$D^0 - \bar{D}^0$ MIXING

Revised January 2006 by D. Asner (Carleton University)

Standard Model contributions to $D^0 - \bar{D}^0$ mixing are strongly suppressed by CKM and GIM factors. Thus the observation of $D^0 - \bar{D}^0$ mixing might be evidence for physics beyond the Standard Model. See Burdman and Shipsey [1] for a review of $D^0 - \bar{D}^0$ mixing, Ref. [2] for a compilation of mixing predictions, and Ref. [3] for later predictions.

**Formalism:** The time evolution of the $D^0 - \bar{D}^0$ system is described by the Schrödinger equation

$$i \frac{\partial}{\partial t} \left( \frac{D^0(t)}{\bar{D}^0(t)} \right) = \left( M - \frac{i}{2} \Gamma \right) \left( \frac{D^0(t)}{\bar{D}^0(t)} \right),$$

where the $M$ and $\Gamma$ matrices are Hermitian, and $CPT$ invariance requires that $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The off-diagonal elements of these matrices describe the dispersive and absorptive parts of $D^0 - \bar{D}^0$ mixing.

The two eigenstates $D_1$ and $D_2$ of the effective Hamiltonian matrix $(M - \frac{i}{2} \Gamma)$ are given by

$$|D_{1,2}⟩ = p |D^0⟩ \pm q |\bar{D}^0⟩.$$  

The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2} \Gamma_{1,2} = \left( M - \frac{i}{2} \Gamma \right) \pm \frac{q}{p} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right),$$

where $m_1$ and $\Gamma_1$ are the mass and width of the $D_1$, etc., and

$$\left| \frac{q}{p} \right|^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}. $$

We define reduced mixing amplitudes $x$ and $y$ by

$$x \equiv 2M_{12}/\Gamma = (m_1 - m_2)/\Gamma = \Delta m/\Gamma$$

and

$$y \equiv \Gamma_{12}/\Gamma = (\Gamma_1 - \Gamma_2)/2\Gamma = \Delta \Gamma/2\Gamma,$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. The mixing rate, $R_M$, is approximately $(x^2 + y^2)/2$. In Eq. (5) and Eq. (6), the middle relation holds.
only in the limit of $CP$ conservation, in which case the subscripts 1 and 2 denote the $CP$-even and $CP$-odd eigenstates.

The parameters $x$ and $y$ are measured in several ways. The most precise constraints are obtained using the time-dependence of $D$ decays. Since $D^0$-$\bar{D}^0$ mixing is a small effect, the identification tag of the initial particle as a $D^0$ or a $\bar{D}^0$ must be extremely accurate. The usual tag is the charge of the distinctive slow pion in the decay sequence $D^{*+} \to D^0 \pi^+$ or $D^{*-} \to \bar{D}^0 \pi^-$. In current experiments, the probability of mistagging is about 0.1%. Another tag of comparable accuracy is identification of one of the $D$’s produced from $\psi(3770) \to D^0 D^0$. Time-dependent analyses are not possible at symmetric charm threshold facilities (the $D^0$ and $\bar{D}^0$ do not travel far enough). However, the quantum coherent $D^0 \bar{D}^0 C = -1$ state provides time-integrated sensitivity [4, 5].

**Time-Dependent Analyses:** We extend the formalism of this Review’s note on “$B^0$–$\bar{B}^0$ Mixing” [6]. In addition to the “right-sign” instantaneous decay amplitudes $\overline{A}_f \equiv \langle f | H | D^0 \rangle$ and $A_{\bar{f}} \equiv \langle \bar{f} | H | D^0 \rangle$ for $CP$ conjugate final states $f$ and $\bar{f}$, we include the “wrong-sign” amplitudes $\overline{A}_{\bar{f}} \equiv \langle \bar{f} | H | \bar{D}^0 \rangle$ and $A_f \equiv \langle f | H | D^0 \rangle$.

