/m_Q^2} + ...\right], \tag{2}$$

here and in the following discussion of exclusive semileptonic decays, m_Q is a generic notation for m_c or m_b. By virtue of Luke's theorem [5], the first term in the non-perturbative expansion in powers of $1/m_Q$ vanishes. QED corrections up to leading-logarithmic order give $\eta_{\rm QED} \approx 1.007$ [6] and QCD radiative corrections to two loops give $\eta_A = 0.960 \pm 0.007$ [7]. Different estimates of the $1/m_Q^2$ corrections, involving terms proportional to $1/m_b^2$, $1/m_c^2$, and $1/(m_b m_c)$, have been performed in a quark model [8,10], with OPE sum rules [11], and, more recently, with an HQET based lattice gauge calculation [12]. The value from this quenched lattice HQET calculation is $\mathcal{F}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$. The errors quoted reflect the statistical accuracy, the matching error, the lattice finite size, the uncertainty in the quark masses, and an estimate of the error induced by the quenched approximation, respectively. The central value obtained with OPE sum rules is similar, $0.900 \pm 0.015 \pm 0.025 \pm 0.025$ [11], where the three errors parameterize different sources of theoretical uncertainty. Here we will use $\mathcal{F}(1) = 0.91 \pm 0.04$ [13], a value that is consistent with all the three determinations discussed above. We have chosen not to rely solely on the value of $\mathcal{F}(1)$ coming from the lattice, because of the difficulties of quantifying the uncertainty induced by the quenched approximation. Recent developments give confidence that this limitation will be overcome in the next few years [15]. Technical advances, such as new improved staggered discretization, may lead to precise value of some "gold-plated" lattice quantities, such as $\mathcal{F}(1)$ in $B \to D^\star \ell \overline{\nu}$. The stated theoretical accuracy will be checked by comparing predicted and measured values of a large number of non-perturbative quantities [17].

The analytical expression of $g(w)$, the universal form factor related to the Isgur-Wise function [18], is not known a-priori, and this introduces an additional uncertainty in the determination of $\mathcal{F}(1)|V_{cb}|$. First measurements of $|V_{cb}|$ were performed assuming a linear approximation for $g(w)$. It has been shown [19] that this assumption is not justified, and that linear fits systematically underestimate the extrapolation at zero recoil ($w = 1$) by about 3%. Most of this effect is related to the curvature of the form factor, and does not depend strongly upon the details of the chosen non-linear shape [19]. All recent published results use a non-linear shape for $g(w)$, approximated with an expansion near $w = 1$ [20]. $g(w)$ is parameterized in terms of the variable ρ^2, which is the slope of the form factor at zero recoil given in Ref. 20.

Experimental techniques to study the decay $B \to D^\star \ell\nu$:

The decay $B \to D^\star \ell\nu$ has been studied in experiments performed at center-of-mass energies equal to the $\Upsilon(4S)$ mass and the Z^0 mass. At the $\Upsilon(4S)$, experiments have the advantage that the w resolution is quite good. The dominant systematic uncertainties arise from background estimation and from the slow pion efficiency evaluation. This efficiency for charged pions is very low near w=1 and increases rapidly as w increases, while for neutral pions it drops slowly. CLEO [22] studies both $D^{\star +}$ and $D^{\star o}$ channels, while Belle [23] and BaBar [24] have so far presented only results based on $D^{\star +}\ell\nu$. In addition, kinematic constraints enable $\Upsilon(4S)$ experiments to identify the final state, including the $D^\star$, without a large contamination from the poorly known semileptonic decays including a hadronic system heavier than $D^\star$, commonly identified as '$D^{\star\star}$'. At LEP, B's are produced with a large momentum (about 30 GeV on average). The large boost produces a broadening in the reconstructed ν 4-momentum, needed to determine w, thus giving a relatively poor resolution and limited physics background rejection capabilities. On the other hand, LEP experiments benefit from an efficiency that is only mildly dependent upon w.

