

## 21. EXPERIMENTAL TESTS OF GRAVITATIONAL THEORY

Revised November 2015, with an additional February 2016 comment on the observation of the first gravitational-wave event, by T. Damour (IHES, Bures-sur-Yvette, France), .

Einstein’s General Relativity, the current “standard” theory of gravitation, describes gravity as a universal deformation of the Minkowski metric:

$$g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda) , \text{ where } \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) . \quad (21.1)$$

General Relativity is classically defined by two postulates. One postulate states that the Lagrangian density describing the propagation and self-interaction of the gravitational field is

$$\mathcal{L}_{\text{Ein}}[g_{\alpha\beta}] = \frac{c^4}{16\pi G_N} \sqrt{g} g^{\mu\nu} R_{\mu\nu}(g_{\alpha\beta}) , \quad (21.2)$$

where  $G_N$  is Newton’s constant,  $g = -\det(g_{\mu\nu})$ ,  $g^{\mu\nu}$  is the matrix inverse of  $g_{\mu\nu}$ , and where the Ricci tensor  $R_{\mu\nu} \equiv R^\alpha_{\mu\alpha\nu}$  is the only independent trace of the curvature tensor

$$R^\alpha_{\mu\beta\nu} = \partial_\beta \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\sigma\beta} \Gamma^\sigma_{\mu\nu} - \Gamma^\alpha_{\sigma\nu} \Gamma^\sigma_{\mu\beta} , \quad (21.3)$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}) , \quad (21.4)$$

A second postulate states that  $g_{\mu\nu}$  couples universally, and minimally, to all the fields of the Standard Model by replacing everywhere the Minkowski metric  $\eta_{\mu\nu}$ . Schematically (suppressing matrix indices and labels for the various gauge fields and fermions and for the Higgs doublet),

$$\begin{aligned} \mathcal{L}_{\text{SM}}[\psi, A_\mu, H, g_{\mu\nu}] = & -\frac{1}{4} \sum \sqrt{g} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu}^a F_{\alpha\beta}^a - \sum \sqrt{g} \bar{\psi} \gamma^\mu D_\mu \psi \\ & - \frac{1}{2} \sqrt{g} g^{\mu\nu} \overline{D_\mu H} D_\nu H - \sqrt{g} V(H) - \sum \lambda \sqrt{g} \bar{\psi} H \psi , \end{aligned} \quad (21.5)$$

where  $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$ , and where the covariant derivative  $D_\mu$  contains, besides the usual gauge field terms, a spin-dependent gravitational contribution. From the total action follow Einstein’s field equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} . \quad (21.6)$$

Here  $R = g^{\mu\nu} R_{\mu\nu}$ ,  $T_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} T^{\alpha\beta}$ , and  $T^{\mu\nu} = (2/\sqrt{g}) \delta \mathcal{L}_{\text{SM}} / \delta g_{\mu\nu}$  is the (symmetric) energy-momentum tensor of the Standard Model matter. The theory is invariant under arbitrary coordinate transformations:  $x'^\mu = f^\mu(x^\nu)$ . To solve the field equations Eq. (21.6), one needs to fix this coordinate gauge freedom. *E.g.*, the “harmonic gauge”

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(which is the analogue of the Lorenz gauge,  $\partial_\mu A^\mu = 0$ , in electromagnetism) corresponds to imposing the condition  $\partial_\nu(\sqrt{g}g^{\mu\nu}) = 0$ .

In this *Review*, we only consider the classical limit of gravitation (*i.e.* classical matter and classical gravity). Quantum gravitational effects are expected (when considered at low energy) to correct the classical action Eq. (21.3) by additional terms involving quadratic and higher powers of the curvature tensor. This suggests that the validity of classical gravity extends (at most) down to length scales of order the Planck length  $L_P = \sqrt{\hbar G_N/c^3} \simeq 1.62 \times 10^{-33}$  cm, *i.e.* up to energy scales of order the Planck energy  $E_P = \sqrt{\hbar c^5/G_N} \simeq 1.22 \times 10^{19}$  GeV. Considering quantum matter in a classical gravitational background also poses interesting challenges, notably the possibility that the zero-point fluctuations of the matter fields generate a nonvanishing vacuum energy density  $\rho_{\text{vac}}$ , corresponding to a term  $-\sqrt{g} \rho_{\text{vac}}$  in  $\mathcal{L}_{\text{SM}}$  [1]. This is equivalent to adding a “cosmological constant” term  $+\Lambda g_{\mu\nu}$  on the left-hand side of Einstein’s equations Eq. (21.6), with  $\Lambda = 8\pi G_N \rho_{\text{vac}}/c^4$ . Recent cosmological observations (see the following *Reviews*) suggest a positive value of  $\Lambda$  corresponding to  $\rho_{\text{vac}} \approx (2.3 \times 10^{-3} \text{eV})^4$ . Such a small value has a negligible effect on the non cosmological tests discussed below.

