27. ACCELERATOR PHYSICS OF COLLIDERS

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27.1. Luminosity

This article provides background for the High-Energy Collider Parameter Tables that follow. The number of events, N_{exp}, is the product of the cross section of interest, σ_{exp}, and the time integral over the instantaneous *luminosity,* $\mathscr{L}$:

$$N_{exp} = \sigma_{exp} \times \int \mathscr{L}(t) dt. \tag{27.1}$$

Today's colliders all employ bunched beams. If two bunches containing n_1 and n_2 particles collide head-on with frequency f, a basic expression for the luminosity is

$$\mathscr{L} = f \frac{n_1 n_2}{4\pi \sigma_x \sigma_y} \tag{27.2}$$

where σ_x and σ_y characterize the rms transverse beam sizes in the horizontal (bend) and vertical directions. In this form it is assumed that the bunches are identical in transverse profile, that the profiles are Gaussian and independent of position along the bunch, and the particle distributions are not altered during bunch crossing.

Whatever the distribution at the source, by the time the beam reaches high energy, the normal form is a useful approximation as suggested by the σ-notation. In the case of an electron storage ring, synchrotron radiation leads to a Gaussian distribution in equilibrium, but even in the absence of radiation the central limit theorem of probability and the diminished importance of space charge effects produces a similar result.

The luminosity may be obtained directly by measurement of the beam properties in Eq. (27.2), but the beam measurements are apt to interfere with data acquisition, so this method though valuable to establish collider performance is not suitable for continuous use. A similar expression to Eq. (27.1) with N_{ref} from a known reference cross section, σ_{ref}, may be used to determine σ_{exp} according to $\sigma_{exp} = (N_{exp}/N_{ref})\sigma_{ref}$.

In the Tables, luminosity is stated in units of cm^{-2}s^{-1}. Integrated luminosity, on the other hand is usually quoted as the inverse of the standard measures of cross section such as femtobarns and, recently, attobarns.

Subsequent sections in this report enlarge briefly on the dynamics behind collider design, comment on the realization of collider performance in a selection of today's facilities, and end with some remarks on future possibilities.

27.2. Beam Dynamics

The first concern of beam dynamics is stability. While a reference particle proceeds along the design, or reference, trajectory other particles in the bunch are to remain close by. Assume that the reference particle carries a right-handed Cartesian coordinate system, with the z-coordinate pointed in the direction of motion along the reference trajectory. The independent variable is the distance s of the reference particle along this trajectory rather than time, and for simplicity this path is taken to be planar. The transverse coordinates are x and y, where $\{x, z\}$ defines the plane of the reference trajectory.

Several time scales are involved, and the approximations used in writing the equations of motion reflect that circumstance. All of today's high energy colliders are alternating gradient synchrotrons [1,2], and the shortest time scale is that associated with transverse stability, the betatron oscillations, so called because of their analysis for the betatron accelerator species years ago. The linearized equations of motion of a particle displaced from the reference particle are

$$\begin{aligned} x'' + K_x x = 0, \qquad & K_x \equiv \frac{e}{p}\frac{\partial B}{\partial x} + \frac{1}{\rho^2} \\ y'' + K_y y = 0, \qquad & K_y \equiv -\frac{e}{p}\frac{\partial B}{\partial x} \\ z' = -x/\rho \end{aligned} \tag{27.3}$$

where the magnetic field $B(s)$ is only in the y direction, contains only dipole and quadrupole terms, and is treated as static here. The radius of curvature due to the field on the reference orbit is ρ; p and e are the particle's momentum and charge respectively. The prime denotes d/ds.

The equations for x and y are those of harmonic oscillators but with restoring force periodic in s, that is, they are instances of Hill's equation. The solution may be written in the form

$$\begin{aligned} x(s) &= A_x\sqrt{\beta_x}\ \cos\psi_x \\ x'(s) &= -\frac{A_x}{\sqrt{\beta_x}}\left[\alpha\cos\psi_x + \sin\psi_x\right] \end{aligned} \tag{27.4}$$

where A_x is a constant of integration, $\alpha \equiv -(1/2)d\beta_x(s)/ds$, and the envelope of the motion is modulated by the *amplitude function*, β_x. A solution of the same form describes the motion in y. The subscripts will be suppressed in the following discussion.

