CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY

Written November 2007 by M. Antonelli (LNF-INFN, Frascati) and G. D’Ambrosio (Naples U., INFN).

CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \bar{K}^0$ system, described by the equation

$$i\frac{d}{dt} \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = \begin{bmatrix} M - i\Gamma/2 \\ K^0 \end{bmatrix} ,$$

where $M$ and $\Gamma$ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [4] for notations and previous literature), allows a very accurate test of CPT symmetry; indeed since CPT requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a CPT-violating piece, $\delta$, in addition to the usual CPT-conserving parameter $\epsilon$:

$$\epsilon_{S,L} = \frac{1}{\sqrt{2(1 + |\epsilon_{S,L}|^2)}} \left[ (1 + \epsilon_{S,L}) K^0 + (1 - \epsilon_{S,L}) \bar{K}^0 \right]$$

$$\equiv \epsilon \mp \delta . \quad (1)$$

Using the phase convention $\Im(\Gamma_{12}) = 0$, we determine the phase of $\epsilon$ to be $\varphi_{SW} \equiv \arctan \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of $K^0$ and $\bar{K}^0$ wave functions, we obtain the Bell-Steinberger relation [5] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact, neglecting $O(\epsilon)$ corrections to the coefficient of the CPT-violating parameter, $\delta$, we can write [4]

$$\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} \left[ \frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i\Im(\delta) \right] = \frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f) A_S^*(f), \quad (2)$$

where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (2); in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S} \langle A_L(i)A_S^*(i) \rangle = \eta_i \, B(K_S \to i),$$

(i = \pi^0\pi^0, \pi^+\pi^-(\gamma), 3\pi^0, \pi^0\pi^+\pi^-(\gamma), (3)

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. 4 has been updated by using the recent measurements of $K_L$ branching ratios from KTeV [6] and NA48 [7,8])

$$\alpha_{\pi^+\pi^-} = \left(1.112 \pm 0.013 + i(1.061 \pm 0.014)\right) \times 10^{-3},$$

$$\alpha_{\pi^0\pi^0} = \left(0.493 \pm 0.007 + i(0.471 \pm 0.007)\right) \times 10^{-3},$$

$$\alpha_{\pi^+\pi^-\pi^0} = \left((0 \pm 2) + i(0 \pm 2)\right) \times 10^{-6},$$

$$|\alpha_{\pi^0\pi^0\pi^0}| < 7 \times 10^{-6} \text{ at } 95\% \text{ CL.} \tag{4}$$

The semileptonic contribution to the right-handed side of Eq. (2) requires the determination of several observables: we define [2,3]

$$A(K^0 \to \pi^-l^+\nu) = A_0(1 - y),$$

$$A(K^0 \to \pi^+l^-\nu) = A_0^*(1 + y^*) (x_+ - x_-)^*,$$

$$A(K^0 \to \pi^+l^-\nu) = A_0^*(1 + y^*),$$

$$A(K^0 \to \pi^-l^+\nu) = A_0(1 - y)(x_+ + x_-), \tag{5}$$

where $x_+$ ($x_-$) describes the violation of the $\Delta S = \Delta Q$ rule in $CPT$-conserving (violating) decay amplitudes, and $y$ parametrizes $CPT$ violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\bar{K}^0)$ beams, CPLEAR has measured $\Re(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [10]. These determinations have been improved in Ref. 4 by including the information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$, where $A_{L,S}$ are the $K_L$ and $K_S$ semileptonic charge asymmetries, respectively, from the PDG [11] and KLOE [12]. Here we are also including the $T$-violating asymmetry measurement from CPLEAR [13].

November 29, 2007 15:00
Table 1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [4] and CPLEAR [13].

