$\pi^\pm \to \ell^\pm\nu\gamma$ AND $K^\pm \to \ell^\pm\nu\gamma$ FORM FACTORS

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In the radiative decays $\pi^\pm \to \ell^\pm\nu\gamma$ and $K^\pm \to \ell^\pm\nu\gamma$, where ℓ is an e or a μ and γ is a real or virtual photon (e^+e^- pair), both the vector and the axial-vector weak hadronic currents contribute to the decay amplitude. Each current gives a structure-dependent term (SD_V and SD_A) from virtual hadronic states, and the axial-vector current also gives a contribution from inner bremsstrahlung (IB) from the lepton and meson. The IB amplitudes are determined by the meson decay constants f_π and f_K [1]. The SD_V and SD_A amplitudes are parameterized in terms of the vector form factor F_V and the axial-vector form factors F_A and R [1–4]:

$$M(\mathrm{SD}_V) = \frac{-eG_F\,V_{qq'}}{\sqrt{2}\,m_P}\epsilon^\mu\,\ell^\nu\,F_V\,\epsilon_{\mu\nu\sigma\tau}\,k^\sigma\,q^\tau\ ,$$

$$M(\mathrm{SD}_A) = \frac{-ie\,G_F V_{qq'}}{\sqrt{2}\,m_P}\,\epsilon^\mu\,\ell^\nu\,\{F_A\,[(s-t)g_{\mu\nu} - q_\mu\,k_\nu] + R\,t\,g_{\mu\nu}\}\ . \tag{1}$$

Here $V_{qq'}$ is the Cabibbo-Kobayashi-Maskawa mixing-matrix element; ϵ^μ is the polarization vector of the photon (or the effective vertex, $\epsilon^\mu = (e/t)\overline{u}(p_-)\gamma^\mu v(p_+)$, of the e^+e^- pair); $\ell^\nu = \overline{u}(p_\nu)\gamma^\nu(1-\gamma_5)v(p_\ell)$ is the lepton-neutrino current; q and k are the meson and photon four-momenta, with $s = q\cdot k$ and $t = k^2 (= (p_+ + p_-)^2)$; and P stands for π or K. In the analysis of data, the s and t dependence of the form factors is neglected, which is a good approximation for pions [2] but not for kaons [4]. The pion vector form factor F_V^π is related via CVC to the π^0 lifetime, $|F_V^\pi| = (1/\alpha)\sqrt{2\Gamma_{\pi^0}/\pi m_{\pi^0}}$ [1]. PCAC relates R to the electromagnetic radius of the meson [2,4], $R^P = \frac{1}{3}m_P f_P \langle r_P^2\rangle$. The calculation of the other form factors, F_A^π, F_V^K, and F_A^K, is model dependent [1,4].

When the photon is real, the partial decay rate can be given analytically [1,5]:

$$\frac{d^2\Gamma_{P\to\ell\nu\gamma}}{dxdy} = \frac{d^2\left(\Gamma_{\mathrm{IB}} + \Gamma_{\mathrm{SD}} + \Gamma_{\mathrm{INT}}\right)}{dxdy}\ , \tag{2}$$

where $\Gamma_{\rm IB}, \Gamma_{\rm SD}$, and $\Gamma_{\rm INT}$ are the contributions from inner bremsstrahlung, structure-dependent radiation, and their interference, and the $\Gamma_{\rm SD}$ term is given by

$$\frac{d^2\Gamma_{\rm SD}}{dxdy} = \frac{\alpha}{8\pi}\Gamma_{P\to\ell\nu}\,\frac{1}{r(1-r)^2}\left(\frac{m_P}{f_P}\right)^2 \times\left[(F_V+F_A)^2\,{\rm SD}^+ + (F_V-F_A)^2\,{\rm SD}^-\right] \ . \quad (3)$$

Here

$$\begin{aligned} {\rm SD}^+ &= (x+y-1-r)\left[(x+y-1)(1-x)-r\right] \ , \\ {\rm SD}^- &= (1-y+r)\left[(1-x)(1-y)+r\right] \ , \end{aligned} \quad (4)$$

where $x = 2E_\gamma/m_P$, $y = 2E_\ell/m_P$, and $r = (m_\ell/m_P)^2$.

In $\pi^\pm \to e^\pm\nu\gamma$ and $K^\pm \to e^\pm\nu\gamma$ decays, the interference terms are small, and thus only the absolute values $|F_A+F_V|$ and $|F_A-F_V|$ can be obtained. In $K^\pm \to \mu^\pm\nu\gamma$ decay, the interference term is important, and thus the signs of F_V and F_A can be obtained. In $\pi^\pm \to \mu^\pm\nu\gamma$ decay, bremsstrahlung completely dominates. In $\pi^\pm \to e^\pm\nu e^+e^-$ and $K^\pm \to \ell^\pm\nu e^+e^-$ decays, all three form factors, F_V, F_A, and R, can be determined.

We give the $\pi^\pm$ form factors F_V, F_A, and R in the Listings below. In the $K^\pm$ Listings, we give the sum F_A+F_V and difference F_A-F_V.

The electroweak decays of the pseudoscalar mesons are investigated to learn something about the unknown hadronic structure of these mesons, assuming a standard $V-A$ structure of the weak leptonic current. The experiments are quite difficult, and it is not meaningful to analyse the results using parameters for both the hadronic structure (decay constants, form factors) and the leptonic weak current (*e.g.*, to add pseudoscalar or tensor couplings to the $V-A$ coupling). Deviations from the $V-A$ interactions are much better studied in purely leptonic systems such as muon decay.

References

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