The amplitude function satisfies

$$2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4, \tag{27.5}$$

and in a region free of magnetic field it should be noted that the solution of Eq. (27.5) is a parabola. Expressing A in terms of x, x' yields

$$\begin{aligned} A^2 &= \gamma x^2 + 2\alpha x x' + \beta x'^2 \\ &= \frac{1}{\beta}\left[x^2 + (\alpha x + \beta x')^2\right] \end{aligned} \tag{27.6}$$

with $\gamma \equiv (1+\alpha^2)/\beta$. In a single pass system such as a linac, the *Courant-Snyder parameters* α, β, γ may be selected to match the $x\,x'$ distribution of the input beam; in a recursive system, the parameters are usually defined by the structure rather than by the beam.

The relationships between the parameters and the structure may be seen by treatment of a simple *lattice* consisting of equally spaced thin lens quadrupoles equal in magnetic field gradient magnitude but alternating in sign. For this discussion, the weak focusing effects of the bending magnets may be neglected. The propagation of $X \equiv \{x, x'\}$ through a repetition period may be written $X_2 = MX_1$, with the matrix $M = FODO$ composed of the matrices

$$F = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \; D = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}, \; O = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix},$$

where f is the magnitude of the focal length and L the lens spacing. Then

$$M = \begin{pmatrix} 1 + \dfrac{L}{f} & 2L + \dfrac{L^2}{f} \\ -\dfrac{L}{f^2} & 1 - \dfrac{L}{f} - \dfrac{L^2}{f^2} \end{pmatrix}. \tag{27.7}$$

The matrix for y is identical in form differing only by a change in sign of terms linear in f. An eigenvector-eigenvalue analysis of the matrix M shows that the motion is stable provided $f > L/2$. While that criterion is easily met, in practice instability may be caused by many other factors. including the beam-beam interaction itself.

Standard focus-drift-defocus-drift, or *FODO*, cells such as characterized in simple form by Eq. (27.7) occupy most of the layout of a large collider ring and may be used to set the scale of the amplitude function and related phase advance. Conversion of Eq. (27.4) to a matrix form equivalent to Eq. (27.7) gives

$$M = \begin{pmatrix} c + \alpha s & \beta s \\ -\gamma s & c - \alpha s \end{pmatrix} \tag{27.8}$$

where $c \equiv \cos \Delta\psi$, $s \equiv \sin \Delta\psi$, and the relation between structure and amplitude function is specified by setting the values of the latter to be the same at both ends of the cell. By comparison of Eq. (27.7) and Eq. (27.8) one finds $c = 1 - L^2/(2f^2)$, so the choice $f = L/\sqrt{2}$ would give a phase advance $\Delta\psi$ of 90 degrees for the standard cell. The amplitude function – a maximum at the focusing quadrupole – would then be $2.7L$, illustrating the relationship of alternating gradient focusing amplitudes to relatively local aspects of the design. Other functions such as injection, extraction, and HEP experiments are included by lattice sections matched to the β, α standard cell parameters at the insertion points.

The phase advances according to $d\psi/ds = 1/\beta$; that is, β also plays the role of a local $\lambda/2\pi$, and the *tune*, ν, is the number of such oscillations per turn about the closed path. In the neighborhood of an interaction point (IP), the beam optics of the ring is configured so as to produce a near focus; the value of the amplitude function at this point is designated β^*.

The motion as it develops with s describes an ellipse in $\{x, x' \equiv dx/ds\}$ phase space the area of which is πA^2, where A is the constant in Eq. (27.4). If the interior of that ellipse is populated by an ensemble of non-interacting particles, that area, given the name *emittance* and denoted by ε, would change only with energy. For a beam with a Gaussian distribution in x, x', the area containing one standard deviation σ_x is the definition of emittance in the Tables:

$$\varepsilon_x \equiv \pi \frac{\sigma_x^2}{\beta_x} , \tag{27.9}$$

with a corresponding expression in the other transverse direction, y. This definition includes 39% of the beam. For most of the entries in the Tables the standard deviation is used as the beam radius.

To complete the coordinates used to describe the motion, we take as the variable conjugate to z the fractional momentum deviation $\delta p/p$ from that of the reference particle. Radiofrequency electric fields in the s direction provide a means for longitudinal oscillations, and the frequency determines the bunch length. The frequency of this system appears in the Tables as does the rms value of $\delta p/p$ characterized as "energy spread" of the beam.

For HEP bunch length is a significant quantity for a variety of reasons, but in the present context if the bunch length becomes larger than β^* the luminosity is adversely affected. This is because β grows parabolically as one proceeds away from the interaction point and so the beam size increases thus lowering the contribution to the luminosity from such locations. This is often called the "hourglass" effect.

The other major external electromagnetic field interaction in the single particle context is the production of synchrotron radiation due to centripetal acceleration, given by the Larmor formula multiplied by a relativistic magnification factor of γ^4 [3]. In the case of electron rings this process determines the equilibrium emittance through a balance between radiation damping and excitation of oscillations, and further serves as the barrier to future higher energy versions in this variety of collider.

