LEPTONIC DECAYS OF CHARGED PSEUDOSCALAR MESONS

Revised September 2013 by J. Rosner (Univ. Chicago) and S. Stone (Syracuse Univ.)

We review the physics of purely leptonic decays of $\pi^\pm$, $K^\pm$, $D^\pm$, $D_s^\pm$, and $B^\pm$ pseudoscalar mesons. The measured decay rates are related to the product of the relevant weak-interaction-based CKM matrix element of the constituent quarks and a strong interaction parameter related to the overlap of the quark and antiquark wave-functions in the meson, called the decay constant f_P. The interplay between theory and experiment is different for each particle. Theoretical predictions of f_B that are needed in the B sector can be tested by measuring f_{D^+} and $f_{D_s^+}$ in the charm sector. The lighter $\pi^\pm$ and $K^\pm$ mesons provide stringent comparisons between experiment and theory due to the accuracy of both the measurements and the theoretical predictions [1].

Introduction: Charged mesons formed from a quark and an antiquark can decay to a charged lepton pair when these objects annihilate via a virtual W boson. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.

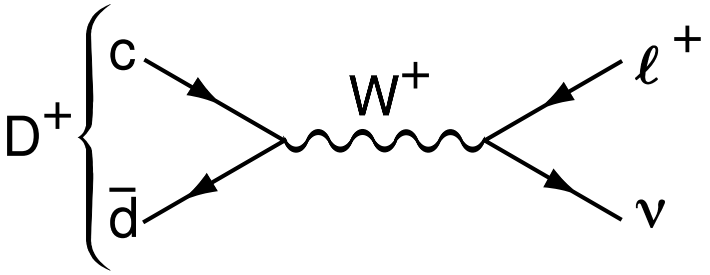

Figure 1: The annihilation process for pure D^+ leptonic decays in the Standard Model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+, K^+, D_s^+, and B^+ mesons. (Charge-conjugate particles and decays are implied.) Let P be

CITATION: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014) (URL: http://pdg.lbl.gov)

any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \to \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 \, m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 \quad . \qquad (1)$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1\bar{q}_2$ in P, and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, proportional to the matrix element of the axial current between the one-P-meson state and the vacuum, and is related to the wave-function overlap of the quark and antiquark.

The decay $P \to \ell\nu$ starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the $\ell^\pm$ must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width. Radiative corrections are needed when the final charged particle is an electron or muon [2].

Measurements of purely leptonic decay branching fractions and lifetimes allow an experimental determination of the product $|V_{q_1 q_2}|\, f_P$. If the CKM element is well known from other measurements, then f_P can be well measured. If, on the other hand, the CKM element is not well measured, having theoretical input on f_P can allow a determination of the CKM element. The importance of measuring $\Gamma(P \to \ell\nu)$ depends on the particle being considered. For example, the measurement of $\Gamma(B^- \to \tau^-\overline{\nu})$ provides an indirect determination of $|V_{ub}|$ provided that f_B is provided by theory. In addition, f_B is crucial for using measurements of B^0-$\overline{B}^0$ mixing to extract information on the fundamental CKM parameters. Knowledge of f_{B_s} is also needed, but it cannot be directly measured as the B_s is neutral, so the violation of the SU(3) relation $f_{B_s} = f_B$ must be estimated theoretically. This difficulty does not occur for D mesons as both the D^+ and D_s^+ are charged, allowing the direct measurement of SU(3) breaking and a direct comparison with theory.

For B^- and D_s^+ decays, the existence of a charged Higgs boson (or any other charged object beyond the Standard Model) would modify the decay rates; however, this would not necessarily be true for the D^+ [3,4]. More generally, the ratio of $\tau\nu$ to $\mu\nu$ decays can serve as one probe of lepton universality [3,5].

As $|V_{ud}|$ has been quite accurately measured in super-allowed β decays [6], with a value of 0.97425(22) [7], measurements of $\Gamma(\pi^+ \to \mu^+\nu)$ yield a value for f_π. Similarly, $|V_{us}|$ has been well measured in semileptonic kaon decays, so a value for f_K from $\Gamma(K^- \to \mu^-\bar{\nu})$ can be compared to theoretical calculations. Lattice gauge theory calculations, however, have been claimed to be very accurate in determining f_K, and these have been used to predict $|V_{us}|$ [8].

