

# $V_{cb}$ and $V_{ub}$ CKM Matrix Elements

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## $V_{cb}$ MEASUREMENTS

For the discussion of  $V_{cb}$  measurements, which is not repeated here, see the review on “Determination of  $|V_{cb}|$  and  $|V_{ub}|$ .”

The CKM matrix element  $|V_{cb}|$  can be determined by studying the rate of the semileptonic decay  $B \rightarrow D^{(*)} \ell \nu$  as a function of the recoil kinematics of  $D^{(*)}$  mesons. Taking advantage of theoretical constraints on the normalization and a linear  $\omega$  dependence of the form factors ( $F(\omega)$ ,  $G(\omega)$ ) provided by Heavy Quark Effective Theory (HQET), the  $|V_{cb}| \times F(\omega)$  and  $\rho^2$  ( $a^2$ ) can be simultaneously extracted from data, where  $\omega$  is the scalar product of the two-meson four velocities,  $F(1)$  is the form factor at zero recoil ( $\omega=1$ ) and  $\rho^2$  is the slope, sometimes denoted as  $a^2$ . Using the theoretical input of  $F(1)$ , a value of  $|V_{cb}|$  can be obtained.

“OUR EVALUATION” is an average using rescaled values of the data listed below. The average and rescaling were performed by the Heavy Flavor Averaging Group (HFLAV) and are described at <http://www.slac.stanford.edu/xorg/hflav/>. The averaging/rescaling procedure takes into account correlations between the measurements.

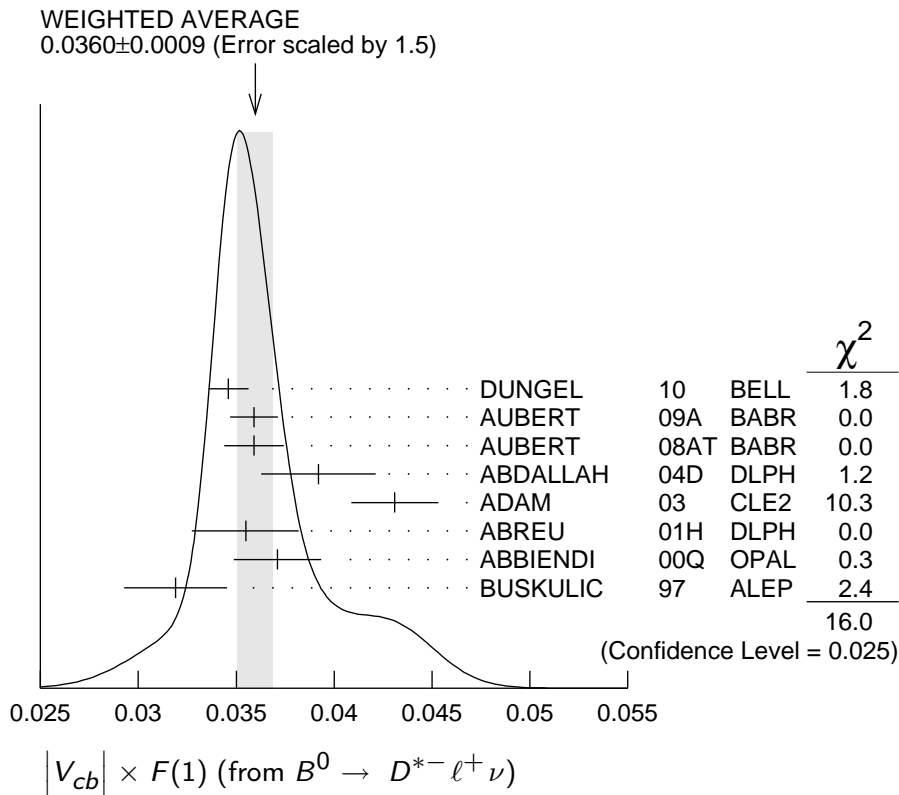
### $|V_{cb}| \times F(1)$ (from $B^0 \rightarrow D^{*-} \ell^+ \nu$ )

| VALUE                                                  | DOCUMENT ID                                            | TECN | COMMENT |
|--------------------------------------------------------|--------------------------------------------------------|------|---------|
| <b><math>0.03581 \pm 0.00045</math> OUR EVALUATION</b> | with $\rho^2=1.207 \pm 0.026$ and a correlation 0.324. |      |         |

The fitted  $\chi^2$  is 30.0 for 23 degrees of freedom.

