POLARIZATION IN B DECAYS

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We review the notation used in polarization measurements of B decays and discuss CP-violating observables in polarization measurements. We look at several examples of vector-vector and vector-tensor B meson decays, while more details about the theory and experimental results in B decays can be found in a separate mini-review [1] in this *Review.*

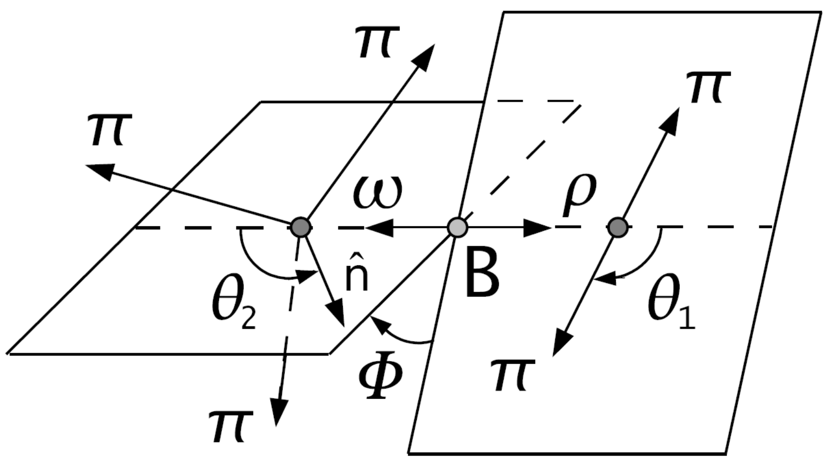

Figure 1: Definition of the helicity angles in the decay $B \to \rho\,\omega$ for the two-body (θ_1) and three-body (θ_2) B-daughter decays, where both angles are defined in the rest-frame of the decaying meson. The normal to the three-body decay plane ($\hat{n}$), and the daughter direction in the two-body decay, serve as analyzers of the polarization.

The angular distribution of the B meson decay to two mesons with non-zero spin is of special interest because it is sensitive to quark-spin alignment in decay transition, and reflects both weak- and strong-interaction dynamics. The angular distribution of decay products can be expressed as a function of three helicity angles which describe the alignment of the particles in the decay chain. The analyzer of the B-daughter polarization is normally chosen for two-body decays, as the direction of the daughters in the center-of-mass of the parent (*e.g.*, $\rho \to 2\pi$) [2], and for three-body decays as the normal to the decay plane [3] (see Fig. 1). An equivalent set of transversity angles is sometimes used in polarization analyses [4]. The differential decay width depends on complex amplitudes A_λ, corresponding to the B-daughter helicity states λ.

Most B-decay polarization analyses are limited to the case when the spin of one of the B-meson daughters is 1. In that case, there are only three independent amplitudes corresponding to $\lambda = 0$ or ± 1 [5], where the last two can be expressed in terms of parity-even and parity-odd amplitudes $A_{\parallel,\perp} = (A_{+1} \pm A_{-1})/\sqrt{2}$. The overall decay amplitude would involve three complex terms proportional to the above amplitudes and the d functions of helicity angles. The exact angular dependence would depend on the quantum numbers of the B-meson daughters and of their decay products, and can be found in the literature [5,6]. The differential decay rate would involve six real quantities α_i, including interference terms,

$$\frac{d\Gamma}{\Gamma\, d\cos\theta_1\, d\cos\theta_2\, d\mathit{\Phi}} = \sum_i \alpha_i \;\; f_i\left(\cos\theta_1,\, \cos\theta_2,\, \mathit{\Phi}\right), \tag{1}$$

where each $f_i\left(\cos\theta_1,\, \cos\theta_2,\, \mathit{\Phi}\right)$ has unique angular dependence specific to particle quantum numbers, and the α_i parameters are defined as:

$$\alpha_1 = \frac{|A_0|^2}{\mathit{\Sigma}|A_\lambda|^2} = f_L\,, \tag{2}$$

$$\alpha_2 = \frac{|A_\parallel|^2 + |A_\perp|^2}{\mathit{\Sigma}|A_\lambda|^2} = (1 - f_L)\,, \tag{3}$$

$$\alpha_3 = \frac{|A_\parallel|^2 - |A_\perp|^2}{\mathit{\Sigma}|A_\lambda|^2} = (1 - f_L - 2\, f_\perp)\,, \tag{4}$$

$$\alpha_4 = \frac{\Im m(A_\perp A_\parallel^*)}{\mathit{\Sigma}|A_\lambda|^2} = \sqrt{f_\perp (1 - f_L - f_\perp)}\, \sin(\phi_\perp - \phi_\parallel)\,, \tag{5}$$