Charmed mesons: Our review of current measurements starts with the charm system. Measurements have been made for $D^+ \to \mu^+\nu$, and $D_s^+ \to \mu^+\nu$ and $\tau^+\nu$. Only an upper limit has been determined for $D^+ \to \tau^+\nu$. Both CLEO-c and BES have made measurements of D^+ decay using e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D's, say the D^-. Counting the number of these events provides the normalization for the branching fraction measurement. They then find a candidate μ^+, and then form the missing-mass squared, $MM^2 = (E_{\rm CM} - E_{D^-})^2 - \left(\overrightarrow{p}_{\rm CM} - \overrightarrow{p}_{D^-} - \overrightarrow{p}_{\mu^+}\right)^2$, taking into account their knowledge of the center-of-mass energy, $E_{\rm CM}$, and momentum, $p_{\rm CM}$, that equals zero in e^+e^- collisions. A peak at zero MM^2 infers the existence of a missing neutrino and hence the $\mu^+\nu$ decay of the D^+. CLEO-c does not explicitly identify the muon, so their data consist of a combination of $\mu^+\nu$ and $\tau^+\nu$, $\tau^+ \to \pi^+\nu$ events. This permits them to do two fits; in one they fit for the individual components, and in the other they fix the ratio of $\tau^+\nu/\mu^+\nu$ events to be that given by the SM expectation. Thus, the latter measurement should be used for SM comparisons and the other for new physics searches. Our average uses the fixed ratio value. The measurements are shown in Table 1.

Table 1: Experimental results for $\mathcal{B}(D^+ \to \mu^+\nu)$, $\mathcal{B}(D^+ \to \tau^+\nu)$, and f_{D^+}. Numbers for f_{D^+} have been extracted using updated values for masses and $|V_{cd}|$ (see text). Radiative corrections are included. Systematic uncertainties arising from the D^+ lifetime and mass are included.

Experiment	Mode	$\mathcal{B}$	f_{D^+} (MeV)
CLEO-c [9]	$\mu^+\nu$	$(3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$	$209.1 \pm 9.3 \pm 2.5$
CLEO-c [9]	$\mu^+\nu$+$\tau^+\nu$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$	$206.2 \pm 8.6 \pm 2.6$
BES [10]	$\mu^+\nu$	$(3.74 \pm 0.21 \pm 0.06) \times 10^{-4}$	$204.0 \pm 5.7 \pm 2.0$
Average	$\mu^+\nu$	$(3.76 \pm 0.18) \times 10^{-4}$	204.6 ± 5.0
CLEO-c [13]	$\tau^+\nu$	$< 1.2 \times 10^{-3}$	

To extract the value of f_{D^+} we use the well-measured D^+ lifetime of 1.040(7) ps. The value of $|V_{cd}|$ is taken to equal to the value of $|V_{us}|$ of 0.2252(9) [7] minus higher-order correction terms [11], which results in $|V_{cd}| = 0.2251(9)$. The $\mu^+\nu$ results include a 1% correction (lowering) of the rate due to the presence of the radiative $\mu^+\nu\gamma$ final-state based on the estimate by Dobrescu and Kronfeld [12].

Before we compare this result with theoretical predictions, we discuss the D_s^+. Measurements of $f_{D_s^+}$ have been made by several groups and are listed in Table 2 [13–17]. We exclude older values obtained by normalizing to D_s^+ decay modes that are not well defined. Many measurements, for example, used the $\phi\pi^+$ mode. This decay is a subset of the $D_s^+ \to K^+K^-\pi^+$ channel which has interferences from other modes populating the K^+K^- mass region near the ϕ, the most prominent of which is the $f_0(980)$. Thus the extraction of effective $\phi\pi^+$ rate is sensitive to the mass resolution of the experiment and the cuts used to define the ϕ mass region [18,19].

To find decays in the $\mu^+\nu$ signal channels, CLEO, BaBar and Belle rely on fully reconstructing all the final-state particles except for neutrinos and using a missing-mass technique to infer the existence of the neutrino. CLEO uses $e^+e^- \to D_sD_s^*$

collisions at 4170 MeV, while Babar and Belle use $e^+e^- \rightarrow DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$. CLEO does a similar analysis as was done for the D^+ above. Babar and Belle do a similar MM^2 calculation by using the reconstructed hadrons, the photon from the D_s^{*+} decay and a detected μ^+. To get the normalization they do a MM^2 fit without the μ^+ and use the signal at the D_s^+ mass squared to determine the total D_s^+ yield.

Table 2: Experimental results for $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu)$, $\mathcal{B}(D_s^+ \rightarrow \tau^+\nu)$, and $f_{D_s^+}$. Numbers for $f_{D_s^+}$ have been extracted using updated values for masses and $|V_{cs}|$ (see text). Radiative corrections and systematic uncertainties for errors on the D_s^+ lifetime and mass have been included. Common systematic errors in each experiment have been taken into account.

