DECAY CONSTANTS OF CHARGED PSEUDOSCALAR MESONS

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Introduction: Charged mesons formed from a quark and an antiquark can decay to a charged lepton pair when these objects annihilate via a virtual W boson [1]. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.

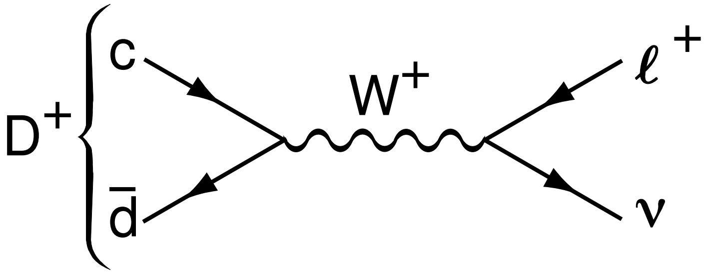

Figure 1: The annihilation process for pure D^+ leptonic decays in the Standard Model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+, K^+, D_s^+, and B^+ mesons. (Charge-conjugate particles and decays are implied.) Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \, m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 \left|V_{q_1 q_2}\right|^2 \quad . \qquad (1)$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1\bar{q}_2$ in P, and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, and is related to the wave-function overlap of the quark and antiquark.

The decay $P^\pm$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the

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finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width.

There is a complication in measuring purely leptonic decay rates. The process $P \to \ell\nu\gamma$ is not simply a radiative correction, although radiative corrections contribute. The P can make a transition to a virtual P^*, emitting a real photon, and the P^* decays into $\ell\nu$, avoiding helicity suppression. The importance of this amplitude depends on the decaying particle and the detection technique. The $\ell\nu\gamma$ rate for a heavy particle such as B decaying into a light particle such as a muon can be larger than the width without photon emission [2]. On the other hand, for decays into a $\tau^\pm$, the helicity suppression is mostly broken and these effects appear to be small.

Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1q_2}|\, f_P$. If the CKM element is well known from other measurements, then f_P can be well measured. If, on the other hand, the CKM element is not well measured, having theoretical input on f_P can allow a determination of the CKM element. The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. For example, the measurement of $\Gamma(B^- \to \tau^-\overline{\nu})$ provides an indirect determination of $|V_{ub}|$ provided that f_B is provided by theory. In addition, f_B is crucial for using measurements of B^0-$\overline{B}^0$ mixing to extract information on the fundamental CKM parameters. Knowledge of f_{B_s} is also needed, but it cannot be directly measured as the B_s is neutral, so the violation of the SU(3) relation $f_{B_s} = f_B$ must be estimated theoretically. This difficulty does not occur for D mesons as both the D^+ and D_s^+ are charged, allowing the direct measurement of SU(3) breaking and a direct comparison with theory.

For B^- and D_s^+ decays, the existence of a charged Higgs boson (or any other charged object beyond the Standard Model) would modify the decay rates; however, this would not necessarily be true for the D^+ [3,4]. More generally, the ratio of $\tau\nu$ to $\mu\nu$ decays can serve as one probe of lepton universality [3,5].

As $|V_{ud}|$ has been quite accurately measured in superallowed β decays [6], with a value of 0.97425(22) [7], measurements of $\Gamma(\pi^+ \to \mu^+\nu)$ yield a value for f_π. Similarly, $|V_{us}|$ has been well measured in semileptonic kaon decays, so a value for f_K from $\Gamma(K^- \to \mu^-\bar{\nu})$ can be compared to theoretical calculations. Lattice gauge theory calculations, however, have been claimed to be very accurate in determining f_K, and these have been used to predict $|V_{us}|$ [8].

D^+ and D_s^+ decay constants: We review current measurements, starting with the charm system. The CLEO collaboration has performed the only measurement of the branching fraction for $D^+ \to \mu^+\nu$ [9]. CLEO uses e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D's, find a candidate muon track of opposite sign to the tag, and then use kinematical constraints to infer the existence of a missing neutrino and hence the $\mu\nu$ decay of the other D. They find $\mathcal{B}(D^+ \to \mu^+\nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$. We use the well-measured D^+ lifetime of 1.040(7) ps, and assuming $|V_{cd}|$ equals $|V_{us}| = 0.2246(12)$ [7] minus higher order correction terms [10], we find $|V_{cd}| = 0.2245(12)$. The CLEO branching fraction result then translates into a value of

$$f_{D^+} = (206.7 \pm 8.5 \pm 2.5)\ \text{MeV} \ .$$

This result includes a 1% correction (lowering) of the rate due to the presence of the radiative $\mu^+\nu\gamma$ final state based on the estimate by Dobrescu and Kronfeld [11].