27.3. Impediments to High Luminosity

Eq. (27.2) can be recast in terms of emittances and amplitude functions as

$$\mathscr{L} = f \frac{n_1 n_2}{4\sqrt{\epsilon_x \, \beta_x^* \, \epsilon_y \, \beta_y^*}} \; . \tag{27.10}$$

So to achieve high luminosity, all one has to do is make high population bunches of low emittance collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible.

Such expressions as Eq. (27.10) of the luminosity are special cases of the more general forms available elsewhere [4]. But while there are no fundamental limits to the process, there are certainly challenges. Here we have space to mention only a few of these. The beam-beam tune shift appears in the Tables. A bunch in beam 1 presents a (nonlinear) lens to a particle in beam 2 resulting in changes to the particle's transverse tune with a range characterized by the parameter [4]

$$\xi_{y,2} = \frac{r_e n_1 \beta_{y,2}^*}{2\pi\gamma_2 \sigma_{y,1}(\sigma_{x,1} + \sigma_{y,1})} \tag{27.11}$$

where $r_e = e^2/(4\pi\varepsilon_0 mc^2)$ is the classical radius of the electron. The transverse oscillations are susceptible to resonant perturbations from a variety of sources such as imperfections in the magnetic guide field, so certain values of the tune must be avoided. Accordingly, the tune spread arising from ξ is limited, but limited to a value difficult to predict. But a glance at the Tables shows that electrons are more forgiving than protons thanks to the damping effects of synchrotron radiation; the ξ-values for the former are about an order of magnitude larger than those for protons.

A subject of present intense interest is the *electron-cloud effect* [5,6]; actually a variety of related processes come under this heading. They typically involve a buildup of electron density in the vacuum chamber due to emission from the chamber walls stimulated by electrons or photons originating from the beam itself. For instance, there is a process closely resembling the multipacting effects familiar from radiofrequency system commissioning. Low energy electrons are ejected from the walls by photons from positron or proton beam-produced synchrotron radiation. These electrons are accelerated toward a beam bunch, but by the time they reach the center of the vacuum chamber the bunch has gone and so the now-energetic electrons strike the opposite wall to produce more secondaries. These secondaries are now accelerated by a subsequent bunch, and so on. Among the disturbances that this electron accumulation can produce is enhancement of the tune spread within the bunch; the near-cancellation of bunch-induced electric and magnetic fields is no longer in effect.

The benefits of low emittance are clear in Eq. (27.10). For electron synchrotrons, radiation damping provides an automatic route. For hadrons, particularly antiprotons, two inventions have played a prominent role. Stochastic cooling [7] was employed first in the $S\bar{p}pS$ and subsequently in the Tevatron. Electron cooling [8] was also used in the Tevatron complex to great advantage. Further innovations are underway due to the needs of potential future projects; these are noted in the final section.

27.4. Comments on Present Facilities

Collider accelerator physics of course goes far beyond the elements of the preceding sections. In this section elaboration is made on various issues associated with some of the recently operating colliders, particularly factors which impact integrated luminosity. The various colliders utilizing hadrons have important unique differences and hence are broken out separately. As space is limited, general references are provided where much further information can be obtained.

27.4.1. ***LHC*** : [9] The superconducting Large Hadron Collider is the world's highest energy collider. Operation for HEP is currently conducted with 3.5 TeV protons in each beam. Progress is rapid and current status is best checked at the Web site referenced in the heading of this subsection. To meet its luminosity goals the LHC will have to contend with a high beam current of 0.5 A, leading to stored energies of several hundred MJ per beam. Component protection, beam collimation, and controlled energy deposition will be given very high priorities. Additionally, at energies of 5-7 TeV per particle, synchrotron radiation will move from being a curiosity to a challenge in a hadron accelerator for the first time. At design beam current the system must remove roughly 7 kW at 1.8 K due to synchrotron radiation. As the photons are emitted their interactions with the vacuum chamber wall can generate free electrons, with consequent "electron cloud" development. Much care was taken to design a special liner for the chamber to mitigate this issue.

The two proton beams are contained in separate pipes throughout most of the circumference, but naturally must be brought together into a single pipe at the interaction points. The large number of bunches, and subsequent short bunch spacing, would lead to approximately 30 head-on collisions through 120 m of common beam pipe at each IP. Thus, a small crossing angle is employed, which reduces the luminosity by about 15%. Still, the bunches moving in one direction will have long-range encounters with the counter-rotating bunches and the resulting perturbations of the particle motion constitute a continued course of study. Initial luminosity measurements were made by the "van der Meer scan" as was done long ago on the ISR [10]. The detectors will have measurements based on a reference cross section; for an example see the discussion in the ATLAS design report [11]. The Tables also show the performance anticipated for Pb-Pb collisions. The ALICE [12] experiment is designed to concentrate on these high energy-density phenomena.

