

## REVIEW OF D-MESON DALITZ PLOT ANALYSES

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The formalism of Dalitz-Plot analysis is reviewed in the preceding note. Recent studies of multi-body decays of charm mesons probe a variety of physics including  $\gamma/\phi_3$ ,  $D^0 - \bar{D}^0$  mixing, searches for  $CP$  violation, doubly Cabibbo-suppressed decays, and properties of S-wave  $\pi\pi$ ,  $K\pi$ , and  $KK$  resonances. In the following, we discuss: (1)  $D^0 \rightarrow K_S^0\pi^+\pi^-$ ; (2) doubly Cabibbo-suppressed decays; and (3)  $CP$  violation.

**$D^0 \rightarrow K_S^0\pi^+\pi^-$ :** Several experiments have analyzed  $D^0 \rightarrow K_S^0\pi^+\pi^-$  decay. A CLEO analysis [1] of this process included ten resonances:  $K_S^0\rho^0$ ,  $K_S^0\omega$ ,  $K_S^0f_0(980)$ ,  $K_S^0f_2(1270)$ ,  $K_S^0f_0(1370)$ ,  $K^*(892)^-\pi^+$ ,  $K_0^*(1430)^-\pi^+$ ,  $K_2^*(1430)^-\pi^+$ ,  $K^*(1680)^-\pi^+$ , and the doubly Cabibbo-suppressed (DCS) mode  $K^*(892)^+\pi^-$ . A BABAR analysis [2–4] added to these ten the  $K^*(1410)^-\pi^+$ ,  $K_S^0\rho(1450)$ , the DCS resonances  $K_0^*(1430)^+\pi^-$  and  $K_2^*(1430)^+\pi^-$ , and two Breit-Wigner  $\pi\pi$  S-wave contributions. A Belle analysis [5–7] included all the components of BABAR and added two more DCS contributions,  $K^*(1410)^+\pi^-$  and  $K^*(1680)^+\pi^-$ .

The primary motivation for the analysis of the decay  $D^0 \rightarrow K_S^0\pi^+\pi^-$  is to study  $D^0 - \bar{D}^0$  oscillations and the CKM angles. The quasi-two-body intermediate states include both  $CP$ -even and  $CP$ -odd eigenstates as well as doubly Cabibbo-suppressed channels. A time-dependent analysis of the Dalitz plot from CLEO [8] and Belle [9] allows simultaneous determination of the strong transition amplitudes and phases, the mixing parameters  $x$  and  $y$  without phase or sign ambiguity, and the  $CP$ -violating parameter  $|q/p|$  and  $\text{Arg}(q/p)$ . See the note on “ $D^0 - \bar{D}^0$  Mixing” for a discussion.

The CKM angle  $\gamma/\phi_3$  [10] and the quark-mixing parameter  $\cos 2\beta/\phi_1$  [11] can be determined with the process  $B^- \rightarrow D^{(*)}K^{(*)-}$  and  $\bar{B}^0 \rightarrow Dh^0$ , respectively, followed by the decay  $D \rightarrow K_S^0\pi^+\pi^-$ . The Belle and BABAR experiments measured  $\gamma/\phi_3$  (Belle [5–7] and BABAR [2–4]) and  $\cos 2\beta/\phi_1$  (Belle [12], BABAR [13]). In these analyses, a large systematic uncertainty

in the relative phase between the  $D^0$  and  $\overline{D}^0$  amplitudes point by point across the Dalitz plot remains to be fully understood.

The CLEO model with only ten submodes does not provide a good description of the higher-statistics BABAR and Belle data samples. An improved description is obtained in two ways: First, by adding more Breit-Wigner resonances, including two  $\pi\pi$  resonances with arbitrary mass and width. Second, following the methodology of FOCUS [14], by applying a  $K$ -matrix model to the  $\pi\pi$  S-wave [9,2].

The quantum entangled production of  $D$ 's from  $\psi(3770)$  enables a model-independent determination of the  $D^0 - \overline{D}^0$  relative phase. Studying  $CP$ -tagged Dalitz plots [15,16] provides sensitivity to the cosine of the relative phase, while studying double-tagged Dalitz plots [16] probes both the cosine and sine of the  $D^0 - \overline{D}^0$  phase difference. CLEO analyzed [17] the  $D^0 \rightarrow K_S^0\pi^+\pi^-$  and  $D^0 \rightarrow K_L^0\pi^+\pi^-$  samples using the  $CP$ -even tag modes  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $K_L^0\pi^0$  (vs.  $K_S^0\pi^+\pi^-$  only), the  $CP$ -odd tag modes  $K_S^0\pi^0$ ,  $K_S^0\eta$ , and the double-tag modes  $(K_S^0\pi^+\pi^-)^2$  and  $(K_S^0\pi^+\pi^-)(K_L^0\pi^+\pi^-)$ . These measurements can reduce the model uncertainty on  $\gamma/\phi_3$  to about  $3^\circ$ .