Before we compare this result with theoretical predictions, we discuss the D_s^+. Measurements of $f_{D_s^+}$ have been made by several groups and are listed in Table 1 [12–16]. We exclude older values obtained by normalizing to D_s^+ decay modes that are not well defined. Many measurements, for example, used the $\phi\pi^+$ mode. This decay is a subset of the $D_s^+ \to K^+K^-\pi^+$ channel which has interferences from other modes populating the K^+K^- mass region near the ϕ, the most prominent of which is the $f_0(980)$. Thus the extraction of effective $\phi\pi^+$ rate is sensitive to the mass resolution of the experiment and the cuts used to define the ϕ mass region [17,18]. The CLEO, BaBar, and Belle $\mu^+\nu$ results rely on fully reconstructing all the final-state particles except for the neutrino and using a missing-mass technique to infer the existence of the neutrino. CLEO uses $e^+e^- \to D_s D_s^*$ collisions at 4170 MeV, while Babar and Belle use $e^+e^- \to DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$.

Table 1: Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu)$, $\mathcal{B}(D_s^+ \to \tau^+\nu)$, and $f_{D_s^+}$. Numbers for $f_{D_s^+}$ have been extracted using updated values for masses and $|V_{cs}|$ (see text). Radiative corrections and systematic uncertainties for errors on the D_s^+ lifetime and mass have been included. Common systematic errors in the CLEO results have been taken into account.

Experiment	Mode	$\mathcal{B}(\%)$	$f_{D_s^+}$ (MeV)
CLEO-c [12]	$\mu^+\nu$	$0.565 \pm 0.045 \pm 0.017$	$257.6 \pm 10.3 \pm 4.3$
BaBar [16]	$\mu^+\nu$	$0.602 \pm 0.038 \pm 0.034$	$265.9 \pm 8.4 \pm 7.7$
Belle [13]	$\mu^+\nu$	$0.638 \pm 0.076 \pm 0.057$	$274 \pm 16 \pm 12$
Average	$\mu^+\nu$	0.589 ± 0.033	263.0 ± 7.3
CLEO-c [12]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$6.42 \pm 0.81 \pm 0.18$	$278.0 \pm 17.5 \pm 4.4$
CLEO-c [14]	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$5.52 \pm 0.57 \pm 0.21$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [15]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.30 \pm 0.47 \pm 0.22$	$252.6 \pm 11.1 \pm 5.2$
BaBar [16]	$\tau^+\nu$ $(e^+/\mu^+\nu\overline{\nu})$	$5.00 \pm 0.35 \pm 0.49$	$245.4 \pm 8.6 \pm 12.2$
Average	$\tau^+\nu$	5.43 ± 0.31	255.7 ± 7.2

When selecting the $\tau^+ \to \pi^+\bar{\nu}$ and $\tau^+ \to \rho^+\bar{\nu}$ decay modes, CLEO uses both calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s^- and the τ^+ decay products. The $\tau^+ \to e^+\nu\bar{\nu}$ mode, however, uses only no extra energy. BaBar measures $\Gamma(D_s^+ \to \tau^+\nu)/\Gamma(D_s^+ \to \overline{K}^0 K^+)$ using the $\tau^+ \to e^+\nu\bar{\nu}$ mode.

We extract the decay constant from the measured branching ratios using the D_s^+ mass of 1.96847(33) GeV, the τ^+ mass of 1.77682(16) GeV, and a lifetime of 0.500(7) ps. We use the first-order correction $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2$ [10]; taking $|V_{ud}| = 0.97425(22)$ [6], and $|V_{cb}| = 0.04$ from an average of exclusive and inclusive semileptonic B decay results as discussed in Ref. [19], we find $|V_{cs}| = 0.97345(22)$. CLEO has included the radiative correction of 1% in the $\mu^+\nu$ rate listed in the Table [11] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that the $\mu^+\nu\gamma$ rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [20]. As this is a small effect we do not attempt to correct the other measurements.

The average decay constant cannot simply be obtained by averaging the values in Table 1 since there are correlated errors between the $\mu^+\nu$ and $\tau^+\nu$ values. Table 2 gives the average values of f_{D_s} where the experiments have included the correlations.

Table 2: Experimental results for $f_{D_s^+}$ taking into account the common systematic errors in the $\mu^+\nu$ and $\tau^+\nu$ measurements.