It is usual to normalize the wrong-sign decay distributions to the integrated rate of right-sign decays and to express time in units of the precisely measured $D^0$ mean lifetime, $\tau_{D^0} = 1/\Gamma = 2/(\Gamma_1 + \Gamma_2)$. Starting from a pure $|D^0\rangle$ or $|\bar{D}^0\rangle$ state at $t = 0$, the time-dependent rates of production of the wrong-sign final states relative to the integrated right-sign states are then

$$r(t) = \frac{|\langle f | H | D^0(t) \rangle|^2}{|A_f|^2} = \frac{q}{p} \left| g_+(t) \chi_f^{-1} + g_-(t) \right|^2$$

and

$$\overline{r}(t) = \frac{|\langle \bar{f} | H | \bar{D}^0(t) \rangle|^2}{|A_{\bar{f}}|^2} = \frac{p}{q} \left| g_+(t) \chi_{\bar{f}} + g_-(t) \right|^2,$$

where

$$\chi_f \equiv q \overline{A}_f / p A_f, \quad \chi_{\bar{f}} \equiv q \overline{A}_{\bar{f}} / p A_{\bar{f}}.$$

[7]
and
\[ g_{\pm}(t) = \frac{1}{2} \left( e^{-iz_{1}t} \pm e^{-iz_{2}t} \right), \quad z_{1,2} = \frac{\lambda_{1,2}}{\Gamma} . \] (10)

Note that a change in the convention for the relative phase of \( D_{0} \) and \( S^{0} \) would cancel between \( q/p \) and \( A_{f}/A_{f} \) and leave \( \chi_{f} \) invariant.

We expand \( r(t) \) and \( \tau(t) \) to second order in time for modes where the ratio of decay amplitudes \( R_{D} = |A_{f}/A_{f}|^{2} \) is very small.

**Semileptonic decays:** In semileptonic \( D \) decays, \( A_{f} = \overline{A_{f}} = 0 \) in the Standard Model. Then in the limit of weak mixing, where \( |ix + y| \ll 1 \), \( r(t) \) is given by

\[ r(t) = |g_{-}(t)|^{2} \left| \frac{q}{p} \right|^{2} \approx \frac{e^{-t}}{4} (x^{2} + y^{2}) t^{2} \left| \frac{q}{p} \right|^{2} . \] (11)

For \( \tau(t) \) one replaces \( q/p \) here with \( p/q \). In the limit of \( CP \) conservation, \( r(t) = \tau(t) \), and the time-integrated mixing rate relative to the time-integrated right-sign decay rate is

\[ R_{M} = \int_{0}^{\infty} r(t) dt = \left| \frac{q}{p} \right|^{2} \frac{x^{2} + y^{2}}{2 + x^{2} - y^{2}} \approx \frac{1}{2} (x^{2} + y^{2}) . \] (12)

Table 1 summarizes results from semileptonic decays.

<table>
<thead>
<tr>
<th>Year</th>
<th>Exper.</th>
<th>Final state(s)</th>
<th>( R_{M} ) (90 (95)% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>Belle(^{a})</td>
<td>( K^{(*)}+e^{-}\overline{\nu}_{e} )</td>
<td>( &lt; 1.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>2005</td>
<td>CLEO(^{b})</td>
<td>( K^{(*)}+e^{-}\overline{\nu}_{e} )</td>
<td>( &lt; 7.8 \times 10^{-3} )</td>
</tr>
<tr>
<td>2004</td>
<td>BABAR(^{c})</td>
<td>( K^{(*)}+e^{-}\overline{\nu}_{e} )</td>
<td>( &lt; 4.2(4.6) \times 10^{-3} )</td>
</tr>
<tr>
<td>2002</td>
<td>FOCUS (^{[7]})</td>
<td>( K^{+}\mu^{-}\overline{\nu}_{\mu} )</td>
<td>( &lt; 1.01(1.31) \times 10^{-3} )</td>
</tr>
<tr>
<td>1996</td>
<td>E791(^{d})</td>
<td>( K^{+}\ell^{-}\overline{\nu}_{\ell} )</td>
<td>( &lt; 5.0 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

See the end of the \( D^{0} \) listings for these references: \(^{a}\)BITENC 05, \(^{b}\)CAWLFIELD 05, \(^{c}\)AUBERT 04, \(^{d}\)AITALA 96C.
Wrong-sign decays to hadronic non-CP eigenstates:
Consider the final state \( f = K^+\pi^- \), where \( A_f \) is doubly Cabibbo-suppressed. The ratio of decay amplitudes is

\[
\frac{A_f}{A_d} = -\sqrt{R_D} e^{-i\delta}, \quad \left| \frac{A_f}{A_d} \right| \sim O(\tan^2\theta_c), \quad (13)
\]

where \( R_D \) is the doubly Cabibbo-suppressed (DCS) decay rate relative to the Cabibbo-favored (CF) rate, the minus sign originates from the sign of \( V_{us} \) relative to \( V_{cd} \), and \( \delta \) is the phase difference between DCS and CF processes not attributed to the first-order electroweak spectator diagram.