In the coming years, an ambitious upgrade program, Super LHC [13], has as its target an order-of-magnitude increase in luminosity.

27.4.2. ***Tevatron*** : [14] The first superconducting synchrotron in history, the Tevatron was the highest energy collider for 25 years. Operation was terminated in September 2011. The route to high integrated luminosity in the Tevatron was governed by the antiproton production rate, the turn-around time to produce another store, and the resulting optimization of store time. The overall reliability of the accelerator complex plays a crucial role, as it can take many hours to produce an adequate number of antiprotons for collisions.

Unlike the LHC, the beams in the Tevatron circulated in a single vacuum pipe and thus were placed on separated orbits which wrap around each other in a helical pattern outside of the interaction regions. Hence, long-range encounters played an important role here as well, though the effects could be different from the LHC where the encounters are more or less "in phase" with each other through a single interaction region. In the Tevatron, the 70 long-range encounters were distributed about the synchrotron and their mitigation was limited by the available aperture.

In recent years the antiproton bunch intensities approached those of the proton bunches, and their emittances were greatly reduced using improved beam cooling, so much so that detrimental effects on the proton beam became apparent. The antiproton beam emittance was adjusted prior to collision conditions to optimize the proton bunch lifetime during the store [15]. The Tevatron ultimately achieved luminosities a factor of 400 over the original design specification.

27.4.3. ***e^+e^- Rings*** : As should be expected, synchrotron radiation plays a major role in the design and optimization of e^+e^- colliders. While vacuum stability and electron clouds can be of concern in the positron rings, synchrotron radiation along with the restoration of longitudinal momentum by the RF system have the positive effect of generating very small transverse beam sizes and small momentum spread. Further reduction of beam size at the interaction points using standard beam optics techniques and successfully contending with high beam currents has led to record luminosities in these rings, far exceeding those of hadron colliders. To maximize integrated luminosity the beam can be "topped off" by injecting new particles without removing existing ones – a feature difficult to imitate in hadron colliders.

Asymmetric energies of the two beams have allowed for the enhancement of B-physics research and for interesting interaction region designs. As the bunch spacing can be quite short, the lepton beams sometimes pass through each other at an angle and hence have reduced luminosity. Recently, however, the invention of high frequency "crab crossing" schemes have produced full restoration of the luminous region. KEK-B has attained over 1 fb^{-1} of integrated luminosity in a single day, and its upgrade plans are aiming for initial luminosities of 8×10^{35} $\text{cm}^{-2}\text{s}^{-1}$ [16].

27.4.4. ***HERA*** : [17] Now decommissioned, HERA was the first facility to employ both applications of superconductivity: magnets and accelerating structures. Its next-generation cold-iron superconducting magnets for the proton beam were the culmination of lessons learned from the Tevatron experience and extensive development of the technology since then. The HERA team felt comfortable with a larger dynamic range of the magnet system, enabling the use of the existing DESY complex for injection. Though the HERA magnets could reach fields consistent with energies above 1 TeV, other accelerator systems precluded operation above 920 GeV.

The lepton beams (positrons or electrons) were provided by the existing complex, and were accelerated to 27.5 GeV using conventional magnets. The interaction region where the beams had common vacuum chambers had the interesting feature that the lepton beam could be manipulated without detrimental effects on the proton beam due to the large difference in magnetic rigidity. A 4-times higher frequency RF system was used at collision to generate shorter bunches, thus helping alleviate the hour glass effect at the collision points. As in any high energy lepton storage ring, the lepton beam naturally would become transversely polarized (within about 40 minutes, for HERA). "Spin rotators" were implemented on either side of an IP to produce longitudinal polarization at the experiment.

27.4.5. ***RHIC*** : [18] The Relativistic Heavy Ion Collider employs superconducting magnets, and collides combinations of fully-stripped ions such as H-H (p-p), Au-Au, Cu-Cu, and d-Au.

The high charge per particle (+79 for gold, for instance) makes intra-beam scattering of particles within the bunch of special concern, even for seemingly modest bunch intensities. Another special feature of accelerating heavy ions in RHIC is that the beams experience a "transition energy" during acceleration – a point where the derivative with respect to momentum of the revolution period is zero. This is more typical of low-energy accelerators, where the necessary phase jump required of the RF system is implemented rapidly and little time is spent near this condition. In the case of RHIC with heavy ions, the superconducting magnets do not ramp very quickly and the period of time spent crossing transition is long and must be dealt with carefully. For p-p operation the beams are always above their transition energy and so this condition is completely avoided.