Experiments determine the product $(\mathcal{F}(1) \cdot |V_{cb}|)^2$ by fitting the measured $d\Gamma/dw$ distribution. Measurements have been published by CLEO [22], Belle [23], DELPHI [25], ALEPH [26], and OPAL [27]. Most recently, a preliminary measurement from BaBar has been presented [24]. At LEP, the dominant source of systematic error is the uncertainty on the contribution to $d\Gamma/dw$ from semileptonic B decays with final states including a hadron system heavier than the $D^\star$. This component includes both narrow orbitally excited charmed mesons and non-resonant or broad species. The existence of narrow resonant states is well established [1], and a signal of a broad resonance has been seen by CLEO [28], and, most recently, by Belle [29], but the decay characteristics of these states in b-hadron semileptonic decays have large uncertainties. The average of ALEPH [30], Belle [33], CLEO [31], and DELPHI [32] narrow state branching fractions show that the ratio $R_{\star\star} = \frac{\mathrm{B}(\overline{B} \to D_2^\star \ell \overline{\nu})}{\mathrm{B}(\overline{B} \to D_1 \ell \overline{\nu})}$ is smaller than one (< 0.6 at 95% C.L. [34]), in disagreement with HQET models where an infinite quark mass is assumed [35], but in agreement with models which take into account finite quark mass corrections [36]. Hence, LEP experiments use the treatment of narrow $D^{\star\star}$ proposed in [36], which accounts for $\mathcal{O}(1/m_c)$ corrections and provides several possible approximations of the form factors that depend on five different expansion schemes, and on three input parameters. To calculate the systematic errors, each proposed scheme is tested, with the relevant input parameters varied over a range consistent with the experimental limit on $R_{\star\star}$. The quoted systematic error is the maximal difference from the central value obtained with this method. Broad resonances or other non-resonant terms may not be modelled correctly with this approach.

To combine the published data, the central values and the errors of $\mathcal{F}(1)|V_{cb}|$ and ρ^2 are re-scaled to the same set of input parameters and their quoted uncertainties [21]. The $\mathcal{F}(1)|V_{cb}|$ values used for this average are extracted using the parametrization in Ref. 22, based on the experimental determinations of the vector and axial form factor ratios R_1 and R_2 [38]. The LEP data, which originally used theoretical values for these ratios, are re-scaled accordingly [37]. Table 1 summarizes the corrected data. The averaging procedure [37] takes into account statistical and systematic correlations between $\mathcal{F}(1)|V_{cb}|$ and ρ^2. Averaging the measurements in Table 1, we get:

$$\mathcal{F}(1)|V_{cb}| = (38.2 \pm 0.5 \pm 0.9) \times 10^{-3}$$

and

$$\rho^2 = 1.56 \pm 0.05 \pm 0.13, \tag{3}$$

with χ^2 per degree of freedom of 22/12. The error ellipses for the corrected measurements and for the world average are shown in Figure 1. They are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two. Since this is a 2 parameter fit, the ellipses correspond to about 37% CL contours. The χ^2 per degree of freedom is 1.8. We have not rescaled the errors.