$$\alpha_5 = \frac{\Re e(A_\parallel A_0^*)}{\mathit{\Sigma}|A_\lambda|^2} = \sqrt{f_L\,(1 - f_L - f_\perp)}\, \cos(\phi_\parallel)\,, \tag{6}$$

$$\alpha_6 = \frac{\Im m(A_\perp A_0^*)}{\mathit{\Sigma}|A_\lambda|^2} = \sqrt{f_\perp\, f_L}\, \sin(\phi_\perp)\,, \tag{7}$$

where the amplitudes have been expressed with the help of polarization parameters f_L, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$ defined in Table 1. Note that the terms proportional to $\Re e(A_\perp A_\parallel^*)$, $\Im m(A_\parallel A_0^*)$, and $\Re e(A_\perp A_0^*)$ are absent in Eqs. (2-7). However, these terms may appear in some cases of the three-body decay of a B-meson daughter, see Ref. 6.

Table 1: Rate, polarization, and CP-asymmetry parameters defined for the B-meson decays to mesons with non-zero spin. Numerical examples are shown for the $B^0 \to \varphi K^*(892)^0$ decay. The first six parameters are defined under the assumption of no CP violation in decay, while they are averaged between the $\overline{B}$ and B parameters in general. The last six parameters involve differences between the $\overline{B}$ and B meson decay parameters. The phase convention δ_0 is chosen with respect to a single A_{00} amplitude from a reference B decay mode, which is $B^0 \to \varphi K_0^*(1430)^0$ for numerical results.

parameter	definition	average
$\mathcal{B}$	$\Gamma/\Gamma_{\rm total}$	$(9.5 \pm 0.8) \times 10^{-6}$
f_L	$\lvert A_0\rvert^2/\Sigma\lvert A_\lambda\rvert^2$	0.484 ± 0.033
$f_\perp$	$\lvert A_\perp\rvert^2/\Sigma\lvert A_\lambda\rvert^2$	0.26 ± 0.04
$\phi_\parallel - \pi$	$\arg(A_\parallel/A_0) - \pi$	-0.81 ± 0.14
$\phi_\perp - \pi$	$\arg(A_\perp/A_0) - \pi$	-0.81 ± 0.14
$\delta_0 - \pi$	$\arg(A_{00}/A_0) - \pi$	-0.36 ± 0.19
A_{CP}	$(\bar{\Gamma} - \Gamma)/(\bar{\Gamma} + \Gamma)$	-0.01 ± 0.06
A_{CP}^0	$(\bar{f}_L - f_L)/(\bar{f}_L + f_L)$	$+0.02 \pm 0.07$
$A_{CP}^\perp$	$(\bar{f}_\perp - f_\perp)/(\bar{f}_\perp + f_\perp)$	-0.11 ± 0.12
$\Delta\phi_\parallel$	$(\bar{\phi}_\parallel - \phi_\parallel)/2$	$+0.10 \pm 0.24$
$\Delta\phi_\perp$	$(\bar{\phi}_\perp - \phi_\perp - \pi)/2$	$+0.04 \pm 0.23$
$\Delta\delta_0$	$(\bar{\delta}_0 - \delta_0)/2$	$+0.21 \pm 0.19$

Overall, six real parameters describe three complex amplitudes A_0, $A_\parallel$, and $A_\perp$. These could be chosen to be the four polarization parameters f_L, $f_\perp$, $\phi_\parallel$, and $\phi_\perp$, one overall size normalization, such as decay rate Γ, or branching fraction $\mathcal{B}$, and one overall phase δ_0. The phase convention is arbitrary for an isolated B decay mode. However, for several B decays, the relative phase could produce meaningful and observable effects through interference with other B decays with the same final states, such as for $B \to VK^*_J$ with $J = 0, 1, 2, 3, 4, \ldots$ The phase could be referenced to the single $B \to VK^*_0$ amplitude A_{00} in such a case, as shown in Table 1. Here V stands for any spin-one vector meson.