**Doubly Cabibbo-Suppressed Decays:** There are two classes of multibody doubly Cabibbo-suppressed (DCS) decays of  $D$  mesons. The first consists of those in which the DCS and corresponding Cabibbo-favored (CF) decays populate distinct Dalitz plots; the pairs  $D^0 \rightarrow K^+\pi^-\pi^0$  and  $D^0 \rightarrow K^-\pi^+\pi^0$ , or  $D^+ \rightarrow K^+\pi^+\pi^-$  and  $D^+ \rightarrow K^-\pi^+\pi^+$ , are examples. Our average of three measurements of  $\Gamma(D^0 \rightarrow K^+\pi^-\pi^0)/\Gamma(D^0 \rightarrow K^-\pi^+\pi^0)$  is  $(2.20 \pm 0.10) \times 10^{-3}$ . Our average of three measurements of  $\Gamma(D^+ \rightarrow K^+\pi^-\pi^+)/\Gamma(D^+ \rightarrow K^-\pi^+\pi^+)$  is  $(6.8 \pm 0.8) \times 10^{-3}$ ; see the Particle Listings.

The second class consists of decays in which the DCS and CF modes populate the same Dalitz plot; for example,  $D^0 \rightarrow K^{*-}\pi^+$  and  $D^0 \rightarrow K^{*+}\pi^-$  both contribute to  $D^0 \rightarrow K_S^0\pi^+\pi^-$ . In this class, the potential for interference of DCS and CF amplitudes increases the sensitivity to the DCS amplitude and allows direct measurement of the relative strong phases between amplitudes. CLEO [1] and Belle [9] have measured the relative phase between  $D^0 \rightarrow K^*(892)^+\pi^-$  and  $D^0 \rightarrow K^*(892)^-\pi^+$  to

be  $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$  and  $(171.9 \pm 1.3)^\circ$  (statistical error only). These results are close to the  $180^\circ$  expected from Cabibbo factors and a small strong phase.

Additionally, Belle [9] has reported results for both the relative phase (statistical errors only) and ratio  $R$  (central values only) of the DCS fit fraction relative to the CF fit fractions for  $K^*(892)^+\pi^-$ ,  $K_0^*(1430)^+\pi^-$ ,  $K_2^*(1430)^+\pi^-$ ,  $K^*(1410)^+\pi^-$ , and  $K^*(1680)^+\pi^-$ . The reported values for  $R$ , in units of  $\tan^4 \theta_c$ , are  $2.94 \pm 0.12$ ,  $22.0 \pm 1.6$ ,  $34 \pm 4$ ,  $87 \pm 13$ , and  $(5 \pm 5) \times 10^2$ . For  $K^+\pi^-$ , the corresponding value for  $R$  is  $(1.28 \pm 0.02) \times \tan^4 \theta_c$ . Similarly, BABAR [2] has reported central values for  $R$  for  $K^*(892)^+\pi^-$ ,  $K_0^*(1430)^+\pi^-$ , and  $K_2^*(1430)^+\pi^-$ . In units of  $\tan^4 \theta_c$ ,  $R$  is  $3.45 \pm 0.31$ ,  $7.7 \pm 3.0$ , and  $1.7 \pm 1.7$ . The systematic uncertainties on these values remain to be evaluated. The large differences in  $R$  among these final states, if significant, could point to an interesting role for hadronic effects that deserves theoretical attention.

(There are other ways, not involving DCS decays, in which  $D^0$  and  $\overline{D}^0$  decays can populate the same Dalitz plot. Examples are  $D^0$  and  $\overline{D}^0$  decays to  $K_S^0 K^+\pi^-$ , or to  $K_S^0 K^-\pi^+$ . These final states can be used to study  $D^0-\overline{D}^0$  mixing and the CKM angle  $\gamma/\phi_3$ .)

**$CP$  Violation:** In the limit of  $CP$  conservation, charge conjugate decays will have the same Dalitz-plot distribution. The  $D^{*\pm}$  tag enables the discrimination between  $D^0$  and  $\overline{D}^0$ . The integrated  $CP$  violation across the Dalitz plot is determined in two ways. The first uses

$$\mathcal{A}_{CP} = \int \left( \frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \right) dm_{ab}^2 dm_{bc}^2 \Bigg/ \int dm_{ab}^2 dm_{bc}^2 , \quad (1)$$

where  $\mathcal{M}$  and  $\overline{\mathcal{M}}$  are the  $D^0$  and  $\overline{D}^0$  Dalitz-plot amplitudes for the three-body decay  $D \rightarrow abc$ , and  $m_{ab}$  ( $m_{bc}$ ) is the invariant mass of  $ab$  ( $bc$ ). The second uses the asymmetry in the efficiency-corrected  $D^0$  and  $\overline{D}^0$  yields,

$$\mathcal{A}_{CP} = \frac{N_{D^0} - N_{\overline{D}^0}}{N_{D^0} + N_{\overline{D}^0}} . \quad (2)$$

These expressions are less sensitive to  $CP$  violation than are the individual resonant submodes [18]. Our Particle Listings give limits on  $CP$  violation for 11  $D^+$ , 25  $D^0$ , and 12  $D_S^+$  decay modes.

The possibility of interference between  $CP$ -conserving and  $CP$ -violating amplitudes provides a more sensitive probe of  $CP$  violation. The constraints on the square of the  $CP$ -violating amplitude obtained in the resonant submodes of  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  range from  $3.5 \times 10^{-4}$  to  $28.4 \times 10^{-4}$  at 95% confidence level [18].

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