Experiment	$f_{D_s^+}$ (MeV)
CLEO-c	$259.0 \pm 6.2 \pm 3.0$
BaBar	$258.8 \pm 6.4 \pm 7.5$
Belle	$273.8 \pm 16.3 \pm 12.2$
Average of $\mu^+\nu + \tau^+\nu$	260.0 ± 5.4

Our experimental average is

$$f_{D_s^+} = (260.0 \pm 5.4)\ \text{MeV}.$$

Furthermore, the ratio of branching fractions is found to be

$$R \equiv \frac{\mathcal{B}(D_s^+ \to \tau^+\nu)}{\mathcal{B}(D_s^+ \to \mu^+\nu)} = 9.2 \pm 0.7,$$

where a value of 9.76 is predicted in the Standard Model. Assuming lepton universality then we can derive improved values for the leptonic decay branching fractions of

$$\mathcal{B}(D_s^+ \to \mu^+\nu) = (5.75 \pm 0.24) \times 10^{-3}, \quad \text{and}$$

$$\mathcal{B}(D_s^+ \to \tau^+\nu) = (5.61 \pm 0.24) \times 10^{-2} \ .$$

The experimentally determined ratio of decay constants is $f_{D_s^+}/f_{D^+} = 1.26 \pm 0.06$.

Table 3: Theoretical predictions of $f_{D_s^+}$, f_{D^+}, and $f_{D_s^+}/f_{D^+}$. Quenched lattice calculations are omitted, while PQL indicates a partially-quenched lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	260.0 ± 5.4	206.7 ± 8.9	1.26 ± 0.06
Lattice (HPQCD) [21]	248.0 ± 2.5	213 ± 4	1.164 ± 0.018
Lattice (FNAL+MILC) [22]	260.1 ± 10.8	218.9 ± 11.3	1.188 ± 0.025
PQL [23]	244 ± 8	197 ± 9	1.24 ± 0.03
QCD sum rules [24]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD sum rules [25]	$245.3 \pm 15.7 \pm 4.5$	$206.2 \pm 7.3 \pm 5.1$	$1.193 \pm 0.025 \pm 0.007$
Field correlators [26]	260 ± 10	210 ± 10	1.24 ± 0.03
Light front [27]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04

Table 3 compares the experimental $f_{D_s^+}$ with theoretical calculations [21–27]. While most theories give values lower than the $f_{D_s^+}$ measurement, the errors are sufficiently large, in most cases, to declare success.

Upper limits on f_{D^+} and f_{D_s} of 230 and 270 MeV, respectively, have been determined using two-point correlation functions by Khodjamirian [28]. Both the D^+ and D_s^+ values are safely below this limit.

Akeroyd and Chen [29] pointed out that leptonic decay widths are modified in two-Higgs-doublet models (2HDM). Specifically, for the D^+ and D_s^+, Eq. (1) is modified by a factor r_q multiplying the right-hand side [30]:

$$r_q = \left[1 + \left(\frac{1}{m_c + m_q}\right)\left(\frac{M_{D_q}}{M_{H^+}}\right)^2 \left(m_c - \frac{m_q \tan^2\beta}{1 + \epsilon_0 \tan\beta}\right)\right]^2 ,$$

where m_{H^+} is the charged Higgs mass, M_{D_q} is the mass of the D meson (containing the light quark q), m_c is the charm quark mass, m_q is the light-quark mass, and $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. In models where the fermion mass arises from coupling to more than one vacuum expectation value ϵ_0 can be non-zero, perhaps as large as 0.01. For the D^+, $m_d \ll m_c$, and the change due to the H^+ is very small. For the D_s^+, however, the effect can be substantial.

A major concern is the need for the Standard Model (SM) value of $f_{D_s^+}$. We can take that from a theoretical model. Our most aggressive choice is that of the unquenched lattice calculation [21], because it claims the smallest error. Since the charged Higgs would lower the rate compared to the SM, in principle, experiment gives a lower limit on the charged Higgs mass. However, the value for the predicted decay constant using this model is 2.0 standard deviations *below* the measurement. If this small discrepancy is to be taken seriously, either (a) the model of Ref. [21] is not representative; (b) no value of m_{H^+} in the two-Higgs doublet model will satisfy the constraint at 99% confidence level; or (c) there is new physics, different from the 2HDM, that interferes constructively with the SM amplitude such as in the R-parity-violating model of Akeroyd and Recksiegel [31].

To sum up, the situation is not clear. To set limits on new physics we need an independent calculation of f_{Ds} with comparable accuracy, and more precise measurements would also be useful.