### 21.1. Experimental tests of the coupling between matter and gravity

The universality of the coupling between  $g_{\mu\nu}$  and the Standard Model matter postulated in Eq. (21.5) (“Equivalence Principle”) has many observable consequences [2]. First, it predicts that the outcome of a local non-gravitational experiment, referred to local standards, does not depend on where, when, and in which locally inertial frame, the experiment is performed. This means, for instance, that local experiments should neither feel the cosmological evolution of the universe (constancy of the “constants”), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance). These predictions are consistent with many experiments and observations. Stringent limits on a possible time variation of the basic coupling constants have been obtained by analyzing a natural fission reactor phenomenon which took place at Oklo, Gabon, two billion years ago [3,4]. These limits are at the  $1 \times 10^{-7}$  level for the fractional variation of the fine-structure constant  $\alpha_{\text{em}}$  [4], and at the  $4 \times 10^{-9}$  level for the fractional variation of the ratio  $m_q/\Lambda_{QCD}$  between the light quark masses and  $\Lambda_{QCD}$  [5]. The determination of the lifetime of Rhenium 187 from isotopic measurements of some meteorites dating back to the formation of the solar system (about 4.6 Gyr ago) yields comparably strong limits [6]. Measurements of absorption lines in astronomical spectra also give stringent limits on the variability of both  $\alpha_{\text{em}}$  and  $\mu = m_p/m_e$  at cosmological redshifts. *E.g.*

$$\Delta\alpha_{\text{em}}/\alpha_{\text{em}} = (1.3 \pm 2.4_{\text{stat}} \pm 1.0_{\text{sys}}) \times 10^{-6} \quad (21.7)$$

at a redshift  $z = 1.6919$  [7], and

$$|\Delta\mu/\mu| < 4 \times 10^{-7} (95\% \text{ C.L.}) , \quad (21.8)$$

at a redshift  $z = 0.88582$  [8]. There are also strong limits (at the  $10^{-5}$  level) on the variation of  $\alpha_{\text{em}}$  and  $\mu = m_p/m_e$  at very large redshifts, such as  $z = 4.22$  [9] and

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$z = 5.2$  [10]. Direct laboratory limits (based on monitoring the frequency ratio of several different atomic clocks) on the present time variation of  $\alpha_{\text{em}}$ ,  $\mu = m_p/m_e$ , and  $m_q/\Lambda_{QCD}$  have reached the levels [11]:

$$\begin{aligned} d\ln(\alpha_{\text{em}})/dt &= (-2.5 \pm 2.6) \times 10^{-17} \text{yr}^{-1}, \\ d\ln(\mu)/dt &= (-1.5 \pm 3.0) \times 10^{-16} \text{yr}^{-1}, \\ d\ln(m_q/\Lambda_{QCD})/dt &= (7.1 \pm 4.4) \times 10^{-15} \text{yr}^{-1}. \end{aligned} \quad (21.9)$$

There are also experimental limits on a possible dependence of coupling constants on the gravitational potential [11,12]. See Ref. 13 for a review of the issue of “variable constants.”

The highest precision tests of the isotropy of space have been performed by looking for possible quadrupolar shifts of nuclear energy levels [14]. The (null) results can be interpreted as testing the fact that the various pieces in the matter Lagrangian Eq. (21.5) are indeed coupled to one and the same external metric  $g_{\mu\nu}$  to the  $10^{-29}$  level. For astrophysical constraints on possible Planck-scale violations of Lorentz invariance, see Ref. 15.

The universal coupling to  $g_{\mu\nu}$  postulated in Eq. (21.5) implies that two (electrically neutral) test bodies dropped at the same location and with the same velocity in an external gravitational field fall in the same way, independently of their masses and compositions. The universality of the acceleration of free fall has been verified at the  $10^{-13}$  level for laboratory bodies, notably Beryllium-Titanium, and Beryllium-Aluminum test bodies [16,17],

$$\begin{aligned} (\Delta a/a)_{\text{BeTi}} &= (0.3 \pm 1.8) \times 10^{-13}, \\ (\Delta a/a)_{\text{BeAl}} &= (-0.7 \pm 1.3) \times 10^{-13}. \end{aligned} \quad (21.10)$$

The universality of free fall has also been verified when comparing the fall of classical and quantum objects ( $6 \times 10^{-9}$  level [18]), or of two quantum objects ( $5 \times 10^{-7}$  level [19]). The gravitational accelerations of the Earth and the Moon toward the Sun have also been verified to agree [20],

$$(\Delta a/a)_{\text{EarthMoon}} = (-0.8 \pm 1.3) \times 10^{-13}. \quad (21.11)$$

The latter result constrains not only how  $g_{\mu\nu}$  couples to matter, but also how it couples to itself [21] (“strong equivalence principle”). See also Ref. 22 for a review of torsion balance experiments.

Finally, Eq. (21.5) also implies that two identically constructed clocks located at two different positions in a static external Newtonian potential  $U(\mathbf{x}) = \sum G_N m/r$  exhibit, when intercompared by means of electromagnetic signals, the (apparent) difference in clock rate,  $\tau_1/\tau_2 = \nu_2/\nu_1 = 1 + [U(\mathbf{x}_1) - U(\mathbf{x}_2)]/c^2 + O(1/c^4)$ , independently of their nature and constitution. This universal gravitational redshift of clock rates has been verified at the  $10^{-4}$  level by comparing a hydrogen-maser clock flying on a rocket up to an altitude  $\sim 10,000$  km to a similar clock on the ground [23]. The redshift due to a height change of only 33 cm has been detected by comparing two optical clocks based on  $^{27}\text{Al}^+$  ions [24].