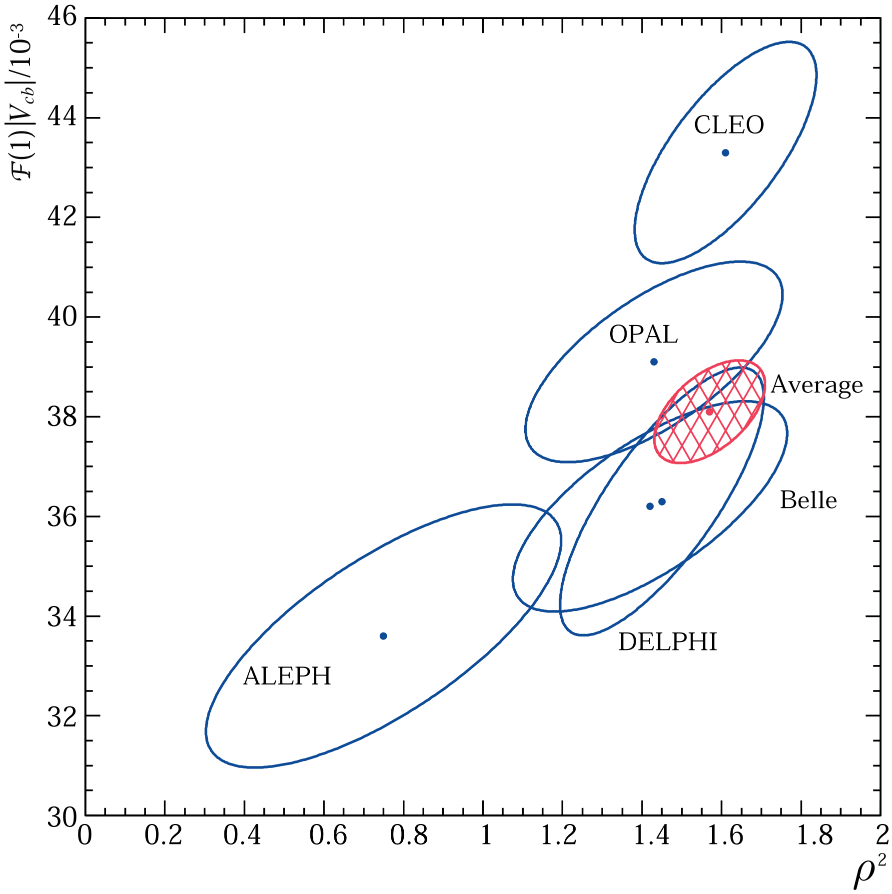

Figure 1: The error ellipses for the corrected measurements and world average for $\mathcal{F}(1)|V_{cb}|$ vs ρ^2. The ellipses are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two. Consequently the ellipses correspond to about 37% CL. See full-color version on color pages at end of book.

Table 1: Experimental results from $B \to D^\star \ell\nu$ analyses after the correction to common inputs and world average. The LEP numbers are corrected to use R_1 and R_2 from CLEO data. ρ^2 is the slope of the form factor at zero recoil as defined in Ref. 20. $\text{Corr}_{\text{stat}}$ is the statistical correlation between $\mathcal{F}(1)|V_{cb}|$ and ρ^2. (* Average of two measurements.)

Exp.	$\mathcal{F}(1)\lvert V_{cb}\rvert(\times 10^3)$	ρ^2	$\text{Corr}_{\text{stat}}$
ALEPH	$33.6 \pm 2.1 \pm 1.6$	$0.75 \pm 0.25 \pm 0.37$	94%
DELPHI*	$36.2 \pm 1.1 \pm 1.8$	$1.42 \pm 0.10 \pm 0.33$	92%
OPAL*	$39.1 \pm 0.9 \pm 1.8$	$1.43 \pm 0.12 \pm 0.31$	89%
Belle	$36.3 \pm 1.9 \pm 1.9$	$1.45 \pm 0.16 \pm 0.20$	91%
CLEO	$43.3 \pm 1.3 \pm 1.8$	$1.61 \pm 0.09 \pm 0.21$	87%
BaBar	$34.1 \pm 0.2 \pm 1.3$	$1.23 \pm 0.02 \pm 0.28$	92%
World average	$38.2 \pm 0.5 \pm 0.9$	$1.56 \pm 0.05 \pm 0.13$	53%

The main contributions to the $\mathcal{F}(1)|V_{cb}|$ systematic error are from the uncertainty on the $B \to D^{\star\star}\ell\nu$ shape and $\text{B}(b \to B_d)$, fully correlated among the LEP experiments, the branching fraction of D and $D^\star$ decays, fully correlated among all the experiments, and the slow pion reconstruction from Belle and CLEO which are uncorrelated, The main contribution to the ρ^2 systematic error is from the uncertainties in the measured values of R_1 and R_2, fully correlated among experiments. Because of the large contribution of this uncertainty to the non-diagonal terms of the covariance matrix, the averaged ρ^2 is higher than one would naively expect.

Using $\mathcal{F}(1) = 0.91 \pm 0.04$ [13], we get $|V_{cb}| = (42.0 \pm 1.1_{\text{exp}} \pm 1.8_{\text{theo}}) \times 10^{-3}$. The dominant error is theoretical, and there are good prospects to reduce it in the next few years [14], [17].

The decay $B \to D\ell\nu$: The study of the decay $B \to D\ell\nu$ poses new challenges both from the theoretical and experimental point of view.