Moreover, *CP* violation can be tested in the angular distribution of the decay as the difference between the B and $\overline{B}$. Each of the six real parameters describing the three complex amplitudes would have a counterpart *CP*-asymmetry term, corresponding to three direct-*CP* asymmetries in three amplitudes, and three *CP*-violating phase differences, equivalent to the phase measurements from the mixing-induced *CP* asymmetries in the time evolution of the B-decays [1]. In Table 1 and Ref. 7, these are chosen to be the direct-*CP* asymmetries in the overall decay rate $\mathcal{A}_{CP}$, in the f_L fraction $\mathcal{A}^0_{CP}$, and in the $f_\perp$ fraction $\mathcal{A}^\perp_{CP}$, and three weak phase differences:

$$\Delta\phi_\parallel = \frac{1}{2}\arg(\bar{A}_\parallel A_0/A_\parallel \bar{A}_0)\,, \tag{8}$$

$$\Delta\phi_\perp = \frac{1}{2}\arg(\bar{A}_\perp A_0/A_\perp \bar{A}_0) - \frac{\pi}{2}\,, \tag{9}$$

$$\Delta\delta_0 = \frac{1}{2}\arg(\bar{A}_{00} A_0/A_{00} \bar{A}_0)\,. \tag{10}$$

The $\frac{\pi}{2}$ term in Eq. (9) reflects the fact that $A_\perp$ and $\bar{A}_\perp$ differ in phase by π if *CP* is conserved. The two parameters $\Delta\phi_\parallel$ and $\Delta\phi_\perp$ are equivalent to triple-product asymmetries constructed from the vectors describing the decay angular distribution [8]. The *CP*-violating phase difference in the reference decay mode is, in the Wolfenstein CKM quark-mixing phase convention,

$$\Delta\phi_{00} = \frac{1}{2}\arg(A_{00}/\bar{A}_{00})\,. \tag{11}$$

This can be measured only together with the mixing-induced phase difference for some of the neutral B-meson decays similar to other mixing-induced *CP* asymmetry measurements [1].

It may not always be possible to have a phase-reference decay mode which would define δ_0 and $\Delta\delta_0$ parameters. In that case, it may be possible to define the phase difference directly similarly to Eq. (11):

$$\Delta\phi_0 = \frac{1}{2}\arg(A_0/\bar{A}_0)\,. \tag{12}$$

One can measure the angles of the CKM unitarity triangle, assuming Standard Model contributions to the $\Delta\phi_0$ and B-mixing phases. Examples include measurements of $\beta = \phi_1$ with $B \to J/\psi K^*$ and $\alpha = \phi_2$ with $B \to \rho\rho$.

Most of the B decays that arise from tree-level $b \to c$ transitions have the amplitude hierarchy $|A_0| > |A_+| > |A_-|$ which is expected from analyses based on quark-helicity conservation [9]. The larger the mass of the vector-meson daughters, the weaker the inequality. The B meson decays to heavy vector particles with charm, such as $B \to J/\psi K^*$, $\psi(2S)K^*$, $\chi_{c1}K^*$, $D^*\rho$, D^*K^*, D^*D^*, and $D^*D_s^*$, show a substantial fraction of the amplitudes corresponding to transverse polarization of the vector mesons ($A_{\pm 1}$), in agreement with the factorization prediction. The detailed amplitude analysis of the $B \to J/\psi K^*$ decays has been performed by the BABAR [10], Belle [11], CDF [12], and CLEO [13] collaborations. Most analyses are performed under the assumption of the absence of direct CP violation. The parameter values are given in the particle listing of this *Review*. The difference between the strong phases $\phi_\parallel$ and $\phi_\perp$ deviates significantly from zero. The recent measurements [10,11] of CP-violating terms similar to those in $B \to \varphi K^*$ [7] shown in Table 1 are consistent with zero.

In addition, the mixing-induced CP-violating asymmetry is measured in the $B^0 \to J/\psi K^{*0}$ decay [1,10,11] where angular analysis allows one to separate CP-eigenstate amplitudes. This allows one to resolve the sign ambiguity of the $\cos 2\beta = \cos 2\phi_1$ term that appears in the time-dependent angular distribution due to interference of parity-even and parity-odd terms. This analysis relies on the knowledge of discrete ambiguities in the strong phases $\phi_\parallel$ and $\phi_\perp$, as discussed below. The BABAR experiment used a novel method based on the dependence on the $K\pi$ invariant mass of the interference between the S- and P-waves to resolve the discrete ambiguity in the determination of the strong phases $(\phi_\parallel, \phi_\perp)$ in $B \to J/\psi K^*$ decays [10]. The result is in agreement with the amplitude hierarchy expectation [9]. The CDF [12] and D0 [14] experiments have studied the $B_s^0 \to J/\psi\varphi$ decay and provided the lifetime, polarization, and phase measurements.