Experiment	Mode	$\mathcal{B}(\%)$	$f_{D_s^+}$ (MeV)
CLEO-c [13]	$\mu^+\nu$	$0.565 \pm 0.045 \pm 0.017$	$257.6 \pm 10.3 \pm 4.3$
BaBar [14]	$\mu^+\nu$	$0.602 \pm 0.038 \pm 0.034$	$265.9 \pm 8.4 \pm 7.7$
Belle [15]	$\mu^+\nu$	$0.531 \pm 0.028 \pm 0.020$	$249 \pm 6.6 \pm 5.0$
Average	$\mu^+\nu$	0.556 ± 0.024	255.6 ± 5.9
CLEO-c [13]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$6.42 \pm 0.81 \pm 0.18$	$278.0 \pm 17.5 \pm 4.4$
CLEO-c [16]	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$5.52 \pm 0.57 \pm 0.21$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [17]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.30 \pm 0.47 \pm 0.22$	$252.6 \pm 11.2 \pm 5.6$
BaBar [14]	$\tau^+\nu$ $(e^+/\mu^+\nu\overline{\nu})$	$5.00 \pm 0.35 \pm 0.49$	$245.4 \pm 8.6 \pm 12.2$
Belle [15]	$\tau^+\nu$ $(\pi^+\overline{\nu})$	$6.04 \pm 0.43^{+0.46}_{-0.40}$	$269.6 \pm 9.6^{+10.4}_{-9.1}$
Belle [15]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$5.37 \pm 0.33^{+0.35}_{-0.31}$	$254.2 \pm 7.8^{+8.5}_{-7.6}$
Belle [15]	$\tau^+\nu$ $(\mu^+\nu\overline{\nu})$	$5.86 \pm 0.37^{+0.34}_{-0.59}$	$265.5 \pm 8.4^{+7.9}_{-13.5}$
Average	$\tau^+\nu$	5.56 ± 0.22	258.3 ± 5.5

When selecting the $\tau^+ \rightarrow \pi^+\bar{\nu}$ and $\tau^+ \rightarrow \rho^+\bar{\nu}$ decay modes, CLEO uses both calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s^- and the τ^+ decay products. The $\tau^+ \rightarrow e^+\nu\bar{\nu}$ mode, however, uses only extra energy. Babar and Belle also use no extra energy to discriminate signal from background in their $\tau^+\nu$ measurements.

We extract the decay constant from the measured branching ratios using the D_s^+ mass of 1.96849(32) GeV, the τ^+ mass of 1.77682(16) GeV, and a lifetime of 0.500(7) ps. We use the first-order correction $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2$ [11]; taking $|V_{ud}| = 0.97425(22)$ [6], and $|V_{cb}| = 0.04$ from an average of exclusive and inclusive semileptonic B decay results as discussed in Ref. [20], and find $|V_{cs}| = 0.97345(22)$. CLEO has included the radiative correction of 1% in the $\mu^+\nu$ rate listed in the Table [12] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that the $\mu^+\nu\gamma$ rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [21]. As this is a small effect we do not attempt to correct the other measurements.

The average decay constant cannot simply be obtained by averaging the values in Table 2 since there are correlated errors between the $\mu^+\nu$ and $\tau^+\nu$ values. Table 3 gives the average values of f_{D_s} where the experiments have included the correlations.

Table 3: Experimental results for $f_{D_s^+}$ taking into account the common systematic errors in the $\mu^+\nu$ and $\tau^+\nu$ measurements.

Experiment	$f_{D_s^+}$ (MeV)
CLEO-c	$259.0 \pm 6.2 \pm 3.0$
BaBar	$258.4 \pm 6.4 \pm 7.5$
Belle	$257.8 \pm 4.2 \pm 4.8$
Average of $\mu^+\nu + \tau^+\nu$	257.5 ± 4.6

Our experimental average is

$$f_{D_s^+} = (257.5 \pm 4.6)\ \text{MeV}.$$

Furthermore, the ratio of branching fractions is found to be

$$R \equiv \frac{\mathcal{B}(D_s^+ \to \tau^+\nu)}{\mathcal{B}(D_s^+ \to \mu^+\nu)} = 10.0 \pm 0.6,$$

where a value of 9.76 is predicted in the Standard Model. Assuming lepton universality then we can derive improved values for the leptonic decay branching fractions of

$$\mathcal{B}(D_s^+ \to \mu^+\nu) = (5.64 \pm 0.20) \times 10^{-3}, \quad \text{and}$$

$$\mathcal{B}(D_s^+ \to \tau^+\nu) = (5.51 \pm 0.20) \times 10^{-2} \ .$$

The experimentally determined ratio of decay constants is $f_{D_s^+}/f_{D^+} = 1.258 \pm 0.038$. Table 4 compares the experimental $f_{D_s^+}$ with theoretical calculations [22–27,30,31]. Most theories give values lower than the $f_{D_s^+}$ measurement. The discrepancies with the models with the smallest quoted uncertainties, both unquenched lattice calculations, are 2.0 standard deviations with HPQCD [22], and 1.9 standard deviations with the preliminary FNAL+MILC prediction [23].