The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{K}_{\mathcal{D}}(w) \mathcal{G}(w)^2, \qquad (4)$$

where w is the inner product of the B and D meson 4-velocities, $\mathcal{K}_{\mathcal{D}}(w)$ is the phase space, and the form factor $\mathcal{G}(w)$ is generally expressed as the product of a normalization factor, $\mathcal{G}(1)$, and a function, $g_D(w)$, constrained by dispersion relations [3].

The strategy to extract $\mathcal{G}(1)|V_{cb}|$ is identical to that used for the $B \to D^\star \ell\nu$ decay. However, in this case there is no suppression of $1/m_Q$ (*i.e.*, no Luke theorem) and corrections and QCD effects on $\mathcal{G}(1)$ are calculated with less accuracy than $\mathcal{F}(1)$ [39,40]. Moreover, $d\Gamma_D/dw$ is more heavily suppressed near $w = 1$ than $d\Gamma_{D^*}/dw$, due to the helicity mismatch between initial and final states. This channel is also much more challenging from the experimental point of view as it is hard to isolate from the dominant $B \to D^\star \ell\nu$ background, as well as from fake D–ℓ combinations. Thus, the extraction of $|V_{cb}|$ from this channel is less precise than the one from the $B \to D^\star \ell\nu$ decay. Nevertheless, the $B \to D\ell\nu$ channel provides a consistency check, and allows a test of heavy-quark symmetry [40] through the measurement of the form factor $\mathcal{G}(w)$, as HQET predicts the ratio $\mathcal{G}(w)/\mathcal{F}(w)$ to be very close to one.

Belle [41] and ALEPH [26] studied the $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ channel, while CLEO [42] studied both $B^+ \to D^0\ell^+\overline{\nu}$ and $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ decays. Averaging the data in Table 2 [37], we get $\mathcal{G}(1)|V_{cb}| = (41.8 \pm 3.7) \times 10^{-3}$ and $\rho_D^2 = 1.15 \pm 0.16$, where ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 20.

The theoretical predictions for $\mathcal{G}(1)$ are consistent: 1.03 ± 0.07 [43], and 1.02 ± 0.08 [40]. A quenched lattice calculation gives $\mathcal{G}(1) = 1.058^{+0.021}_{-0.017}$ [44], where the errors do not include the uncertainties induced by the quenching approximation and lattice spacing. An unquenched value should be available in the next few years [15]. A recent study of the decay $B \to D\ell\overline{\nu}$ in the context of heavy quark sum rules [16] argues that this

Table 2: Experimental results after the correction to common inputs and world average. ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 20.

Exp.	$\mathcal{G}(1)\lvert V_{cb}\rvert(\times 10^3)$	ρ_D^2
ALEPH	$39.3 \pm 10.0 \pm 6.5$	$0.97 \pm 0.98 \pm 0.38$
Belle	$41.8 \pm 4.4 \pm 5.2$	$1.12 \pm 0.22 \pm 0.14$
CLEO	$44.4 \pm 5.8 \pm 3.5$	$1.27 \pm 0.25 \pm 0.14$
World average	$41.8 \pm 2.5 \pm 2.7$	$1.15 \pm 0.13 \pm 0.09$

channel can provide an alternative very precise determination of $|V_{cb}|$.*

Using $\mathcal{G}(1) = 1.04 \pm\ 0.06$, we get $|V_{cb}| = (40.2 \pm 3.6_{\rm exp} \pm 2.3_{\rm theo}) \times 10^{-3}$, consistent with the value extracted from $B \rightarrow D^{\star}\ell\nu$ decay, but with a larger uncertainty.

The experiments have also measured the differential decay rate distribution to extract the ratio $\mathcal{G}(w)/\mathcal{F}(w)$. The data are compatible with a universal from factor as predicted by HQET.