We characterize the violation of \( CP \) in the mixing amplitude, the decay amplitude, and the interference between mixing and decay, by real-valued parameters \( A_M, A_D, \) and \( \phi \). We adopt a parametrization similar to that of Nir [8] and CLEO [GODANG 00] and express these quantities in a way that is convenient to describe the three types of \( CP \) violation:

\[
\left| \frac{q}{p} \right| = 1 + A_M, \quad (14)
\]

\[
\chi_f^{-1} \equiv \frac{pA_f}{qA_d} = -\sqrt{R_D}(1 + A_D) \frac{1}{(1 + A_M)} e^{-i(\delta + \phi)}, \quad (15)
\]

\[
\chi_f \equiv \frac{qA_d}{pA_f} = -\sqrt{R_D}(1 + A_M) \frac{1}{(1 + A_D)} e^{-i(\delta - \phi)}. \quad (16)
\]

In general, \( \chi_f \) and \( \chi_f^{-1} \) are independent complex numbers. To leading order,

\[
\begin{align*}
\tau(t) &= e^{-t} \times \left[ R_D(1 + A_D)^2 \\
+ \sqrt{R_D}(1 + A_M)(1 + A_D) y_+ t + \frac{(1 + A_M)^2 R_M t^2}{2} \right] \quad (17) \\
\end{align*}
\]

and

\[
\begin{align*}
\tau(t) &= e^{-t} \times \left[ \frac{R_D}{(1 + A_D)^2} \\
+ \frac{\sqrt{R_D}}{(1 + A_D)(1 + A_M)} y_+ t + \frac{R_M}{2(1 + A_M)^2} t^2 \right]. \quad (18)
\end{align*}
\]
Here
\[ y'_\pm \equiv y' \cos \phi \pm x' \sin \phi = y \cos(\delta \mp \phi) - x \sin(\delta \mp \phi) \]  
(19)

\[ y' \equiv y \cos \delta - x \sin \delta, \quad x' \equiv x \cos \delta + y \sin \delta, \]  
(20)

and \( R_M \) is the mixing rate relative to the time-integrated right-sign rate.

The three terms in Eq. (17) and Eq. (18) probe the three fundamental types of \( CP \) violation. In the limit of \( CP \) conservation, \( A_M, A_D, \) and \( \phi \) are all zero, and then

\[ r(t) = \tau(t) = e^{-t} \left( R_D + \sqrt{R_D} y't + \frac{1}{2} R_M t^2 \right), \]  
(21)

and the time-integrated wrong-sign rate relative to the integrated right-sign rate is

\[ R = \int_0^\infty r(t) \, dt = R_D + \sqrt{R_D} y' + R_M. \]  
(22)

The ratio \( R \) is the most readily accessible experimental quantity. Table 2 gives recent measurements of \( R \) in \( D^0 \to K^+\pi^- \) decay. The average of these results, \( R = (0.376 \pm 0.009) \% \), is about two standard deviations from the average of earlier, less precise results, \( R = (0.81 \pm 0.23) \% \), which we have omitted.

<table>
<thead>
<tr>
<th>Year</th>
<th>Exper.</th>
<th>Technique</th>
<th>( R \times 10^{-3} )</th>
<th>( A_D % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>Belle\textsuperscript{a}</td>
<td>( e^+e^- \to \Upsilon(4S) )</td>
<td>3.77 ± 0.08 ± 0.05</td>
<td>—</td>
</tr>
<tr>
<td>2005</td>
<td>FOCUS\textsuperscript{b}</td>
<td>( \gamma ) BeO</td>
<td>4.29 ± 0.63 ± 0.28 18.0 ± 14.0 ± 4.1</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>BABAR\textsuperscript{c}</td>
<td>( e^+e^- \to \Upsilon(4S) )</td>
<td>3.57 ± 0.22 ± 0.27 9.5 ± 6.1 ± 8.3</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>CLEO\textsuperscript{d}</td>
<td>( e^+e^- \to \Upsilon(4S) )</td>
<td>3.32\textsuperscript{+0.63}<em>{-0.65} ± 0.40 2\textsuperscript{+19}</em>{-20} ± 1</td>
<td></td>
</tr>
</tbody>
</table>

See the end of the \( D^0 \) listings for these references: \( \textsuperscript{a} \)ZHANG 06, \( \textsuperscript{b} \)LINK 05, \( \textsuperscript{c} \)AUBERT 03Z, \( \textsuperscript{d} \)GODANG 00.