Table 4: Theoretical predictions of $f_{D_s^+}$, f_{D^+}, and $f_{D_s^+}/f_{D^+}$. Quenched lattice calculations are omitted, while PQL indicates a partially-quenched lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	257.5 ± 4.6	204.6 ± 5.0	1.258 ± 0.038
Lattice (HPQCD) [22]	$246.0 \pm 0.7 \pm 3.5$	$208.3 \pm 1.0 \pm 3.3$	$1.187 \pm 0.004 \pm 0.012$
Lattice (FNAL+MILC) [23]	$246.4 \pm 0.5 \pm 3.6$	$209.2 \pm 3.0 \pm 3.6$	1.175 ± 0.019
PQL [24]	244 ± 8	197 ± 9	1.24 ± 0.03
QCD sum rules [25]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD sum rules [26]	$245.3 \pm 15.7 \pm 4.5$	$206.2 \pm 7.3 \pm 5.1$	$1.193 \pm 0.025 \pm 0.007$
QCD sum rules [27]	246 ± 6	204 ± 6	1.21 ± 0.04
QCD sum rules [28](I)	241 ± 12	208 ± 11	1.16 ± 0.07
QCD sum rules [28](II)	258 ± 13	211 ± 14	1.22 ± 0.08
QCD sum rules [29]	238^{+13}_{-23}	201^{+12}_{-13}	$1.15^{+0.04}_{-0.05}$
Field correlators [30]	260 ± 10	210 ± 10	1.24 ± 0.03
Light front [31]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04

Upper limits on f_{D^+} and f_{D_s} of 230 and 270 MeV, respectively, have been determined using two-point correlation functions by Khodjamirian [32]. The D^+ result is safely below this limit, while the average D_s^+ result is also, but older results [1] not used in our average are often above the limit.

Akeroyd and Chen [33] pointed out that leptonic decay widths are modified in two-Higgs-doublet models (2HDM). Specifically, for the D^+ and D_s^+, Eq. (1) is modified by a factor r_q multiplying the right-hand side [34]:

$$r_q = \left[1 + \left(\frac{1}{m_c + m_q} \right) \left(\frac{M_{D_q}}{M_{H^+}} \right)^2 \left(m_c - \frac{m_q \tan^2\beta}{1 + \epsilon_0 \tan\beta} \right) \right]^2 ,$$

where m_{H^+} is the charged Higgs mass, M_{D_q} is the mass of the D meson (containing the light quark q), m_c is the charm quark mass, m_q is the light-quark mass, and $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. In models where the fermion mass arises from coupling to more than one vacuum expectation value ϵ_0 can be non-zero, perhaps as large as 0.01. For the D^+, $m_d \ll m_c$, and the change due to the H^+ is very small. For the D_s^+, however, the effect can be substantial.

In order to investigate the possible presence of new physics we need to specify a SM value of $f_{D_s^+}$. We can only use a theory prediction. Our most aggressive choice is that of the unquenched lattice calculation [22], because it claims the smallest error. Since the charged Higgs would lower the rate compared to the SM, in principle, experiment gives a lower limit on the charged Higgs mass. However, the value for the predicted decay constant using this model is 2.0 standard deviations *below* the measurement. If this small discrepancy is to be taken seriously, either (a) the model of Ref. [22] is not representative; (b) no value of m_{H^+} in the two-Higgs doublet model will satisfy the constraint at 99% confidence level; or (c) there is new physics, different from the 2HDM, that interferes constructively with the SM amplitude such as in the R-parity-violating model of Akeroyd and Recksiegel [35].

To sum up, the standard model calculations are now consistent with the data and new physics effects are small. Limits can be placed on new particles depending on the specific model.

The B^- meson: The Belle and BaBar collaborations have found evidence for $B^- \to \tau^- \overline{\nu}$ decay in $e^+e^- \to B^-B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and photon candidates, and examining the extra energy in the event which should be close to zero for a real τ^- decay to $e^-\nu\bar{\nu}$ or $\mu^-\nu\bar{\nu}$ opposite a B^+ tag. While the BaBar results have remained unchanged, Belle did a reanalysis of their data using the hadronic B decay sample. The branching fraction changed from $1.79\,^{+0.56}_{-0.49}\,^{+0.46}_{-0.51} \times 10^{-4}$ [36] to $0.72^{+0.27}_{-0.25} \pm 0.11 \times 10^{-4}$ [37]. This change demonstrates the difficulty of the analysis. It is unfortunate that other results have not been updated. The results are listed in Table 5.

Table 5: Experimental results for $\mathcal{B}(B^- \to \tau^-\overline{\nu})$.