The amplitude hierarchy $|A_0| \gg |A_+| \gg |A_-|$ was expected in the B decays to light vector particles in both the penguin transition [15,16] and the tree-level transition [9]. There is confirmation by BABAR and BELLE experiments of predominantly longitudinal polarization in the tree-level $b \to u$ transition, such as $B^0 \to \rho^+\rho^-$ [17], $B^+ \to \rho^0\rho^+$ [18], and $B^+ \to \omega\rho^+$ [19], which is consistent with the analysis of the quark helicity conservation [9]. Because the longitudinal amplitude dominates the decay, a detailed amplitude analysis is not possible with current B samples, and limits on the transverse amplitude fraction are obtained. Only limits have been set on the $B^0 \to \omega\rho^0, \omega\omega$ [19] and evidence found for $B^0 \to \rho^0\rho^0$ [20] decays, still indicating that $b \to d$ penguin pollution is small in the charmless, strangeless vector-vector B decays.

The interest in the polarization and CP-asymmetry measurements in penguin transition, such as $b \to s$ decays $B \to \varphi K^*$, ρK^*, ωK^*, or $B_s^0 \to \varphi\varphi$, and $b \to d$ decay $B \to K^*\bar{K}^*$, is mainly motivated by their potential sensitivity to physics beyond the Standard Model. The decay amplitudes for $B \to \varphi K^*$ have been measured by the BABAR and Belle experiments [7,21,22]. The fractions of longitudinal polarization $f_L = 0.50 \pm 0.05$ for the $B^+ \to \varphi K^{*+}$ decay, and $f_L = 0.484 \pm 0.033$ for the $B^0 \to \varphi K^{*0}$ decay, indicate significant departure from the naive expectation of predominant longitudinal polarization, and suggest other contributions to the decay amplitude, previously neglected, either within the Standard Model, such as penguin annihilation [24] or QCD rescattering [25], or from physics beyond the Standard Model [26]. The complete set of twelve amplitude parameters measured in the $B^0 \to \varphi K^{*0}$ decay are given in Table 1. Several other parameters could be constructed from the above twelve parameters, as suggested in Ref. 27.

The discrete ambiguity in the phase $(\phi_\parallel, \phi_\perp, \Delta\phi_\parallel, \Delta\phi_\perp)$ measurements has been resolved by BABAR in favor of $|A_+| \gg |A_-|$ through interference between the S- and P-waves of $K\pi$. The search for vector-tensor $B \to \varphi K_J^*$ decays with $J = 2, 3, 4$ revealed a large fraction of longitudinal polarization in the decay $B \to \varphi K_2^*(1430)$ with $f_L = 0.85 \pm 0.08$ [7,28].

Like $B \to \varphi K^*$, the decays $B \to \rho K^*$ and $B \to \omega K^*$ may be sensitive to New Physics. Measurements of the longitudinal polarization fraction in $B^+ \to \rho^0 K^{*0}$ and $B^+ \to \rho^+ K^{*0}$ [29] reveal a polarization anomaly similar to $B \to \varphi K^*$. At the same time, first measurement of the polarization in the $b \to d$ penguin decay $B^0 \to K^{*0}\bar{K}^{*0}$ indicates a large fraction of longitudinal polarization $f_L = 0.81^{+0.12}_{-0.13}$ [30]. There is also evidence for the $B^0_s \to \varphi\varphi$ decay [31].

The three-body smileptonic B-meson decays, such as $B \to V l_1 l_2$, share many features with the two-body $B \to VV$ decays. Their differential decay width can be parameterized with the two helicity angles defined in the V and $(l_1 l_2)$ frames and with the azimuthal angle, as defined in Fig. 1. However, since the $(l_1 l_2)$ pair does not come from an on-shell particle, the angular distribution is unique to each point in the dilepton mass m_{ll} spectrum. The polarization measurements as a function of m_{ll} provide complementary information on physics beyond the Standard Model, as discussed for $B \to K^* l^+ l^-$ decay in Ref. 32, though the current data in this mode [33] are not yet sufficient for precise tests.

The examples of the angular distributions and observables in $B \to K^* l^+ l^-$ are discussed in Ref. 32. With the present statistics only two angular observables have been measured in this decay when integrated over certain ranges of the dilepton mass m_{ll} [33]. One parameter is the fraction of longitudinal polarization F_L, which is determined by the K^* angular distribution and is similar to f_L defined for exclusive two-body decays. The other parameter is the forward-backward asymmetry of the lepton pair A_{FB}, which is the asymmetry of the decay rate with positive and negative values of $\cos\theta_1$.

In summary, there has been considerable recent interest in the polarization measurements of B-meson decays because they reveal both weak- and strong-interaction dynamics [24–26,34]. New measurements will further elucidate the pattern of spin alignment measurements in rare B decays, and further test the Standard Model and strong interaction dynamics, including the non-factorizable contributions to the B-decay amplitudes.

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