The contributions to \( R \)—allowing for \( CP \) violation—can be extracted by fitting the \( D^0 \to K^+\pi^- \) and \( \bar{D}^0 \to K^-\pi^+ \) decay rates. Table 2 gives the constraints on \( A_D \) with \( x' = y' = 0 \). Table 3 summarizes the results for \( y' \) and \( x'^2/2 \). Figure 1 shows the two-dimensional allowed regions. No meaningful constraints on \( A_M \) and \( \phi \) have been reported.
Table 3: Results from studies of the time dependence \( r(t) \).

<table>
<thead>
<tr>
<th>Year</th>
<th>Exper.</th>
<th>( y' ) (95% C.L.)</th>
<th>( x'^2/2 ) (95% C.L.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>Belle(^a)</td>
<td>(-2.8 &lt; y' &lt; 2.1%)</td>
<td>(&lt; 0.036 %)</td>
</tr>
<tr>
<td>2005</td>
<td>FOCUS(^b)</td>
<td>(-11.2 &lt; y' &lt; 6.7%)</td>
<td>(&lt; 0.40 %)</td>
</tr>
<tr>
<td>2003</td>
<td>BABAR(^c)</td>
<td>(-5.6 &lt; y' &lt; 3.9%)</td>
<td>(&lt; 0.11 %)</td>
</tr>
<tr>
<td>2000</td>
<td>CLEO(^d)</td>
<td>(-5.8 &lt; y' &lt; 1.0%)</td>
<td>(&lt; 0.041 %)</td>
</tr>
</tbody>
</table>

See the end of the \( D^0 \) listings for these references: \(^a\)ZHANG 06, \(^b\)LINK 05, \(^c\)AUBERT 03Z, \(^d\)GODANG 00.

Extraction of the amplitudes \( x \) and \( y \) from the results in Table 3 requires knowledge of the relative strong phase \( \delta \), a subject of theoretical discussion \([4,9–11]\). In most cases, it appears difficult for theory to accommodate \( \delta > 25^\circ \), although the judicious placement of a \( K \pi \) resonance could allow \( \delta \) to be as large as \( 40^\circ \).

A quantum interference effect that provides useful sensitivity to \( \delta \) arises in the decay chain \( \psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (f_{cp})(K^+\pi^-) \), where \( f_{cp} \) denotes a \( CP \) eigenstate from \( D^0 \) decay, such as \( K^+K^- \)[1, 16]. Here, the amplitude triangle relation

\[
\sqrt{2} A(D^+_\pm \rightarrow K^-\pi^+) = A(D^0 \rightarrow K^-\pi^+) \pm A(D^0 \rightarrow K^-\pi^+),
\]

(23)

where \( D^\pm \) denotes a \( CP \) eigenstate, implies that

\[
\cos \delta = \frac{B(D^+_+ \rightarrow K^-\pi^+) - B(D^-_- \rightarrow K^-\pi^+)}{2\sqrt{R_D} B(D^0 \rightarrow K^-\pi^+)},
\]

(24)

neglecting \( CP \) violation and exploiting \( R_D \ll \sqrt{R_D} \).

The strong phase \( \delta \) might also be determined by constructing amplitude quadrangles from a complete set of branching fraction measurements of the other DCS \( D \) decays to two pseudoscalars [12]. This analysis would have to assume that the amplitudes from both \( \Delta I = 1 \) and \( \Delta I = 0 \) that populate the total \( I = 1/2 \) \( K \pi \) state have the same strong phase relative to the amplitude that populates the total \( I = 3/2 \) \( K \pi \) state.