Experiment	Tag	$\mathcal{B}$ (units of 10^{-4})
Belle [37]	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
Belle [38]	Semileptonic	$1.54\,^{+0.38}_{-0.37}\,^{+0.29}_{-0.31}$
Belle [37]	Average	0.96 ± 0.26
BaBar [39]	Hadronic	$1.83\,^{+0.53}_{-0.49} \pm 0.24$
BaBar [40]	Semileptonic	$1.7 \pm 0.8 \pm 0.2$
BaBar [39]	Average	1.79 ± 0.48
	Our average	1.14 ± 0.23

There are large backgrounds under the signals in all cases. The systematic errors are also quite large. Thus, the significances are not that large. Belle quotes 3.0σ and 3.6σ for their hadronic and semileptonic tags, while BaBar quotes 3.3σ and 2.3σ for these tags. More accuracy would be useful to investigate the effects of new physics.

We extract a SM value using Eq. (1). Here theory provides a value of $f_B = (190.6 \pm 4.7)$ MeV [41]. We also need a value for $|V_{ub}|$. Here significant differences arise between using inclusive charmless semileptonic decays and the exclusive decay $B \to \pi\ell^+\nu$ [42]. The inclusive decays give rise to a value of $|V_{ub}| = (4.41 \pm 0.22) \times 10^{-3}$ while the exclusive measurements yield $|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$, where the errors are dominantly theoretical [43]. Their average, enlarging the error in the standard manner because the results differ, is $|V_{ub}| = (4.01 \pm 0.56) \times 10^{-3}$. Using these values and the PDG values for the B^+ mass and lifetime, we arrive at the SM prediction for the $\tau^-\bar{\nu}$ branching fraction of $(1.03 \pm 0.29) \times 10^{-4}$. This value is now consistent with the average.

It is instructive to examine the correlation between the CKM angle β and $\mathcal{B}(B^- \to \tau^-\bar{\nu})$. The CKM fitter group provides a fit to a large number of measurements involving heavy quark transitions [44]. The black point in Fig. 2 shows the directly measured values from 2012, while the predictions from their fit without the direct measurements are also shown. There is about a factor of two discrepancy between the old measured average value of $\mathcal{B}(B^- \to \tau^-\bar{\nu})$ and the fit prediction. The (purple) dashed point shows the new Belle measurement only, and is consistent with the prediction, as is the new average.

Charged pions and kaons: We now discuss the determination of charged pion and kaon decay constants. The sum of branching fractions for $\pi^- \to \mu^-\bar{\nu}$ and $\pi^- \to \mu^-\bar{\nu}\gamma$ is 99.98770(4)%. The two modes are difficult to separate experimentally, so we use this sum, with Eq. (1) modified to include photon emission and radiative corrections [45]. The branching fraction together with the lifetime 26.033(5) ns gives

$$f_{\pi^-} = (130.41 \pm 0.03 \pm 0.20)\ \text{MeV}\ \ .$$

The first error is due to the error on $|V_{ud}|$, 0.97425(22) [6]; the second is due to the higher-order corrections, and is much larger.

Similarly, the sum of branching fractions for $K^- \to \mu^-\bar{\nu}$ and $K^- \to \mu^-\bar{\nu}\gamma$ is 63.55(11)%, and the lifetime is 12.3840(193) ns [46]. Measurements of semileptonic kaon decays provide a