III. $|V_{cb}|$ determination from inclusive B semileptonic decays

Alternatively, $|V_{cb}|$ can be extracted from the inclusive semileptonic width, requiring measurements of both the B lifetimes and the semileptonic branching fraction B($B \rightarrow X_c\ell\nu$) [45,46]. In this case, quark-hadron duality bridges the gap between theoretical calculations and experimental observables [47]. The modern theoretical formulation based on the Operator Product Expansion (OPE) determines the inclusive decay amplitudes in inverse powers of $1/m_Q$ [45]. Non-perturbative

* Ref. [16] uses heavy quark sum rules to relate exclusive form factors and inclusive semileptonic width and argues that in the approximation $\mu_\pi^2 - \mu_G^2 << \mu_\pi^2$, many power corrections vanish to all orders in $1/m_Q$. The parameter μ_π^2 represents the expectation value of the leading local heavy quark kinetic operator and μ_G^2 parameterizes the corresponding expectation value of the chromomagnetic operator. A more extensive discussion of the theoretical treatment of inclusive semileptonic decays is given in the next section.

corrections to the leading term, given by the spectator decay amplitude, arise only to order $1/m_b^2$. Quark-hadron duality is an important *ab initio* assumption in these calculations [47]. As M. Shifman put it [47], "It is fair to say that (short of the full solution of QCD) understanding and controlling the accuracy of the quark-hadron duality is one of the most important and challenging problem for the QCD practitioner today." In other words, as pointed out by Shifman and Buchalla [49], "duality violation parameterize our ignorance." Models can give estimates of the uncertainty induced by duality violations [48], [50]. These models need to have a clear physical interpretation and must be tested, in their key features, against experimental data [47]. The models quoted before imply different power suppression of duality violations. This issue needs to be resolved with further theoretical effort in defining clear and unambiguous quantitative tests of duality violations complemented by an experimental program to validate them.

The coefficients of the $1/m_b$ power terms are expectation values of operators that include non-perturbative physics. Relationships that are valid up to $1/m_b^2$ include four such parameters: the expectation value of the kinetic operator, corresponding to the average of the square of the heavy-quark momentum inside the hadron, the expectation value of the chromomagnetic operator, and the heavy-quark masses (m_b and m_c). The expectation value of the kinetic operator is introduced in the literature as μ_π^2 [45,46] or $-\lambda_1$ [51,52], and the expectation value of the chromomagnetic operator as μ_G^2 [45,46], or $3\lambda_2$ [51,52]. The two notations reflect a difference in the approach used to handle the energy scale μ used to separate long-distance from short-distance physics. HQET is most commonly renormalized in a mass-independent scheme, thus making the quark masses the pole masses of the underlying theory (QCD). The second group of authors prefer the definition of the non-perturbative operators using a mass scale $\mu \approx 1$ GeV.

The semileptonic width expression in Ref. 53 has been used to extract $|V_{cb}|$ from the semileptonic branching fraction measured by CLEO, and to measure the heavy-quark expansion (HQE) parameters $\overline{\Lambda}$ and λ_1, as discussed below. The most recent version of the alternative formulation can be found in Ref. 9.

The quark masses are related to the corresponding meson masses through [8]:

$$m_b = \overline{M}_B - \overline{\Lambda} + \frac{\lambda_1}{2\overline{M}_B}, \quad (5)$$

where $\overline{M}_B$ is the spin averaged $B - B^\star$ mass ($\overline{M}_B = 5.3134$ GeV/c^2). A similar equation relates m_c and $\overline{M}_D$. The parameter $\overline{\Lambda}$ represents the energy of the light quark and gluons. From the equations relating m_b and m_c to the corresponding spin-averaged meson masses, experimentalists usually derive the constraint on m_c to be used in the theoretical formulae. It has been pointed out [9] that it may be opportune to replace this constraint with an independent experimental determination of m_c.

HQE and moments in semileptonic decays:

Experimental determinations of the HQE parameters are important in several respects. In particular, redundant determinations of these parameters may uncover inconsistencies, or point to violation of some important assumptions inherent in these calculations. The parameter λ_2 can be extracted from the $B^\star - B$ mass splitting, whereas the other parameters need more elaborate measurements.

The CLEO collaboration determines the parameter $\overline{\Lambda}$ from the first moment of the γ energy in the decay $b \to s\gamma$, which gives the average energy of the γ emitted in this transition. Using the formalism of Ref. 53, they obtain $\overline{\Lambda} = 0.35 \pm 0.07 \pm 0.10$ GeV [54].