The Dalitz-plot analyses of DCS \( D \) decays to a pseudoscalar and a vector allow the measurement of the relative strong phase
Figure 1: Allowed regions in the $x'y'$ plane. The allowed region for $y$ is the average of the results from E791$^a$, FOCUS$^b$, CLEO$^c$, BABAR$^d$, and Belle$^e$. Also shown is the limit from $D^0 \to K^{(*)}\ell\nu$ from Belle$^f$ and limits from $D \to K\pi$ from CLEO$^g$, BABAR$^h$, Belle$^i$ and FOCUS$^j$. The CLEO, BABAR and Belle results allow $CP$ violation in the decay and mixing amplitudes, and in the interference between these two processes. The FOCUS result does not allow $CP$ violation. We assume $\delta = 0$ to place the $y$ results. A non-zero $\delta$ would rotate the $D^0 \to CP$ eigenstates confidence region clockwise about the origin by $\delta$. All results are consistent with the absence of mixing. See the end of the $D^0$ listings for these references: $^a$AITALA 99E, $^b$LINK 00, $^c$CSORNA 02, $^d$AUBERT 03P, $^e$ABE 02I, $^f$BITENC 05, $^g$GODANG 00, $^h$AUBERT 03Z, $^i$ZHANG 06, $^j$LINK 05. See full-color version on color pages at end of book.
between some amplitudes, providing additional constraints to the amplitude quadrangle [13] and thus the determination of the strong phase difference between the relevant DCS and CF amplitudes. In $D^0 \to K_S^0 \pi^+\pi^-$, the DCS and CF decay amplitudes populate the same Dalitz plot, which allows direct measurement of the relative strong phase. CLEO has measured the relative phase between $D^0 \to K^*(892)^+\pi^-$ and $D^0 \to K^*(892)^-\pi^+$ to be $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$ [MURAMATSU 02], consistent with the $180^\circ$ expected from Cabibbo factors and a small strong phase.

There are several results for $R$ measured in multibody final states with nonzero strangeness. Here $R$, defined in Eq. (22), becomes an average over the Dalitz space, weighted by experimental efficiencies and acceptance. Table 4 summarizes the results.

**Table 4:** Results for $R$ in $D^0 \to K^{(*)+}\pi^-(n\pi)$.

<table>
<thead>
<tr>
<th>Year</th>
<th>Exper.</th>
<th>$D^0$ final state</th>
<th>$R$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>Belle$^a$</td>
<td>$K^+\pi^-\pi^+\pi^-$</td>
<td>$0.320 \pm 0.019^{+0.018}_{-0.013}$</td>
</tr>
<tr>
<td>2005</td>
<td>Belle$^a$</td>
<td>$K^+\pi^0\pi^0$</td>
<td>$0.229 \pm 0.017^{+0.013}_{-0.009}$</td>
</tr>
<tr>
<td>2002</td>
<td>CLEO$^b$</td>
<td>$K^{**}\pi^-$</td>
<td>$0.5 \pm 0.2^{+0.6}_{-0.1}$</td>
</tr>
<tr>
<td>2001</td>
<td>CLEO$^c$</td>
<td>$K^+\pi^-\pi^+\pi^-$</td>
<td>$0.41^{+0.12}_{-0.11} \pm 0.04$</td>
</tr>
<tr>
<td>2001</td>
<td>CLEO$^d$</td>
<td>$K^+\pi^0\pi^0$</td>
<td>$0.43^{+0.11}_{-0.10} \pm 0.07$</td>
</tr>
<tr>
<td>1998</td>
<td>E791$^e$</td>
<td>$K^+\pi^-\pi^+\pi^-$</td>
<td>$0.68^{+0.34}_{-0.33} \pm 0.07$</td>
</tr>
</tbody>
</table>

See the end of the $D^0$ listings for these references: $^a$TIAN 05, $^b$MURAMATSU 02, $^c$DYTMAN 01, $^d$BRANDENBURG 01, $^e$AITALA 98.

For multibody final states, Eqs. (13)–(22) apply to one point in the Dalitz space. Although $x$ and $y$ do not vary across the space, knowledge of the resonant substructure is needed to extrapolate the strong phase difference $\delta$ from point to point. Both the sign and magnitude of $x$ and $y$ may be measured using the time-dependent resonant substructure of multibody $D^0$ decays. CLEO has performed a time-dependent Dalitz-plot analysis of $D^0 \to K^0_S\pi^+\pi^-$, and reports $(-4.5 < x < 9.3)\%$ and
$(-6.4 < y < 3.6)\%$ at the 95% confidence level, without phase or sign ambiguity [ASNER 05], as shown in Figure 2.