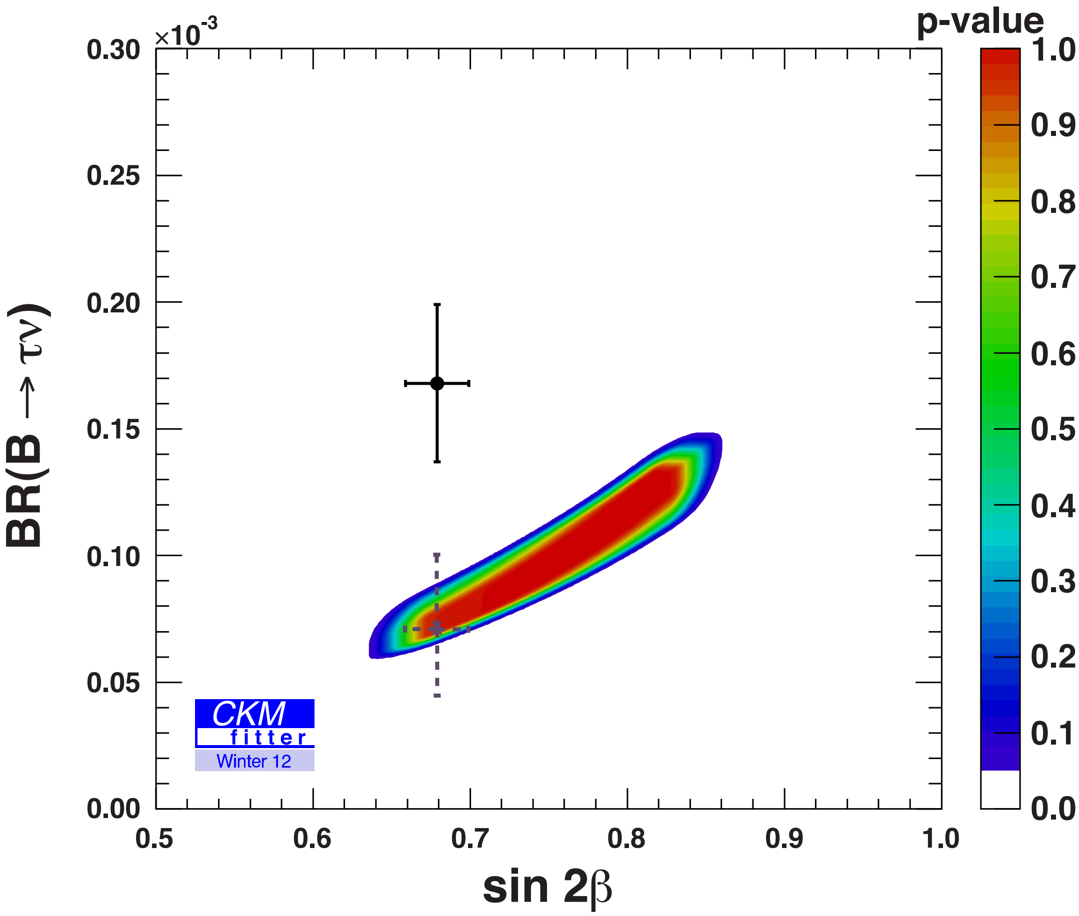

Figure 2: Measured versus predicted values of $\mathcal{B}(B^- \to \tau^- \overline{\nu})$ versus $\sin 2\beta$ from the CKM fitter group. The solid (black) point with error bars shows the old (2012) measured average value, the dashed (purple) point the new Belle measurement, while the predictions are in colors, with the color being related to the confidence level. (Adopted from the CKM Fitter group.)

value for the product $f_+(0)|V_{us}|$, where $f_+(0)$ is the form factor at zero four-momentum transfer between the initial state kaon and the final state pion. We use a value for $f_+(0)|V_{us}|$ of 0.2163(5) [46]. The $f_+(0)$ must be determined theoretically. The two most recent determinations from lattice QCD are 0.9667(23)(33) [47] and $0.9599(34)(^{+31}_{-43})$ [48], whose average is $f_+(0) = 0.9638(30)$. This is more precise than the classic

Table 6: Lattice calculations of f_K/f_π and extracted values of $|V_{us}|/|V_{ud}|$.

Group	f_K/f_π	$\lvert V_{us}\rvert/\lvert V_{ud}\rvert$
HPQCD [52]	1.1916 ± 0.0021	0.23160(54)
Laiho and Van de Water [53]	$1.202 \pm 0.011 \pm 0.013$	–
BMW [54]	$1.192 \pm 0.007 \pm 0.006$	0.2315(19)
MILC [55]	$1.1947 \pm 0.0026 \pm 0.0037$	0.2309(10)
RBC/UKQCD [56]	$1.204 \pm 0.007 \pm 0.025$	–

Leutwyler-Roos calculation $f_+(0) = 0.961 \pm 0.008$ [49]. The result is $|V_{us}| = 0.2244(9)$, which is consistent with the hyperon decay value of 0.2250(27) [50].

Experimental branching ratios provide the ratio [52]

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^-}} = 0.27598(35)(25) \ ,$$

where the first error is due to branching fractions and the second is due to electromagnetic corrections. With $|V_{ud}| = 0.97425(22)$, f_{π^-} as given above, and $|V_{us}| = 0.2244(9)$, we then find

$$f_{K^-} = (156.2 \pm 0.2 \pm 0.6 \pm 0.3) \ \text{MeV} \ .$$

The first uncertainty is due to the error on Γ; the second is due to the CKM factor $|V_{us}|$, and the third is due to the higher-order corrections. The largest source of error in these corrections depends on the QCD part, which is based on one calculation in the large N_c framework. A large part of the additional uncertainty vanishes in the ratio of the K^- and π^- decay constants, which is

$$f_{K^-}/f_{\pi^-} = 1.198 \pm 0.002 \pm 0.005 \pm 0.001 \ .$$

The first uncertainty is due to the measured decay rates; the second is due to the uncertainties on the CKM factors; the third is due to the errors in the radiative correction ratio. These measurements can be used in conjunction with calculations of f_K/f_π in order to find a value for $|V_{us}|/|V_{ud}|$ [51]. Recent lattice predictions of f_K/f_π are shown in Table 6.

These calculations are in agreement with our experimental average. Together with the precisely measured $|V_{ud}|$, these results can be used to find an independent measure of $|V_{us}|$ [8,46].