The parameter λ_1 can be determined from of the first moment of the mass M_X of the hadronic system recoiling against the $\ell - \overline{\nu}$ pair. The relationship between the first moment of $M_1 \equiv < M_X^2 - M_D^2 >$ and the parameters $\overline{\Lambda}$ and λ_1 is given in Ref. 55.

The measured value for $< M_X^2 - M_D^2 >$ [55] is 0.251 ± 0.066 GeV^2. This constraint, combined with the measurement of the mean photon energy in $b \to s\gamma$, implies a value of $\lambda_1 = -0.24 \pm 0.11$ GeV^2, to order $1/M_B^3$ and $\beta_0 \alpha_s^2$ in ($\overline{\mathrm{MS}}$).

The shape of the lepton spectrum provides further constraints on OPE. Moments of the lepton momentum with a cut $p_\ell^{CM} \geq 1.5$ GeV/c have been measured by the CLEO collaboration [62]. The two approaches give consistent results, although the technique used to extract the OPE parameters has still relatively large uncertainties associated with the $1/m_b^3$ form factors. The sensitivity to $1/m_b^3$ corrections depends upon which moments are considered. Bauer and Trott [61] have performed an extensive study of the sensitivity of lepton energy moments to non-perturbative effects. In particular, they have proposed "duality moments," very insensitive to neglected higher order terms. The comparison between the CLEO measurement of these moments [62] and the predicted values shows a very impressive agreement:

$$D_3 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.5190 \pm 0.0007 & \text{(T)} \\ 0.5193 \pm 0.0008 & \text{(E)} \end{cases}$$

$$D_4 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.6034 \pm 0.0008 & \text{(T)} \\ 0.6036 \pm 0.0006 & \text{(E)} \end{cases} \qquad (6)$$

(where "T" and "E" denote theory and experiment, respectively).

More recently, both CLEO and BaBar explored the moments of the hadronic mass M_X^2 with lower lepton momentum cuts. In order to identify the desired semileptonic decay from background processes including cascade decays, continuum leptons and fake leptons, CLEO performs a fit for the contributions of signal and backgrounds to the full three-dimensional differential decay rate distribution as a function of the reconstructed quantities q^2, M_X^2, $\cos\theta_{W\ell}$. The signal includes the components $B \to D\ell\overline{\nu}$, $B \to D^\star\ell\overline{\nu}$, $B \to D^{\star\star}\ell\overline{\nu}$, $B \to X_c\ell\overline{\nu}$ non-resonant, and $B \to X_u\ell\overline{\nu}$. The backgrounds considered are: secondary leptons, continuum leptons and fake leptons. BaBar uses a sample where the hadronic decay of one B is fully reconstructed and the charged lepton from the other B is identified. In this case the main sources of systematic errors are the uncertainties related to the detector modelling and reconstruction.

Figure 2 shows the extracted $< M_X^2 - \overline{M}_D^2 >$ moments as a function of the minimum lepton momentum cut from these two measurements, as well as the original measurement with $p_\ell \geq 1.5$ GeV/c. The results are compared with theory bands that reflect experimental errors, $1/m_b^3$ correction uncertainties and uncertainties in the higher order QCD radiative corrections [56]. The CLEO and BaBar results are consistent and show an improved agreement with theoretical predictions with respect to earlier preliminary results [57]. Moments of the M_X distribution without an explicit lepton momentum cut have been extracted from preliminary DELPHI data [58] and give consistent results.