**Figure 2:** Allowed regions in the $xy$ plane. No assumption is made regarding $\delta$. The allowed region for $y$ is the average of the results from E791$^a$, FOCUS$^b$, CLEO$^c$, BABAR$^d$, and Belle$^e$. Also shown is the limit from $D^0 \rightarrow K^{(*)} \ell \nu$ from Belle$^f$. The CLEO experiment has constrained $x$ and $y$ with the time-dependent Dalitz-plot analysis of $D^0 \rightarrow KS^{0}\pi^{+}\pi^{-}$, $g$. All results are consistent with the absence of mixing. See the end of the $D^0$ listings for these references: $^a$AITALA 99E, $^b$LINK 00, $^c$CSORNA 02, $^d$AUBERT 03P, $^e$ABE 02I, $^f$BITENC 05, $^g$ASNER 05.
Decays to CP Eigenstates: When the final state $f$ is a CP eigenstate, there is no distinction between $f$ and $\bar{f}$, and then $A_f = A_{\bar{f}}$ and $\bar{A}_f = \bar{A}_{\bar{f}}$. We denote final states with CP eigenvalues $\pm 1$ by $f_\pm$. In analogy with Eqs. (7)–(8), the decay rates to CP eigenstates are then

$$r_\pm(t) = \frac{|\langle f_\pm | H | D^0(t) \rangle|^2}{|A_\pm|^2}$$

$$= \frac{1}{4} |h_\pm(t)\left(\frac{A_\pm}{A_\pm} \mp \frac{q}{p}\right) + h_\mp(t)\left(\frac{A_\mp}{A_\pm} \mp \frac{q}{p}\right)|^2,$$

$$\propto \frac{1}{|p|^2} |h_\pm(t) + \eta_\pm h_\mp(t)|^2,$$  \hspace{1cm} (25)

and

$$\tau_\pm(t) = \frac{|\langle f_\pm | H | D^0(t) \rangle|^2}{|A_\pm|^2} \propto \frac{1}{|q|^2} |h_\pm(t) - \eta_\pm h_\mp(t)|^2,$$  \hspace{1cm} (26)

where

$$h_\pm(t) = g_+(t) \pm g_-(t) = e^{-iz_\pm t},$$  \hspace{1cm} (27)

and

$$\eta_\pm \equiv \frac{pA_\pm \mp qa_\pm}{pA_\pm \mp qa_\pm} = \frac{1 \mp \chi_\pm}{1 \pm \chi_\pm}.$$  \hspace{1cm} (28)

The variable $\eta_\pm$ describes CP violation; it can receive contributions from each of the three fundamental types of CP violation.

The quantity $y$ may be measured by comparing the rate for decays to non-CP eigenstates such as $D^0 \to K^-\pi^+$ with decays to CP eigenstates such as $D^0 \to K^+K^-$ [11]. A positive $y$ would make $K^+K^-$ decays appear to have a shorter lifetime than $K^-\pi^+$ decays. The decay rate for a $D^0$ into a CP eigenstate is not described by a single exponential in the presence of CP violation.

In the limit of weak mixing, where $|ix + y| \ll 1$, and small CP violation, where $|A_M|$, $|A_D|$, and $|\sin \phi| \ll 1$, the time
dependence of decays to $CP$ eigenstates is proportional to a single exponential:

$$r_{\pm}(t) \propto \exp\left(-[1 \pm \frac{p}{q}] (y \cos \phi - x \sin \phi) t\right),$$  \hspace{1cm} (29)

$$\bar{r}_{\pm}(t) \propto \exp\left(-[1 \pm \frac{q}{p}] (y \cos \phi + x \sin \phi) t\right),$$  \hspace{1cm} (30)

$$r_{\pm}(t) + \bar{r}_{\pm}(t) \propto e^{-(1 \pm y_{CP})t}. \hspace{1cm} (31)$$

Here

$$y_{CP} = y \cos \phi \left[ \frac{1}{2} \left( \left| \frac{p}{q} \right| + \left| \frac{q}{p} \right| \right) + \frac{A_{\text{prod}}}{2} \left( \left| \frac{p}{q} \right| - \left| \frac{q}{p} \right| \right) \right]$$

$$- x \sin \phi \left[ \frac{1}{2} \left( \left| \frac{p}{q} \right| - \left| \frac{q}{p} \right| \right) + \frac{A_{\text{prod}}}{2} \left( \left| \frac{p}{q} \right| + \left| \frac{q}{p} \right| \right) \right],$$  \hspace{1cm} (32)

and

$$A_{\text{prod}} \equiv \frac{N(D^0) - N(D^0)}{N(D^0) + N(D^0)}$$  \hspace{1cm} (33)

is defined as the production asymmetry of the $D^0$ and $D^0$.