<table>
<thead>
<tr>
<th></th>
<th>value</th>
<th>Correlations coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Re(\delta)$</td>
<td>$(3.0 \pm 2.3) \times 10^{-4}$</td>
<td>1</td>
</tr>
<tr>
<td>$\Im(\delta)$</td>
<td>$(-0.66 \pm 0.65) \times 10^{-2}$</td>
<td>-0.21 1</td>
</tr>
<tr>
<td>$\Re(x_-)$</td>
<td>$(-0.30 \pm 0.21) \times 10^{-2}$</td>
<td>-0.21 -0.60 1</td>
</tr>
<tr>
<td>$\Im(x_+)$</td>
<td>$(0.02 \pm 0.22) \times 10^{-2}$</td>
<td>-0.38 -0.14 0.47 1</td>
</tr>
<tr>
<td>$A_S + A_L$</td>
<td>$(-0.40 \pm 0.83) \times 10^{-2}$</td>
<td>-0.10 -0.63 0.99 0.43 1</td>
</tr>
</tbody>
</table>

The value $A_S + A_L$ in Table 1 can be directly included in the semileptonic contributions to the Bell Steinberger relations in Eq. (2)

\[
\sum_{\pi\ell\nu} \langle A_L(\pi\ell\nu)A_S^*(\pi\ell\nu) \rangle \\
= 2\Gamma(K_L \rightarrow \pi\ell\nu) \left( (\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta)) \right) \\
= 2\Gamma(K_L \rightarrow \pi\ell\nu) \left( (A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta)) \right) .
\]

Defining

\[
\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi\ell\nu} \langle A_L(\pi\ell\nu)A_S^*(\pi\ell\nu) \rangle + 2i \frac{\tau_{KS}}{\tau_{KL}} B(K_L \rightarrow \pi\ell\nu)\Im(\delta),
\]

we find:

\[
\alpha_{\pi\ell\nu} = \left( (-0.2 \pm 0.5) + i(0.1 \pm 0.5) \right) \times 10^{-5} .
\]

Inserting the values of the $\alpha$ parameters into Eq. (2), we find

\[
\Re(\epsilon) = (161.2 \pm 0.6) \times 10^{-5} ,
\]

\[
\Im(\delta) = (-0.6 \pm 1.9) \times 10^{-5} .
\]

The complete information on Eq. (8) is given in Table 2.
Table 2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

<table>
<thead>
<tr>
<th></th>
<th>value</th>
<th>Correlations coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Re(\epsilon)$</td>
<td>$(161.2 \pm 0.6) \times 10^{-5}$</td>
<td>1</td>
</tr>
<tr>
<td>$\Im(\delta)$</td>
<td>$(-0.6 \pm 1.9) \times 10^{-5}$</td>
<td>$-0.26$ 1</td>
</tr>
<tr>
<td>$\Re(\delta)$</td>
<td>$(2.5 \pm 2.3) \times 10^{-4}$</td>
<td>$-0.08$ $-0.09$ 1</td>
</tr>
<tr>
<td>$\Re(x_-)$</td>
<td>$(-4.2 \pm 1.7) \times 10^{-3}$</td>
<td>$-0.13$ 0.17 $-0.43$ 1</td>
</tr>
</tbody>
</table>

Now the agreement with CPT conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 30% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 1.

The process giving the largest contribution to the size of the allowed region is $K_L \rightarrow \pi^+\pi^-$, through the uncertainty on $\phi_{+-}$.

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \bar{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\bar{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{SW} e^{i\phi_{SW}} [1 + O(\epsilon)].$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\bar{K}^0})$, $\Delta \Gamma = (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$ plane is shown in the bottom panel of Fig. 1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. 11) and in the limit $\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$ we obtain

$$-5.1 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 5.1 \times 10^{-19} \text{ GeV} \text{ at 95 % C.L.}$$
Figure 1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the $\Delta M$, $\Delta \Gamma$ plane.
References

1. See the “CP Violation in Meson Decays,” in this Review.
8. We thank G. Isidori and M. Palutan for their contribution to the original analysis [4] performed with KLOE data.