The B^+ decay constant: The Belle and BaBar collaborations have found evidence for $B^- \to \tau^- \overline{\nu}$ decay in $e^+e^- \to B^-B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and or photon candidates, and examining the extra energy in the event which should be close to zero for a real τ^- decay to $e^-\nu\bar{\nu}$ or $\mu^-\nu\bar{\nu}$ opposite a B^+ tag. The results are listed in Table 4.

Table 4: Experimental results for $\mathcal{B}(B^- \to \tau^-\overline{\nu})$. We have computed an average for the two Belle measurements assuming that the systematic errors are fully correlated.

Experiment	Tag	$\mathcal{B}$ (units of 10^{-4})
Belle [32]	Hadronic	$1.79^{+0.56+0.46}_{-0.49-0.51}$
Belle [33]	Semileptonic	$1.54^{+0.38+0.29}_{-0.37-0.31}$
Belle	Our average	1.62 ± 0.40
BaBar [34]	Hadronic	$1.80^{+0.57}_{-0.54} \pm 0.26$
BaBar [35]	Semileptonic	$1.7 \pm 0.8 \pm 0.2$
BaBar	Average [34]	1.76 ± 0.49
	Our average	1.68 ± 0.31

There are large backgrounds under the signals in all cases. The systematic errors are also quite large, on the order of 20%. Thus, the significance of the signals is not that large. Belle quotes 3.5σ and 3.6σ for their hadronic and semileptonic tags, while BaBar quotes 3.3σ and 2.3σ for these tags. We note that the four central values are remarkably close to the average considering the large errors on all the measurements. More accuracy would be useful to investigate the effects of new physics.

We extract a SM value using Eq. (1). Here theory provides a value of $f_B = (194 \pm 9)$ MeV [36]. We also need a value for $|V_{ub}|$. Here significant differences arise between using inclusive charmless semileptonic decays and the exclusive decay $B \to \pi\ell^+\nu$ [37]. The inclusive decays give rise to a value of $|V_{ub}| = (4.27 \pm 0.38) \times 10^{-3}$ while the exclusive measurements yield $|V_{ub}| = (3.38 \pm 0.36) \times 10^{-3}$, where the errors are dominantly theoretical [38]. Their average, enlarging the error in the standard manner because the results differ, is $|V_{ub}| = (3.80 \pm 0.44) \times 10^{-3}$. Using these values and the PDG values for the B^+ mass and lifetime, we arrive at the SM prediction for the $\tau^-\bar{\nu}$ branching fraction of $(0.96 \pm 0.24) \times 10^{-4}$. This value is about a factor of two smaller than the measurements. There is a 6.6% probability that the data and the SM prediction are consistent.

This difference is more clearly seen by examining the correlation between the CKM angle β and $\mathcal{B}(B^- \to \tau^-\bar{\nu})$. The CKM fitter group provides a fit to a large number of measurements involving heavy quark transitions [39]. The point in Fig. 2 shows the directly measured values, while the predictions from their fit without the direct measurements are also shown. There is about a factor of two discrepancy between the measured value of $\mathcal{B}(B^- \to \tau^-\overline{\nu})$ and the fit prediction.