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### 21.2. Tests of the dynamics of the gravitational field in the weak field regime

The effect on matter of one-graviton exchange, *i.e.*, the interaction Lagrangian obtained when solving Einstein’s field equations Eq. (21.6) written in, say, the harmonic gauge at first order in  $h_{\mu\nu}$ ,

$$\square h_{\mu\nu} = -\frac{16\pi G_N}{c^4}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) + O(h^2) + O(hT) , \quad (21.12)$$

reads  $-(8\pi G_N/c^4)T^{\mu\nu}\square^{-1}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu})$ . For a system of  $N$  moving point masses, with

free Lagrangian  $L^{(1)} = \sum_{A=1}^N -m_A c^2 \sqrt{1 - \mathbf{v}_A^2/c^2}$ , this interaction, expanded to order  $v^2/c^2$ , reads (with  $r_{AB} \equiv |\mathbf{x}_A - \mathbf{x}_B|$ ,  $\mathbf{n}_{AB} \equiv (\mathbf{x}_A - \mathbf{x}_B)/r_{AB}$ )

$$\begin{aligned} L^{(2)} = & \frac{1}{2} \sum_{A \neq B} \frac{G_N m_A m_B}{r_{AB}} \left[ 1 + \frac{3}{2c^2}(\mathbf{v}_A^2 + \mathbf{v}_B^2) - \frac{7}{2c^2}(\mathbf{v}_A \cdot \mathbf{v}_B) \right. \\ & \left. - \frac{1}{2c^2}(\mathbf{n}_{AB} \cdot \mathbf{v}_A)(\mathbf{n}_{AB} \cdot \mathbf{v}_B) + O\left(\frac{1}{c^4}\right) \right]. \end{aligned} \quad (21.13)$$

The two-body interactions, Eq. (21.13), exhibit  $v^2/c^2$  corrections to Newton’s  $1/r$  potential induced by spin-2 exchange (“gravito-magnetism”). Consistency at the “post-Newtonian” level  $v^2/c^2 \sim G_N m/rc^2$  requires that one also considers the three-body interactions induced by some of the three-graviton vertices and other nonlinearities (terms  $O(h^2)$  and  $O(hT)$  in Eq. (21.12)),

$$L^{(3)} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G_N^2 m_A m_B m_C}{r_{AB} r_{AC} c^2} + O\left(\frac{1}{c^4}\right) . \quad (21.14)$$

All currently performed gravitational experiments in the solar system, including perihelion advances of planetary orbits, the bending and delay of electromagnetic signals passing near the Sun, and very accurate ranging data to the Moon obtained by laser echoes, are compatible with the post-Newtonian results Eqs. (21.12)–(21.14). The “gravito-magnetic” interactions  $\propto v_A v_B$  contained in Eq. (21.13) are involved in many of these experimental tests. They have been particularly tested in lunar laser ranging data [20], in the LAGEOS satellite observations [25,26], and in the dedicated Gravity Probe B mission [27]. The recently launched LARES satellite promises to improve the accuracy of such tests [26].

Similar to what is done in discussions of precision electroweak experiments, it is useful to quantify the significance of precision gravitational experiments by parameterizing plausible deviations from General Relativity. Here, we shall focus on the simplest, and most conservative deviations from Einstein’s pure spin-2 theory defined by adding

new, bosonic light or massless, macroscopically coupled fields. [For discussions of less conservative deviations from Einstein’s theory (modified newtonian dynamics, massive gravity, higher-order gravity,  $f(R)$ -gravity, Lorentz-violating theories,...) and their confrontation with experiment, see [28,29,30]. ] The possibility of new gravitational-strength couplings leading (on small, and possibly large, scales) to deviations from Einsteinian (and Newtonian) gravity is suggested by String Theory [31], and by Brane World ideas [32]. For reviews of experimental constraints on Yukawa-type additional interactions, see Refs. [22,33,17]. Experiments have set limits on non-Newtonian forces down to 0.056 mm [34].

Here, we shall focus on the parametrization of long-range deviations from relativistic gravity obtained by adding a strictly massless (*i.e.* without self-interaction  $V(\varphi) = 0$ ) scalar field  $\varphi$  coupled to the trace of the energy-momentum tensor  $T = g_{\mu\nu}T^{\mu\nu}$  [35]. The most general such theory contains an arbitrary function  $a(\varphi)$  of the scalar field, and can be defined by the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{tot}}[g_{\mu\nu}, \varphi, \psi, A_\mu, H] = & \frac{c^4}{16\pi G} \sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi) \\ & + \mathcal{L}_{\text{SM}}[\psi, A_\mu, H, \tilde{g}_{\mu\nu}] , \end{aligned} \quad (21.15)$$

where  $G$  is a “bare” Newton constant, and where the Standard Model matter is coupled not to the “Einstein” (pure spin-2) metric  $g_{\mu\nu}$ , but to the conformally related (“Jordan-Fierz”) metric  $\tilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$ . The scalar field equation  $\square_g \varphi = -(4\pi G/c^4)\alpha(\varphi)T$  displays  $\alpha(\varphi) \equiv \partial a(\varphi)/\partial \varphi$  as the basic (field-dependent) coupling between  $\varphi$  and matter [36]. The one-parameter ( $\omega$ ) Jordan-Fierz-Brans-Dicke theory [35] is the special case  $a(\varphi) = \alpha_0 \varphi$  leading to a field-independent coupling  $\alpha(\varphi) = \alpha_0$  (with  $\alpha_0^2 = 1/(2\omega + 3)$ ). The addition of a self-interaction term  $V(\varphi)$  in Eq. (21.15) introduces new phenomenological possibilities; notably the “chameleon mechanism” [37].