The possibility of $CP$ violation has been considered in the limit of weak mixing and small $CP$ violation. In this limit there is no sensitivity to $CP$ violation in direct decay. Belle [14] and BABAR [AUBERT 03P] have measure $A_{\Gamma}$, where

$$A_{\Gamma} \equiv \frac{r_{\pm}(t) - \bar{r}_{\pm}(t)}{r_{\pm}(t) + \bar{r}_{\pm}(t)} \approx A_M \cos \phi - x \sin \phi,$$

allowing $CP$ violation in interference and mixing.

In the limit of $CP$ conservation, $A_{\pm} = \pm A_{\bar{\pm}}$, $\eta_{\pm} = 0$, $y = y_{CP}$, and

$$r_{\pm}(t) \left| A_{\pm} \right|^2 = \bar{r}_{\pm}(t) \left| A_{\pm} \right|^2 \propto e^{-(1 \pm y_{CP})t}. \hspace{1cm} (34)$$

All measurements of $y$ and $A_{\Gamma}$ are relative to the $D^0 \to K^- \pi^+$ decay rate. Table 5 summarizes the current status of measurements. The average of the six $y_{CP}$ measurements is $0.90 \pm 0.42\%$. 

July 27, 2006 11:28
Table 5: Results for $y$ from $D^0 \rightarrow K^+K^-$ and $\pi^+\pi^-$. 

<table>
<thead>
<tr>
<th>Year</th>
<th>Exper.</th>
<th>$D^0$ final state(s)</th>
<th>$y_{CP}$($%$)</th>
<th>$A_{\Gamma} (\times 10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>Belle [14]</td>
<td>$K^+K^-$</td>
<td>$1.15 \pm 0.69 \pm 0.38$</td>
<td>$-2.0 \pm 6.3 \pm 3.0$</td>
</tr>
<tr>
<td>2003</td>
<td>BABAR$^a$</td>
<td>$K^+K^-, \pi^+\pi^-$</td>
<td>$0.8 \pm 0.4^{+0.5}_{-0.4}$</td>
<td>$-8 \pm 6 \pm 2$</td>
</tr>
<tr>
<td>2001</td>
<td>CLEO$^b$</td>
<td>$K^+K^-, \pi^+\pi^-$</td>
<td>$-1.1 \pm 2.5 \pm 1.4$</td>
<td>$-$</td>
</tr>
<tr>
<td>2001</td>
<td>Belle$^c$</td>
<td>$K^+K^-$</td>
<td>$-0.5 \pm 1.0^{+0.7}_{-0.8}$</td>
<td>$-$</td>
</tr>
<tr>
<td>2000</td>
<td>FOCUS$^d$</td>
<td>$K^+K^-$</td>
<td>$3.4 \pm 1.4 \pm 0.7$</td>
<td>$-$</td>
</tr>
<tr>
<td>1999</td>
<td>E791$^e$</td>
<td>$K^+K^-$</td>
<td>$0.8 \pm 2.9 \pm 1.0$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

See the end of the $D^0$ listings for these references: $^a$AUBERT 03P, $^b$CSORNA 02, $^c$ABE 02I, $^d$LINK 00, $^e$AITALA 99E.

Substantial work on the integrated $CP$ asymmetries in decays to $CP$ eigenstates indicates that $A_{CP}$ is consistent with zero at the few percent level [15]. The expression for the integrated $CP$ asymmetry that includes the possibility of $CP$ violation in mixing is

$$A_{CP} = \frac{\Gamma(D^0 \rightarrow f^{\pm}) - \Gamma(\overline{D}^0 \rightarrow f^{\pm})}{\Gamma(D^0 \rightarrow f^{\pm}) + \Gamma(\overline{D}^0 \rightarrow f^{\pm})} \tag{35}$$

$$= \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} + 2\text{Re}(\eta^{\pm}). \tag{36}$$

**Coherent $D^0\overline{D}^0$ Analyses:** Measurements of $R_D$, $\cos \delta$, $x$, and $y$ can be made simultaneously in a combined fit to the single-tag (ST) and double-tag (DT) yields or individually by a series of “targeted” analyses [16, 17].