We gratefully acknowledge support of the U. S. National Science Foundation and the U. S. Department of Energy through Grant No. DE-FG02-90ER40560. We thank A. Khodjamirian, J. Laiho, W. Marciano, S. Narison, and Z.-G. Wang for useful discussions and references.

References

1. This article is an update of the one prepared for the PDG 2012 edition; see J. L. Rosner and S. Stone in J. Beringer *et al.*, Phys. Rev. D **86**, 946 (2012).
2. Most predictions for the rate for $B^- \to \mu^-\bar{\nu}\gamma$ are in the range of 1 to 20 times the rate for $B^- \to \mu^-\bar{\nu}$. See G. Burdman, T. Goldman, and D. Wyler, Phys. Rev. **D51**, 111 (1995); P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. **B372**, 331 (1996); *ibid.*, **386**, 328 (1996); A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. **B358**, 129 (1995); G. Eilam, I. Halperin, and R. Mendel, Phys. Lett. **B361**, 137 (1995); D. Atwood, G. Eilam, and A. Soni, Mod. Phys. Lett. **A11**, 1061 (1996); C.Q. Geng and C.C. Lih, Phys. Rev. **D57**, 5697 (1998) and Mod. Phys. Lett. **A15**, 2087 (2000); G.P. Korchemsky, D. Pirjol, and T. M. Yan, Phys. Rev. **D61**, 114510 (2000); C.W. Hwang, Eur. Phys. J. **C46**, 379 (2006).
3. W.-S. Hou, Phys. Rev. **D48**, 2342 (1993).
4. See, for example, A.G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003); A.G. Akeroyd, Prog. Theor. Phys. **111**, 295 (2004).
5. J.L. Hewett, `hep-ph/9505246`, presented at *Lafex International School on High Energy Physics (LISHEP95)*, Rio de Janeiro, Brazil, Feb. 6-22, 1995.
6. I.S. Towner and J.C. Hardy, Phys. Rev. **C79**, 055502 (2009); *ibid.* **77**, 025501 (2008).
7. E. Blucher and W. J. Marciano, "V_{ud}, V_{us}, the Cabibbo Angle and CKM Unitarity" in J. Beringer *et al.*, (Particle Data Group), Phys. Rev. D **86**, 852 (2012).
8. G. Colangelo *et al.*, Eur. Phys. J. **C71**, 1695 (2011); W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004); A. Jüttner, PoS LAT2007, 014 (2007) `[arXiv:0711.1239]`.

9. B. I. Eisenstein *et al.* (CLEO Collab.), Phys. Rev. **D78**, 052003 (2008). See also M. Artuso *et al.* (CLEO Collab.), Phys. Rev. Lett. **95**, 251801 (2005).
10. G. Huang (BESIII Collab.), "Recent Results from BESIII," presented at FPCP 2012, Hefei, China May, 2012, `[arXiv:1209.4813]`.
11. J. Charles *et al.*, Eur. Phys. J. **C41**, 1 (2005).
12. B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008).
13. J.P. Alexander *et al.* (CLEO Collab.), Phys. Rev. **D79**, 052001 (2009); M. Artuso *et al.* (CLEO Collab.), Phys. Rev. Lett. **99**, 071802 (2007).
14. P. del Amo Sanchez *et al.* (BaBar Collab.), Phys. Rev. D **82**, 091103 (2010) `[arXiv:1008.4080]`. We do not use the previous BaBar result from a subsample of data as reported in J. P. Lees *et al.* (BaBar Collab.), `[arXiv:1003.3063]`.
15. A. Zupanc *et al.*, (Belle Collab.), `[arXiv:1307.6240]`.
16. P. Naik *et al.* (CLEO Collab.), Phys. Rev. **D80**, 112004 (2009).
17. P.U.E. Onyisi *et al.* (CLEO Collab.), Phys. Rev. **D79**, 052002 (2009); K. M. Ecklund *et al.* (CLEO Collab.), Phys. Rev. Lett. **100**, 161801 (2008).
18. See J. Alexander *et al.* (CLEO Collab.), Phys. Rev. Lett. **100**, 161804 (2008).
19. We have not included a BaBar result: $\mathcal{B}(D_s^+ \to \mu^+\nu) = (6.67 \pm 0.83 \pm 0.26 \pm 0.66) \times 10^{-3}$ and $f_{D_s^+} = (281 \pm 17 \pm 7 \pm 14)$ MeV based on $\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.71 \pm 0.46)\%$. These measurements determined the ratio of the leptonic decay to a hadronic decay, usually $\Gamma(D_s^+ \to \ell^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$. See B. Aubert *et al.* (BABAR Collab.), Phys. Rev. Lett. **98**, 141801 (2007).
20. M. Artuso, E. Barberio, and S. Stone, PMC Physics A, 3:3 (2009) `[arXiv:0902.3743]`.
21. See most papers in Ref. 2 and C.D. Lü and G.L. Song, Phys. Lett. **B562**, 75 (2003).
22. H. Na *et al.*, (HPQCD Collab.), Phys. Rev. D **86**, 054510 (2012).
23. A. Bazavov *et al.* (Fermilab/MILC Collab.), `[arXiv:1112.3051]`, submitted to Phys. Rev. D.
24. B. Blossier *et al.*, JHEP **0907**, 043 (2009) `[arXiv:0904.0954]`.
25. J. Bordes, J. Peñarrocha, and K. Schilcher, JHEP **0511**, 014 (2005).