In the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system, the addition of  $\varphi$  modifies Einstein’s predictions only through the appearance of two “post-Einstein” dimensionless parameters:  $\bar{\gamma} = -2\alpha_0^2/(1 + \alpha_0^2)$  and  $\bar{\beta} = +\frac{1}{2}\beta_0\alpha_0^2/(1 + \alpha_0^2)^2$ , where  $\alpha_0 \equiv \alpha(\varphi_0)$ ,  $\beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$ ,  $\varphi_0$  denoting the vacuum expectation value of  $\varphi$ . These parameters show up also naturally (in the form  $\gamma_{\text{PPN}} = 1 + \bar{\gamma}$ ,  $\beta_{\text{PPN}} = 1 + \bar{\beta}$ ) in phenomenological discussions of possible deviations from General Relativity [2]. The parameter  $\bar{\gamma}$  measures the admixture of spin 0 to Einstein’s graviton, and contributes an extra term  $+\bar{\gamma}(\mathbf{v}_A - \mathbf{v}_B)^2/c^2$  in the square brackets of the two-body Lagrangian Eq. (21.13). The parameter  $\bar{\beta}$  modifies the three-body interaction Eq. (21.14) by an overall multiplicative factor  $1 + 2\bar{\beta}$ . Moreover, the combination  $\eta \equiv 4\bar{\beta} - \bar{\gamma}$  parameterizes the lowest order effect of the self-gravity of orbiting masses by modifying the Newtonian interaction energy terms in Eq. (21.13) into  $G_{AB}m_A m_B/r_{AB}$ , with a body-dependent gravitational “constant”  $G_{AB} = G_N[1 + \eta(E_A^{\text{grav}}/m_A c^2 + E_B^{\text{grav}}/m_B c^2) + O(1/c^4)]$ , where  $G_N = G \exp[2a(\varphi_0)](1 + \alpha_0^2)$  and where  $E_A^{\text{grav}}$  denotes the gravitational binding energy of body  $A$ .

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The best current limits on the post-Einstein parameters  $\bar{\gamma}$  and  $\bar{\beta}$  are (at the 68% confidence level):

$$\bar{\gamma} = (2.1 \pm 2.3) \times 10^{-5} , \quad (21.16)$$

deduced from the additional Doppler shift experienced by radio-wave beams connecting the Earth to the Cassini spacecraft when they passed near the Sun [38], and

$$|\bar{\beta}| < 7 \times 10^{-5} , \quad (21.17)$$

from a study of the global sensitivity of planetary ephemerides to post-Einstein parameters [39]. More stringent limits on  $\bar{\gamma}$  are obtained in models (*e.g.*, string-inspired ones [31]) where scalar couplings violate the Equivalence Principle.

### 21.3. Tests of the dynamics of the gravitational field in the radiative and/or strong field regimes

The discovery of pulsars (*i.e.*, rotating neutron stars emitting a beam of radio noise) in gravitationally bound orbits [40,41] has opened up an entirely new testing ground for relativistic gravity, giving us an experimental handle on the regime of radiative and/or strong gravitational fields. In these systems, the finite velocity of propagation of the gravitational interaction between the pulsar and its companion generates damping-like terms at order  $(v/c)^5$  in the equations of motion [43]. These damping forces are the local counterparts of the gravitational radiation emitted at infinity by the system (“gravitational radiation reaction”). They cause the binary orbit to shrink and its orbital period  $P_b$  to decrease. The remarkable stability of pulsar clocks has allowed one to measure the corresponding very small orbital period decay  $\dot{P}_b \equiv dP_b/dt \sim -(v/c)^5 \sim -10^{-12}$  in several binary systems, thereby giving us a direct experimental confirmation of the propagation properties of the gravitational field, and, in particular, an experimental confirmation that the speed of propagation of gravity  $c_g$  is equal to the velocity of light  $c$  to better than a part in a thousand. [See, also, [42] for tight constraints on the difference  $c - c_g$ , when it is assumed positive.] In addition, the surface gravitational potential of a neutron star  $h_{00}(R_{\text{NS}}) \simeq 2Gm/c^2 R_{\text{NS}} \simeq 0.4$  being a factor  $\sim 10^8$  higher than the surface potential of the Earth, and a mere factor 2.5 below the black hole limit ( $h_{00}(R_{\text{BH}}) = 1$ ), pulsar data have allowed one to obtain several accurate tests of the strong-gravitational-field regime, as we discuss next.