The “comprehensive” analysis simultaneously measures mixing and DCS parameters by examining various ST and DT rates. Due to quantum correlations in the $C = -1$ and $C = +1$ $D^0\overline{D}^0$ pairs produced in the reactions $e^+e^- \rightarrow D^0\overline{D}^0(\pi^0)$ and $e^+e^- \rightarrow D^0\overline{D}^0\gamma(\pi^0)$, respectively, the time-integrated $D^0\overline{D}^0$ decay rates are sensitive to interference between amplitudes for indistinguishable final states. The size of this interference is governed by the relevant amplitude ratios and can include contributions from $D^0-\overline{D}^0$ mixing.
Table 6: CLEO-c results from time-integrated yields at $\psi(3770) \rightarrow D\bar{D}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CLEO-c fitted value</th>
<th>Other results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (Table 5)</td>
<td>$-0.058 \pm 0.066$</td>
<td>$(0.90 \pm 0.42)%$</td>
</tr>
<tr>
<td>$\cos \delta_{K\pi}$</td>
<td>$1.09 \pm 0.66$</td>
<td></td>
</tr>
<tr>
<td>$R_M$ (Table 1)</td>
<td>$(1.7 \pm 1.5) \times 10^{-3}$</td>
<td>$&lt;0.1%$ (95% C.L.)</td>
</tr>
<tr>
<td>$x^2/2$ (Table 3)</td>
<td>&lt;0.44% @ (95% C.L.)</td>
<td>&lt;0.036% (95% C.L.)</td>
</tr>
</tbody>
</table>

The following categories of final states are considered:

- **$f$ or $\bar{f}$**: Hadronic states accessed from either $D^0$ or $\bar{D}^0$ decay but that are not $CP$ eigenstates. An example is $K^-\pi^+$, which results from Cabibbo-favored $D^0$ transitions or DCS $\bar{D}^0$ transitions.

- **$\ell^+$ or $\ell^-$**: Semileptonic or purely leptonic final states, which, in the absence of mixing, tag unambiguously the flavor of the parent $D$.

- **$S_+$ or $S_-$**: $CP$-even and $CP$-odd eigenstates, respectively.

The decay rates for $D^0\bar{D}^0$ pairs to all possible combinations of the above categories of final states are calculated in Ref. [4], for both $C = -1$ and $C = +1$, reproducing the work of Refs. [5, 10]. Such $D^0\bar{D}^0$ combinations, where both $D$ final states are specified, are double tags. In addition, the rates for single tags, where either the $D^0$ or $\bar{D}^0$ is identified and the other neutral $D$ decays generically are given in Ref. [4].

CLEO-c has reported results using 281 pb$^{-1}$ of $e^+e^- \rightarrow \psi(3770)$ data [18], where the quantum coherent $D^0\bar{D}^0$ pairs are in the $C = -1$ state. The values of $y$, $R_M$, and $\cos \delta$ are determined from a combined fit to the ST (hadronic only) and DT yields. The hadronic final states included in the analysis are $K^-\pi^+ (f)$, $K^+\pi^- (\bar{f})$, $K^-K^+(S_+)$, $\pi^+\pi^- (S_+)$, $K_S^0\pi^0\pi^0 (S_+)$, and $K_S^0\pi^0 (S_-)$. Both of the two flavored final states, $K^-\pi^+$ and $K^+\pi^-$, can be reached via CF or DCS transitions.

Semileptonic DT yields are also included, where one $D$ is fully reconstructed in one of the hadronic modes listed above, and the other $D$ is partially reconstructed, requiring that only the electron be found. When the electron is accompanied by a flavor tag ($D \rightarrow K^-\pi^+$ or $K^+\pi^-$), only the “right-sign” DT
sample, where the electron and kaon charges are the same, is used. Extraction of the DCS “wrong-sign” semileptonic yield is not feasible with the current CLEO-c data sample, and the parameter $r_{K\pi}$ is constrained to the world average. Table 6 shows the results of the fit to the CLEO-c data.

References

6. See the review on $B^0-\overline{B^0}$ mixing in this Review.


15. See the tabulation of $A_{CP}$ results in the decays of $D^0$ and $D^+$ in this Review.


18. W. Sun, for the CLEO Collaboration, To appear in the proceedings of Particles and Nuclei International Conference (PANIC 05), Santa Fe, New Mexico, 24-28 Oct 2005, hep-ex/0603031.