26. W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. **B701**, 82 (2011).
27. S. Narison, Phys. Lett. B **718**, 1321 (2013), `[arXiv:1209.2023]`; *ibid.*, **721**, 269 (2013).
28. Z.-G. Wang, `[arXiv:1301.1399]`. Cases I and II correspond to two different choices of heavy quark masses, QCD order, and renormalization scale..
29. P. Gelhausen *et al.*, Phys. Rev, D **88**, 014015 (2013) `[arXiv:1305.5432]`..
30. A.M. Badalian *et al.*, Phys. Rev. **D75**, 116001 (2007); see also A.M. Badalian and B.L.G. Bakker, `[hep-ph/0702229]`.
31. C.-W. Hwang, Phys. Rev. **D81**, 054022 (2010) `[arXiv:0910.0145]`.
32. A. Khodjamirian, Phys. Rev. **D79**, 031503 (2009).
33. A.G. Akeroyd and C.H. Chen, Phys. Rev. **D75**, 075004 (2007).
34. A.G. Akeroyd and F. Mahmoudi, JHEP **0904**, 121 (2009).
35. A.G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003).
36. K. Ikado *et al.* (Belle Collab.), Phys. Rev. Lett. **97**, 251802 (2006).
37. K. Ikado *et al.* (Belle Collab.), Phys. Rev. Lett. **110**, 131801 (2013)..
38. K. Hara *et al.* (Belle Collab.), Phys. Rev. **D82**, 071101R (2010).
39. J.P. Lees *et al.* (BaBar Collab.), Phys. Rev. **D88**, 031102 `[arXiv:1207.0698]`.
40. B. Aubert *et al.* (BaBar Collab.), Phys. Rev. **D81**, 051101R (2010).
41. We have averaged the values of A. Bazavov *et al.* (Fermilab Lattice and MILC Collabs.), Phys. Rev. **D85**, 114506 (2012) ($f_B = 196.9 \pm 5.5 \pm 7.0$ MeV), and H. Na *et al.* (HPQCD Collab.), Phys. Rev. **D86**, 034506 (2012) ($f_B = 189 \pm 3.1 \pm 3.2$ MeV), assuming that 3.2 MeV of the systematic uncertainty is fully correlated. See also S. Narison [27–29], who find respective values of $f_B = 206 \pm 7$ MeV, 189 ± 15 MeV(I), or 190 ± 17 MeV(II), and 207^{+17}_{-9} MeV using QCD sum rules.
42. See the discussion in M. Artuso, E. Barberio, and S. Stone, Ref. 20.
43. R. Kowalewski and T. Mannel, in J. Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 1111 (2012).

44. J. Charles *et al.* (CKMfitter Group), Phys. Rev. **D85**, 1 (2011) `[arXiv:1106.4041 [hep-ph]]`, updated results and plots available at: `http://ckmfitter.in2p3.fr`.
45. W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993); V. Cirigliano and I. Rosell, JHEP **0710**, 005 (2007).
46. Flavianet Working Group on Precise SM Tests in K Decays, M. Antonelli *et al.*, Eur. Phys. J. **C69**, 399 (2010).
47. A. Bazavov *et al.* (Fermilab Lattice and MILC Collab.), Phys. Rev. D **87**, 073012 (2013).
48. P. A. Boyle *et al.* (RBC-UKQCD Collab.), Eur. Phys. J. C **69**, 159 (2010).
49. H. Leutwyler and M. Roos, Z. Phys. **C25**, 91 (1984).
50. N. Cabibbo, E.C. Swallow, and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004).
51. W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004), `[hep-ph/0402299]`.
52. R. J. Dowdall *et al.*, (HPQCD Collab.), `[arXiv:1303.1670]`.
53. J. Laiho and R. Van de Water, `[arXiv:1112.4861]`, in PoS Lattice2011, 293 (2011).
54. S. Durr *et al.*, Phys. Rev. D **81**, 054507 (2010), `[arXiv:1001.4692]`.
55. A. Bazavov *et al.* (MILC Collab.), Phys. Rev. Lett. **110**, 172003 (2013) `[arXiv:1301.5855]`.
56. Y. Aoki, *et al.* (RBC/UKQCD Collab.), Phys. Rev. D **83**, 074508 (2011) `[arXiv:1011.0892]`.