Binary pulsar timing data record the times of arrival of successive electromagnetic pulses emitted by a pulsar orbiting around the center of mass of a binary system. After correcting for the Earth motion around the Sun and for the dispersion due to propagation in the interstellar plasma, the time of arrival of the  $N$ th pulse  $t_N$  can be described by a generic, parameterized “timing formula” [44] whose functional form is common to the whole class of tensor-scalar gravitation theories:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p); \{p^K\}; \{p^{PK}\}] . \quad (21.18)$$

Here,  $T_N$  is the pulsar proper time corresponding to the  $N$ th turn given by  $N/2\pi = \nu_p T_N + \frac{1}{2} \dot{\nu}_p T_N^2 + \frac{1}{6} \ddot{\nu}_p T_N^3$  (with  $\nu_p \equiv 1/P_p$  the spin frequency of the pulsar, *etc.*),

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$\{p^K\} = \{P_b, T_0, e, \omega_0, x\}$  is the set of ‘‘Keplerian’’ parameters (notably, orbital period  $P_b$ , eccentricity  $e$ , periastron longitude  $\omega_0$  and projected semi-major axis  $x = a \sin i/c$ ), and  $\{p^{PK}\} = \{k, \gamma_{\text{timing}}, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$  denotes the set of (separately measurable) ‘‘post-Keplerian’’ parameters. Most important among these are: the fractional periastron advance per orbit  $k \equiv \dot{\omega} P_b / 2\pi$ , a dimensionful time-dilation parameter  $\gamma_{\text{timing}}$ , the orbital period derivative  $\dot{P}_b$ , and the ‘‘range’’ and ‘‘shape’’ parameters of the gravitational time delay caused by the companion,  $r$  and  $s$ .

Without assuming any specific theory of gravity, one can phenomenologically analyze the data from any binary pulsar by least-squares fitting the observed sequence of pulse arrival times to the timing formula Eq. (21.18). This fit yields the ‘‘measured’’ values of the parameters  $\{\nu_p, \dot{\nu}_p, \ddot{\nu}_p\}$ ,  $\{p^K\}$ ,  $\{p^{PK}\}$ . Now, each specific relativistic theory of gravity predicts that, for instance,  $k$ ,  $\gamma_{\text{timing}}$ ,  $\dot{P}_b$ ,  $r$  and  $s$  (to quote parameters that have been successfully measured from some binary pulsar data) are some theory-dependent functions of the Keplerian parameters and of the (unknown) masses  $m_1$ ,  $m_2$  of the pulsar and its companion. For instance, in General Relativity, one finds (with  $M \equiv m_1 + m_2$ ,  $n \equiv 2\pi/P_b$ )

$$\begin{aligned}
 k^{\text{GR}}(m_1, m_2) &= 3(1 - e^2)^{-1} (G_N M n / c^3)^{2/3} , \\
 \gamma_{\text{timing}}^{\text{GR}}(m_1, m_2) &= e n^{-1} (G_N M n / c^3)^{2/3} m_2 (m_1 + 2m_2) / M^2 , \\
 \dot{P}_b^{\text{GR}}(m_1, m_2) &= - (192\pi/5) (1 - e^2)^{-7/2} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \\
 &\quad \times (G_N M n / c^3)^{5/3} m_1 m_2 / M^2 , \\
 r(m_1, m_2) &= G_N m_2 / c^3 , \\
 s(m_1, m_2) &= n x (G_N M n / c^3)^{-1/3} M / m_2 .
 \end{aligned} \tag{21.19}$$

In tensor-scalar theories, each of the functions  $k^{\text{theory}}(m_1, m_2)$ ,  $\gamma_{\text{timing}}^{\text{theory}}(m_1, m_2)$ ,  $\dot{P}_b^{\text{theory}}(m_1, m_2)$ , *etc.*, is modified by quasi-static strong field effects (associated with the self-gravities of the pulsar and its companion), while the particular function  $\dot{P}_b^{\text{theory}}(m_1, m_2)$  is further modified by radiative effects (associated with the spin 0 propagator) [36,45,46].

Let us give some highlights of the current experimental situation. In the first discovered binary pulsar PSR 1913+16 [40,41], it has been possible to measure with accuracy *three* post-Keplerian parameters:  $k$ ,  $\gamma_{\text{timing}}$  and  $\dot{P}_b$ . The three equations  $k^{\text{measured}} = k^{\text{theory}}(m_1, m_2)$ ,  $\gamma_{\text{timing}}^{\text{measured}} = \gamma_{\text{timing}}^{\text{theory}}(m_1, m_2)$ ,  $\dot{P}_b^{\text{measured}} = \dot{P}_b^{\text{theory}}(m_1, m_2)$  determine, for each given theory, three curves in the two-dimensional mass plane. This yields *one* (combined radiative/strong-field) test of the specified theory, according to whether the three curves meet at one point, as they should. After subtracting a small ( $\sim 10^{-14}$  level in  $\dot{P}_b^{\text{obs}} = (-2.423 \pm 0.001) \times 10^{-12}$ ), but significant, ‘‘galactic’’ perturbing effect (linked to galactic accelerations and to the pulsar proper motion) [47], one finds that General Relativity passes this  $(k - \gamma_{\text{timing}} - \dot{P}_b)_{1913+16}$  test with complete success at the  $10^{-3}$  level [41,48,49]

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$$\left[ \frac{\dot{P}_b^{\text{obs}} - \dot{P}_b^{\text{gal}}}{\dot{P}_b^{\text{GR}}[k^{\text{obs}}, \gamma_{\text{timing}}^{\text{obs}}]} \right]_{1913+16} = 0.997 \pm 0.002 . \quad (21.20)$$