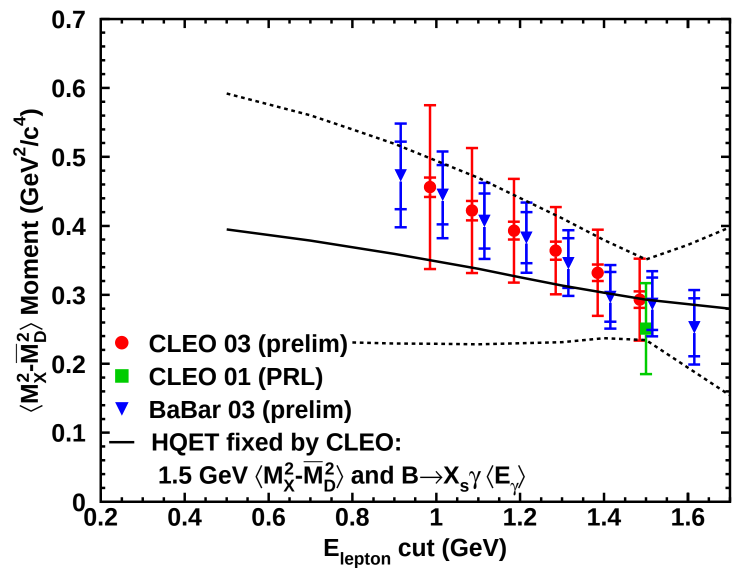

Figure 2: The results of the recent CLEO analysis [59] compared to previous measurements [55,60] and the HQET prediction. The theory bands shown in the figure reflect the variation of the experimental errors on the two constraints, the variation of the third-order HQET parameters by the scale $(0.5\ \text{GeV})^3$, and variation of the size of the higher order QCD radiative corrections [56]. See full-color version on color pages at end of book.

Experimental determination of the semileptonic branching fraction:

The value of B($B \to X_c \ell\nu$) has been measured both at the $\Upsilon(4S)$ and LEP.

Experiments taking data at the Υ(4S) center-of-mass energy determine the inclusive semileptonic branching fraction through a lepton tagged sample. In this approach, a di-lepton sample is studied, and the charge correlation between the two leptons is used to disentangle leptons coming from the direct decay $B \to X_c\ell\nu$ and the dominant background at low lepton momenta, the cascade decay $B \to X_c \to X_s\ell\nu$. This method was pioneered by the ARGUS collaboration [63] to measure the electron spectrum from $B \to X_c\ell\nu$ down to 0.6 GeV/c. Thus, it reduces the model dependence of the extracted semileptonic branching fraction very substantially. Experimental data are summarized in Table 3. The systematic error is dominated by experimental uncertainties: lepton identification efficiency, fake rate determination, and tracking efficiencies contribute to 3% of this overall error. The remaining error is a sum of several small corrections associated with the uncertainty in the mixing parameter, and additional background estimates [64]. BaBar [65] and Belle [66] have studied the inclusive electron spectrum with the same technique.

Table 3: B($b \to \ell$) measurement from experiments at $\Upsilon(4S)$ center-of-mass energy and their average. The errors quoted reflect statistical, and systematic uncertainties. These measurements are largely model independent.

Experiment	B($b \to \ell\nu$)%
ARGUS	$9.75 \pm 0.50 \pm 0.39$
CLEO	$10.49 \pm 0.17 \pm 0.43$
Belle	$10.96 \pm 0.12 \pm 0.50$
BaBar	$10.91 \pm 0.18 \pm 0.29$
Υ(4S) Average	10.73 ± 0.28

Combining $\Upsilon(4S)$ results [1], we obtain: $\mathrm{B}(b \to X\ell\nu) = (10.73\pm0.28)\%$. Upon subtracting $\mathrm{B}(b \to u\ell\nu) = (0.17\pm0.05)\%$, we get: $\mathrm{B}(b \to X_c\ell\nu) = (10.56 \pm 0.28)\%$. Using τ_{B^+}, τ_{B^0} [1], and the ratio between charged and neutral B pair production $f_{+-}/f_{00} = 1.044 \pm 0.05$ [21], we obtain the semileptonic width $\Gamma(b \to X_c\ell\nu) = (0.434 \pm 0.011 \pm 0.003) \times 10^{-10}$ MeV, where the second error includes the uncertainties from $\mathrm{B}(b \to u\ell\nu)$, and the model dependence. A common value for the ratio f_{+-}/f_{00} between B^+B^- and $B^0\overline{B}^0$ final states produced at the $\Upsilon(4S)$ is used here. This parameter is very sensitive to the precise value of the center-of-mass energy and beam energy spread [70] and thus as more precise data become available, it is important to check that it is appropriate to average f_{+-}/f_{00} at different machines.