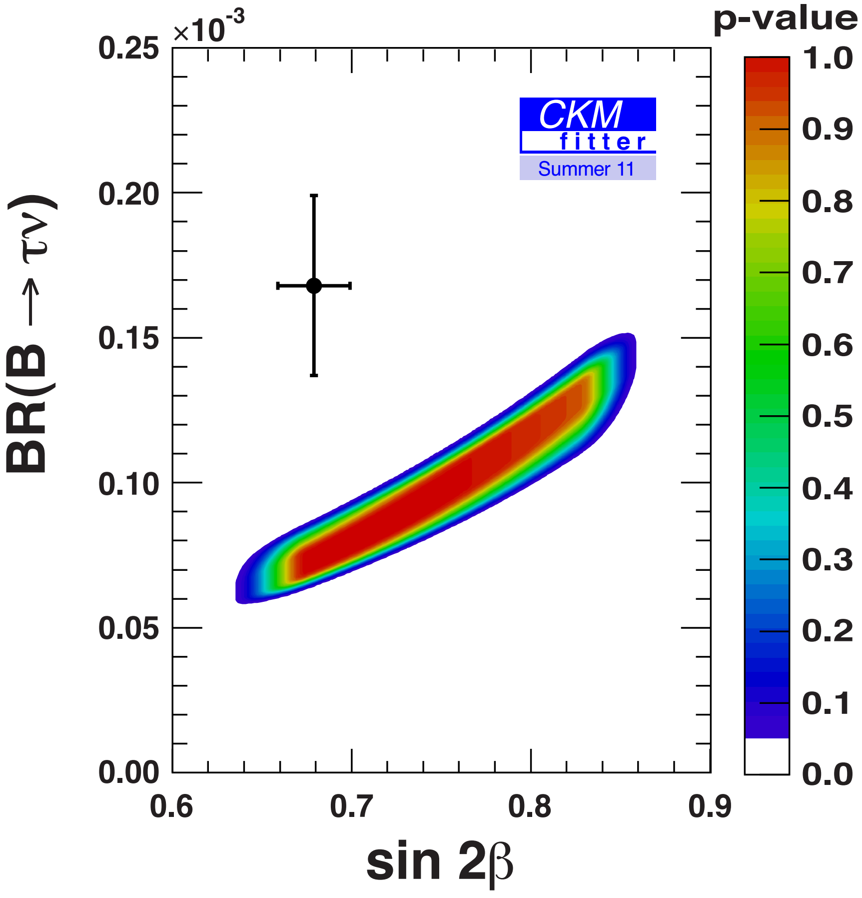

Figure 2: Measured and predicted values of $\mathcal{B}(B^- \to \tau^-\overline{\nu})$ versus $\sin 2\beta$ from the CKM fitter group [39]. The point with error bars shows the measured values, while the predictions are in shaded contours, with the shading related to the confidence level.

π^+ and K^+ decay constants: The sum of branching fractions for $\pi^- \to \mu^-\bar{\nu}$ and $\pi^- \to \mu^-\bar{\nu}\gamma$ is 99.98770(4)%. The two modes are difficult to separate experimentally, so we use this sum, with Eq. (1) modified to include photon emission and radiative corrections [40]. The branching fraction together with the lifetime 26.033(5) ns gives

$$f_{\pi^-} = (130.41 \pm 0.03 \pm 0.20)\ \text{MeV}\ .$$

The first error is due to the error on $|V_{ud}|$, 0.97425(22) [6]; the second is due to the higher-order corrections, and is much larger.

Similarly, the sum of branching fractions for $K^- \to \mu^-\bar{\nu}$ and $K^- \to \mu^-\bar{\nu}\gamma$ is 63.55(11)%, and the lifetime is 12.3840(193) ns [41]. Measurements of semileptonic kaon decays provide a value for the product $f_+(0)|V_{us}|$, where $f_+(0)$ is the form-factor at zero four-momentum transfer between the initial state kaon and the final state pion. We use a value for $f_+(0)|V_{us}|$ of 0.21664(48) [41]. The $f_+(0)$ must be determined theoretically. We follow Blucher and Marciano [7] in using the lattice calculation $f_+(0) = 0.9644 \pm 0.0049$ [42], since it appears to be more precise than the classic Leutwyler-Roos calculation $f_+(0) = 0.961 \pm 0.008$ [43]. [Other recent averages are 0.956 ± 0.008 [49] and 0.9588 ± 0.0044 [44].] Using the value from Ref. [42], the result is $|V_{us}| = 0.2246 \pm 0.0012$, consistent with the hyperon decay value of 0.2250 ± 0.0027 [45]. We derive

$$f_{K^-} = (156.1 \pm 0.2 \pm 0.8 \pm 0.2)\ \text{MeV}\ .$$

The first error is due to the error on Γ; the second is due to the CKM factor $|V_{us}|$, and the third is due to the higher-order corrections. The largest source of error in these corrections depends on the QCD part, which is based on one calculation in the large N_c framework. We have doubled the quoted error here; this would probably be unnecessary if other calculations were to come to similar conclusions. A large part of the additional uncertainty vanishes in the ratio of the K^- and π^- decay constants, which is

$$f_{K^-}/f_{\pi^-} = 1.197 \pm 0.002 \pm 0.006 \pm 0.001\ .$$

The first error is due to the measured decay rates; the second is due to the uncertainties on the CKM factors; the third is due to the uncertainties in the radiative correction ratio.

These measurements have been used in conjunction with calculations of f_K/f_π in order to find a value for $|V_{us}|/|V_{ud}|$. Three recent lattice predictions of f_K/f_π are 1.189 ± 0.007 [46], $1.192 \pm 0.007 \pm 0.006$ [47], and $1.197 \pm 0.002^{+0.003}_{-0.007}$ [48], yielding an average by the FLAG group of 1.195 ± 0.005 [49]. (A new average 1.1872 ± 0.0041 is quoted with statistical errors only [50]) . Together with the precisely measured $|V_{ud}|$, this gives an independent measure of $|V_{us}|$ [8,41].

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