Here  $\dot{P}_b^{\text{GR}}[k^{\text{obs}}, \gamma_{\text{timing}}^{\text{obs}}]$  is the result of inserting in  $\dot{P}_b^{\text{GR}}(m_1, m_2)$  the values of the masses predicted by the two equations  $k^{\text{obs}} = k^{\text{GR}}(m_1, m_2)$ ,  $\gamma_{\text{timing}}^{\text{obs}} = \gamma_{\text{timing}}^{\text{GR}}(m_1, m_2)$ . This yields experimental evidence for the reality of gravitational radiation damping forces at the  $(-3 \pm 2) \times 10^{-3}$  level.

The discovery of the binary pulsar PSR 1534+12 [50] has allowed one to measure *five* post-Keplerian parameters:  $k$ ,  $\gamma_{\text{timing}}$ ,  $r$ ,  $s$ , and (with less accuracy)  $\dot{P}_b$  [51,52]. This allows one to obtain *three* (five observables minus two masses) tests of relativistic gravity. Two among these tests probe strong field gravity, without mixing of radiative effects [51]. General Relativity passes all these tests within the measurement accuracy. The most precise of the new, pure strong-field tests is the one obtained by combining the measurements of  $k$ ,  $\gamma_{\text{timing}}$ , and  $s$ . Using the most recent data [52], one finds agreement at the  $(2 \pm 2) \times 10^{-3}$  level:

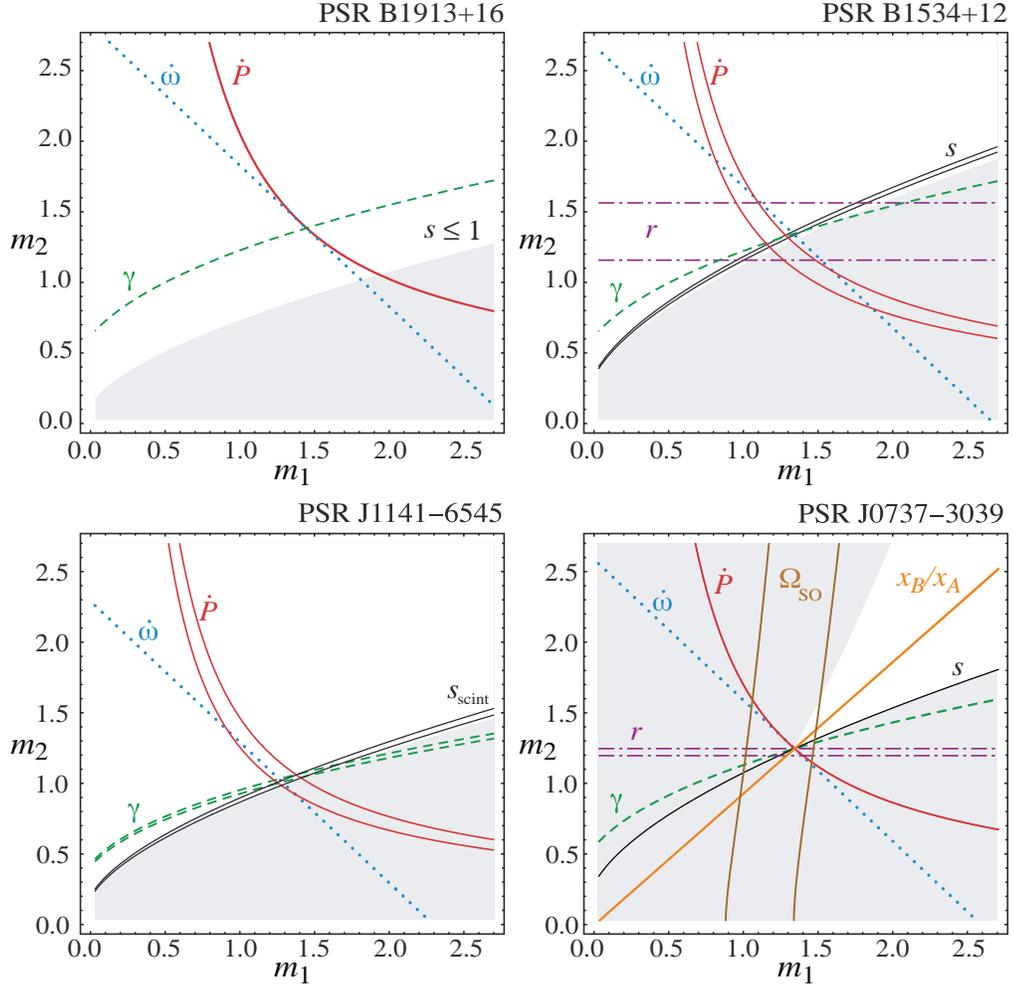
$$\left[ \frac{s^{\text{obs}}}{s^{\text{GR}}[k^{\text{obs}}, \gamma_{\text{timing}}^{\text{obs}}]} \right]_{1534+12} = 1.002 \pm 0.002 . \quad (21.21)$$

The discovery of the binary pulsar PSR J1141–6545 [53] (whose companion is probably a white dwarf) has allowed one to measure *four* observable parameters:  $k$ ,  $\gamma_{\text{timing}}$ ,  $\dot{P}_b$  [54,55], and the parameter  $s$  [56,55]. The latter parameter (which is equal to the sine of the inclination angle,  $s = \sin i$ ) was consistently measured in two ways: from a scintillation analysis [56], and from timing measurements [55]. General Relativity passes all the corresponding tests within measurement accuracy. See Fig. 21.1 which uses the (more precise) scintillation measurement of  $s = \sin i$ .