RHIC is also distinctive in its ability to accelerate and collide polarized proton beams. As proton beam polarization must be maintained from its low-energy source, successful acceleration through the myriad of depolarizing resonance conditions in high energy circular accelerators has taken years to accomplish. An energy of 250 GeV per proton with ∼48% final polarization per beam has been realized.

27.5. Future Prospects

Present design activity emphasizes a lepton collider as the next major HEP project contingent upon the initial results from the LHC. Synchrotron radiation precludes a higher energy successor to LEP. Four alternatives are noted in this section: two approaches to an electron-positron linear collider, a muon ring collider, and potential use of plasma acceleration.

27.5.1. ***Electron-Positron Linear Colliders*** : A major problem confronting a high energy, high luminosity single pass collider design is the power requirement, so measures must be taken to keep the demand within bounds as illustrated in a transformed Eq. (27.2) as developed in the *TESLA Design Report* [19]:

$$\mathscr{L} = \frac{1}{4\pi r_e^{3/2}} \frac{P_b}{E_{cm}} \left(\frac{\pi \delta_E}{\gamma \varepsilon_y}\right)^{1/2} H_D. \tag{27.12}$$

Here, P_b is the total power of both beams and E_{cm} their cms energy. Management of P_b leads to an upward push on the product of collision frequency and bunch population with an attendant rise in the energy radiated due to the electromagnetic field of one bunch acting on the particles of the other. The fractional spread in the collision energy that results from this radiation is represented by δ_E and keeping a significant fraction of the luminosity within a percent or so of the nominal energy represents a design goal. A consequence is the use of flat beams, where δ_E is managed by the beam width, and luminosity adjusted by the beam height, thus the explicit appearance of the vertical emittance ε_y. The final factor in Eq. (27.12), H_D, represents the enhancement of luminosity due to the pinch effect during bunch crossing.

The approach designated by the International Linear Collider (ILC) is presented in the Tables, and the contrast with the collision-point parameters of the circular colliders is striking, though reminiscent in direction of those of the SLAC Linear Collider that are no longer shown. The ILC *Reference Design Report* [20] has a baseline cms energy of 500 GeV with upgrade provision for 1 TeV, and luminosity comparable to the LHC. The ILC is based on superconducting accelerating structures of the 1.3 GHz TESLA variety.

At CERN, a design effort is underway on the Compact Linear Collider (CLIC), each linac of which is itself a two-beam accelerator, in that a high energy, low current beam is fed by a low energy, high current driver [21]. The CLIC design employs normal conducting 12 GHz accelerating structures at a gradient of 100 MeV/m, some three times the current capability of the superconducting ILC cavities. The design cms energy is 3 TeV.

27.5.2. ***Muon Collider*** : The muon to electron mass ratio of 210 implies less concern about synchrotron radiation by a factor of about 2×10^9 and its 1.6 μs lifetime means that it will last for some $150B$ turns in a ring about half of which is occupied by bend magnets with average field B (tesla). Design effort became serious in the mid 1990s and a collider outline emerged quickly [22].

Removal of the synchrotron radiation barrier reduces collider facility scale to a level compatible with on-site placement at some locations. If a Higgs particle is detected the $(m_\mu/m_e)^2$ cross section advantage in s-channel production would be valuable. And a neutrino factory could potentially be realized in the course of construction [23].

The challenges to luminosity achievement were clear and very attractive for R&D: targetting, collection, and emittance reduction are three that come immediately to mind. The proton source needs to deliver a beam power of several MW, collection would be aided by magnetic fields common on neutron stars (though scaled back for application on earth), and the emittance requirements have inspired fascinating investigations into phase space manipulation that are finding application in other facilities. A summary of the status may be found in a presentation to the HEPAP P5 Panel [24].

27.5.3. ***Plasma Acceleration*** : At the 1956 CERN Symposium, a paper by Veksler in which he suggested acceleration of protons to the TeV scale using a bunch of electrons anticipated current interest in plasma acceleration [25]. A half-century later this is more than a suggestion, with the demonstration, as a striking example, of energy enhancement of 28.5 GeV at SLAC [26].

How plasma acceleration will find application in an HEP facility is not yet clear, given the necessity of coordinating multiple plasma chambers. Active R&D is underway; for recent discussion of parameters for a laser-plasma based electron positron collider, see, for example, relevant papers in an Advanced Accelerator Concepts Workshop [27].

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