At LEP, B^0, B^-, B_s, and b baryons are produced, so the measured inclusive semileptonic branching ratio is an average over the different hadron species. Assuming that the semileptonic widths of all b hadrons are equal, the following relation holds:

$$\begin{aligned}
\mathrm{B}(b \to X_c l\nu)_{\mathrm{LEP}} = \\
& f_{B^0}\frac{\Gamma(B^0 \to X_c l\nu)}{\Gamma(B^0)} + f_{B^-}\frac{\Gamma(B^- \to X_c l\nu)}{\Gamma(B^-)} \\
& + f_{B_s}\frac{\Gamma(B_s \to X_c l\nu)}{\Gamma(B_s)} + f_{\Lambda_b}\frac{\Gamma(\Lambda_b \to X_c l\nu)}{\Gamma(\Lambda_b)} \\
& = \Gamma(B \to X_c\ell\nu)\tau_b \ , \qquad (7)
\end{aligned}$$

where τ_b is the average b-hadron lifetime. Taking into account the present precision of LEP measurements of b-baryon semileptonic branching ratios and lifetimes, the estimate uncertainty for a possible difference for the width of b baryons is 0.13%. The average LEP value for $\mathrm{B}(b \to X\ell\nu) = (10.59 \pm 0.30)\%$ is taken from a fit [71], which combines the semileptonic branching ratios, the $B^0-\overline{B}^0$ mixing parameter $\overline{\chi}_b$, and $R_b = \Gamma(Z \to b\overline{b})/\Gamma(Z \to \mathrm{had})$. Ref. 72 shows that the main contribution to the model uncertainty is the composition of the semileptonic width, including the narrow, wide and non-resonant $D^{\star\star}$ states. B_s and b baryons are about 20% of the total signal, and their contribution to the uncertainty of the spectrum is small. In this average, we use the modelling error quoted by Ref. 72, rather than the error from the combined fit, as the ALEPH procedure is based on more recent information. The dominant errors in the combined branching fraction are the modelling of semileptonic decays (2.6%) and the detector related items (1.3%).

Subtracting $B(b \to u\ell\nu)$ from the LEP semileptonic branching fraction, we get: $B(b \to X_c\ell\nu) = (10.52 \pm 0.32)\%$, and using τ_b [1]: $\Gamma(b \to X_c\ell\nu) = (0.439 \pm 0.010 \pm 0.011) \times 10^{-10}$ MeV, where the systematic error 0.011×10^{-10}MeV reflects the $B(b \to u\ell\nu)$ uncertainty and the model dependence.

Combining the LEP and the $\Upsilon(4S)$ semileptonic widths, we get: $\Gamma(b \to X_c\ell\nu) = (0.44 \pm 0.01) \times 10^{-10}$ MeV, which is used in the formula of Ref. 55 to get:

$$|V_{cb}|_{\rm incl} = (41.0 \pm 0.5_{\rm exp} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\rm theo}) \times 10^{-3} \ , \qquad (8)$$

where the first error is experimental, and the second is from the measured value of λ_1 and $\overline{\Lambda}$, assumed to be universal up to higher orders. The third error is from $1/m_b^3$ corrections and from the ambiguity in the α_s scale definition. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio.

IV. Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^\star\ell\nu$ decay is:

$$|V_{cb}|_{\rm exclusive} = (42.0 \pm 1.1_{\rm exp} \pm 1.9_{\rm theo}) \times 10^{-3} \ , \qquad (9)$$

where the first error is experimental and the second error is from the $1/m_Q^2$ corrections to $\mathcal{F}(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\rm incl} = (41.0 \pm 0.5_{\rm exp} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\rm theo}) \times 10^{-3}. \qquad (10)$$

In addition, non-quantified uncertainties are associated with a possible quark-hadron duality violations when using the inclusive method. A first conservative assessment of these uncertainties may be obtained from the difference between the two values of $|V_{cb}|$ extracted from $B \to D^{\star}\ell\overline{\nu}$ and from inclusive measurements. These data imply about 6% uncertainty for non-quantified assumptions in the inclusive determination. This result is largely affected by the quantified theoretical errors in the two determinations and thus does not give a very stringent bound.

High precision tests of lattice gauge theory calculations and more refined experimental assessments of quark-hadron duality in inclusive semileptonic decays are needed to achieve the ultimate accuracy in our knowledge of V_{cb}.

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