The discovery of the remarkable *double* binary pulsar PSR J0737–3039 A and B [57,58] has led to the measurement of *seven* independent parameters [59,60,61]: five of them are the post-Keplerian parameters  $k$ ,  $\gamma_{\text{timing}}$ ,  $r$ ,  $s$  and  $\dot{P}_b$  entering the relativistic timing formula of the fast-spinning pulsar PSR J0737–3039 A, a sixth is the ratio  $R = x_B/x_A$  between the projected semi-major axis of the more slowly spinning companion pulsar PSR J0737–3039 B, and that of PSR J0737–3039 A. [The theoretical prediction for the ratio  $R = x_B/x_A$ , considered as a function of the (inertial) masses  $m_1 = m_A$  and  $m_2 = m_B$ , is  $R^{\text{theory}} = m_1/m_2 + O((v/c)^4)$  [44], independently of the gravitational theory considered.] Finally, the seventh parameter  $\Omega_{\text{SO,B}}$  is the angular rate of (spin-orbit) precession of PSR J0737–3039 B around the total angular momentum [60,61]. These seven measurements give us *five* tests of relativistic gravity [59,62,63]. General Relativity passes all those tests with flying colors (see Fig. 21.1). Let us highlight here two of them (from [63]).

One test is a new confirmation of the reality of gravitational radiation at the  $10^{-3}$  level

$$\left[ \frac{\dot{P}_b^{\text{obs}}}{\dot{P}_b^{\text{GR}}[k^{\text{obs}}, R^{\text{obs}}]} \right]_{0737-3039} = 1.000 \pm 0.001 . \quad (21.22)$$



**Figure 21.1:** Illustration of the *eleven* tests of relativistic gravity obtained in the four different binary pulsar systems PSR1913+16 (one test), PSR1534+12 (3 tests), PSR J1141–6545 (2 tests), and PSR J0737–3039 A,B (5 tests). Each curve (or strip) in the mass plane corresponds to the interpretation, within General Relativity, of some observable parameter among:  $\dot{P}_b$ ,  $k \equiv \dot{\omega}P_b/2\pi$ ,  $\gamma_{\text{timing}}$ ,  $r$ ,  $s = \sin i$ ,  $\Omega_{\text{SO,B}}$  and  $R$ . (Figure updated from [75]; courtesy of G. Esposito-Farèse.)

Another one is a new,  $5 \times 10^{-4}$  level, strong-field confirmation of General Relativity:

$$\left[ \frac{s^{\text{obs}}}{s^{\text{GR}}[k^{\text{obs}}, R^{\text{obs}}]} \right]_{0737-3039} = 1.0000 \pm 0.0005 . \quad (21.23)$$

Fig. 21.1 illustrates all the tests of strong-field and radiative gravity derived from the above-mentioned binary pulsars: (3 – 2 =) one test from PSR1913+16, (5 – 2 =) 3 tests from PSR1534+12, (4 – 2 =) 2 tests from PSR J1141–6545, and (7 – 2 =) 5 tests from PSR J0737–3039. [See, also, [64] for additional, less accurate, and partially discrepant, tests of relativistic gravity.]

## 10 21. Experimental tests of gravitational theory

Data from several nearly circular binary systems (made of a neutron star and a white dwarf) have also led to strong-field confirmations (at the  $4.6 \times 10^{-3}$  level) of the ‘strong equivalence principle,’ *i.e.*, the fact that neutron stars and white dwarfs fall with the same acceleration in the gravitational field of the Galaxy [65,66,67]. The measurements of  $\dot{P}_b$  in some pulsar-white dwarf systems lead to strong constraints on the variation of Newton’s  $G_N$ , and on the existence of gravitational dipole radiation [68,69,70,71,72]. In addition, arrays of millisecond pulsars are sensitive detectors of ultra low frequency gravitational waves ( $f \sim 10^{-9} - 10^{-8}$  Hz) [73]. Such waves might be generated by supermassive black-hole binary systems, by cosmic strings and/or during the inflationary era. The sensitivity of pulsar timing arrays is comparable to predicted gravitational wave signal levels and has recently obtained the most stringent current limit on the energy density of a stochastic relic background of gravitational waves, namely (using standard notation, as in the following *Review* on Big-Bang Cosmology)  $\Omega_{gw}(f)h^2 < 4.2 \times 10^{-10}$  [74].

The constraints on tensor-scalar theories provided by the various binary-pulsar “experiments” have been analyzed in [51,46,75,76,70] and shown to exclude a large portion of the parameter space allowed by solar-system tests. The most stringent tests follow from the measurement of the orbital period decay  $\dot{P}_b$  of the low-eccentricity 8.5-hour pulsar-white dwarf system PSR J1738+0333 with [70]

$$\left[ \dot{P}_b^{\text{obs}} - \dot{P}_b^{\text{gal}} - \dot{P}_b^{\text{GR}} \right]_{1738+0333} = (2.0 \pm 3.7) \times 10^{-15}. \quad (21.24)$$

Asymmetric binary systems are strong emitters of dipolar gravitational radiation in tensor-scalar theories, with  $\dot{P}_b$  scaling (modulo matter-scalar couplings) like  $m_1 m_2 / (m_1 + m_2)^2 (v/c)^3$  ( $\sim 10^{-9}$  for PSR J1738+0333), instead of the smaller quadrupolar radiation  $\dot{P}_b \sim (v/c)^5$  [2,36]. Thereby, the result Eq. (21.24) constrains the basic matter-scalar coupling  $\alpha_0^2$  more strongly, over most of the parameter space, than the best current solar-system limits Eq. (21.16), Eq. (21.17) (namely below the  $10^{-5}$  level) [70]. In the particular case of the Jordan-Fierz-Brans-Dicke theory, the pulsar bound on  $\alpha_0^2$  is (when choosing an equation of state of medium stiffness)  $\alpha_0^2 < 2 \times 10^{-5}$ , which is within a factor two of the Cassini bound Eq. (21.16) (where  $\bar{\gamma} = -2\alpha_0^2 / (1 + \alpha_0^2)$ ).

Measurements over several years of the pulse profiles of various pulsars have detected secular profile changes compatible with the prediction [77] that the general relativistic spin-orbit coupling should cause a secular change in the orientation of the pulsar beam with respect to the line of sight (“geodetic precession”). Such confirmations of general-relativistic spin-orbit effects were obtained in PSR 1913+16 [78], PSR B1534+12 [52], PSR J1141–6545 [79], PSR J0737–3039 [60,61] and PSR J1906+0746 [80]. In some cases (notably PSR 1913+16 and PSR J1906+0746) the secular change in the orientation of the pulsar beam is expected to lead to the disappearance of the beam (as seen on the Earth) on a human time scale (the second pulsar in the double system PSR J0737–3039 has already disappeared in March 2008 and is expected to reappear around 2035 [61]).

The first observation on September 14, 2015, by the two detectors of the Laser Interferometer Gravitational-wave Observatory (LIGO), of a gravitational-wave signal, and its subsequent analysis by the LIGO/Virgo collaboration [81], has opened up a novel

testing ground for relativistic gravity. This transient signal is most readily interpreted as the gravitational-wave signal emitted ( $\sim 400$  Mpc away) by the last  $\sim 3$  inspiralling orbits and the merger of a binary black hole. Thanks to the rather high signal-to-noise ratio ( $\sim 24$ ) of the LIGO observations, one could test consistency with General Relativity in several ways, notably via the good global agreement between the full observed signal and the signal predicted by both analytical [82] and numerical [83] calculations of the gravitational waveform emitted by coalescing black holes. At this early stage, this agreement does not lead to quantitatively accurate tests of relativistic gravity competing with those discussed above. However, this observation has already brought new, deep qualitative confirmations of General Relativity, namely: (i) the first observation, in the wave zone, of gravitational waves; and (ii) the first direct evidence of the existence of black holes via the observation of their merger, followed by an abrupt shut-off of the gravitational-wave signal. Future gravitational-wave observations are expected to be able to probe more deeply relativistic gravity, notably in testing the specific, Einsteinian transverse-traceless quadrupolar nature of gravitational waves, as well as the emission, just after the merger of two black holes, of the characteristic ringing modes of the final, perturbed black hole [84].

The tests considered above have examined the gravitational interaction on scales between a fraction of a millimeter and a few astronomical units. The general relativistic action on light and matter of an external gravitational field have been verified on much larger scales in many gravitational lensing systems [85]. Some tests on cosmological scales are also available [86]. Beyond the various quantitative limits on various parametrized theoretical models discussed in the latter reference, one should remember the massive (strong-field-type) qualitative verification of General Relativity embodied in the fact that relativistic cosmological models give an accurate picture of the Universe over a period during which the spatial metric has been blown up by a gigantic factor, say  $(1+z)^2 \sim 10^{19}$  between Big Bang nucleosynthesis and now.

## 21.4. Conclusions

All present experimental tests are compatible with the predictions of the current “standard” theory of gravitation: Einstein’s General Relativity. The universality of the coupling between matter and gravity (Equivalence Principle) has been verified around the  $10^{-13}$  level. Solar system experiments have tested the weak-field predictions of Einstein’s theory at the few  $10^{-5}$  level. The propagation properties (in the near zone) of relativistic gravity, as well as several of its strong-field aspects, have been verified at the  $10^{-3}$  level (or better) in several binary pulsar experiments. The existence of gravitational waves (in the wave zone), and a direct observational proof of the existence of coalescing black holes, have been obtained by interferometric detectors of gravitational radiation. Recent laboratory experiments have set strong constraints on sub-millimeter modifications of Newtonian gravity. Quantitative confirmations of General Relativity have also been obtained on astrophysical and cosmological scales (though a skeptic might wish to keep in mind the two “dark clouds” of current cosmology, namely the need to assume dark matter and a cosmological constant).

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