**$0.0360 \pm 0.0009$  OUR AVERAGE** Error includes scale factor of 1.5. See the ideogram below.

|                                                                                |                        |      |      |                                    |
|--------------------------------------------------------------------------------|------------------------|------|------|------------------------------------|
| 0.0346 $\pm$ 0.0002 $\pm$ 0.0010                                               | <sup>1</sup> DUNGEL    | 10   | BELL | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0359 $\pm$ 0.0002 $\pm$ 0.0012                                               | <sup>2</sup> AUBERT    | 09A  | BABR | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0359 $\pm$ 0.0006 $\pm$ 0.0014                                               | <sup>3</sup> AUBERT    | 08AT | BABR | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0392 $\pm$ 0.0018 $\pm$ 0.0023                                               | <sup>4</sup> ABDALLAH  | 04D  | DLPH | $e^+ e^- \rightarrow Z^0$          |
| 0.0431 $\pm$ 0.0013 $\pm$ 0.0018                                               | <sup>5</sup> ADAM      | 03   | CLE2 | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0355 $\pm$ 0.0014 $\begin{smallmatrix} +0.0023 \\ -0.0024 \end{smallmatrix}$ | <sup>6</sup> ABREU     | 01H  | DLPH | $e^+ e^- \rightarrow Z$            |
| 0.0371 $\pm$ 0.0010 $\pm$ 0.0020                                               | <sup>7</sup> ABBIENDI  | 00Q  | OPAL | $e^+ e^- \rightarrow Z$            |
| 0.0319 $\pm$ 0.0018 $\pm$ 0.0019                                               | <sup>8</sup> BUSKULIC  | 97   | ALEP | $e^+ e^- \rightarrow Z$            |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●  |                        |      |      |                                    |
| 0.0344 $\pm$ 0.0003 $\pm$ 0.0011                                               | <sup>9</sup> AUBERT    | 08R  | BABR | Repl. by AUBERT 09A                |
| 0.0355 $\pm$ 0.0003 $\pm$ 0.0016                                               | <sup>10</sup> AUBERT   | 05E  | BABR | Repl. by AUBERT 08R                |
| 0.0377 $\pm$ 0.0011 $\pm$ 0.0019                                               | <sup>11</sup> ABDALLAH | 04D  | DLPH | $e^+ e^- \rightarrow Z^0$          |
| 0.0354 $\pm$ 0.0019 $\pm$ 0.0018                                               | <sup>12</sup> ABE      | 02F  | BELL | Repl. by DUNGEL 10                 |
| 0.0431 $\pm$ 0.0013 $\pm$ 0.0018                                               | <sup>13</sup> BRIERE   | 02   | CLE2 | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0328 $\pm$ 0.0019 $\pm$ 0.0022                                               | ACKERSTAFF             | 97G  | OPAL | Repl. by ABBIENDI 00Q              |
| 0.0350 $\pm$ 0.0019 $\pm$ 0.0023                                               | <sup>14</sup> ABREU    | 96P  | DLPH | Repl. by ABREU 01H                 |
| 0.0351 $\pm$ 0.0019 $\pm$ 0.0020                                               | <sup>15</sup> BARISH   | 95   | CLE2 | Repl. by ADAM 03                   |
| 0.0314 $\pm$ 0.0023 $\pm$ 0.0025                                               | BUSKULIC               | 95N  | ALEP | Repl. by BUSKULIC 97               |



<sup>1</sup> Uses fully reconstructed  $D^{*-} \ell^+ \nu$  events ( $\ell = e$  or  $\mu$ ).

<sup>2</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu \ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.22 \pm 0.02 \pm 0.07$ .

<sup>3</sup> Measured using the dependence of  $B^- \rightarrow D^{*0} e^- \bar{\nu}_e$  decay differential rate and the form factor description by CAPRINI 98 with  $\rho^2 = 1.16 \pm 0.06 \pm 0.08$ .

<sup>4</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.32 \pm 0.15 \pm 0.33$ .

<sup>5</sup> Average of the  $B^0 \rightarrow D^*(2010)^- \ell^+ \nu$  and  $B^+ \rightarrow \bar{D}^*(2007) \ell^+ \nu$  modes with  $\rho^2 = 1.61 \pm 0.09 \pm 0.21$  and  $f_{+-} = 0.521 \pm 0.012$ .

<sup>6</sup> ABREU 01H measured using about 5000 partial reconstructed  $D^*$  sample with a  $\rho^2 = 1.34 \pm 0.14^{+0.24}_{-0.22}$ .

<sup>7</sup> ABBIENDI 00Q: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples with a  $\rho^2 = 1.21 \pm 0.12 \pm 0.20$ . The statistical and systematic correlations between  $|V_{cb}| \times F(1)$  and  $\rho^2$  are 0.90 and 0.54 respectively.

<sup>8</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^{*\pm}$  with a  $\rho^2 = 0.31 \pm 0.17 \pm 0.08$ . The statistical correlation is 0.92.

<sup>9</sup> Measured using fully reconstructed  $D^*$  sample and a simultaneous fit to the Caprini-Lellouch-Neubert form factor parameters:  $\rho^2 = 1.191 \pm 0.048 \pm 0.028$ ,  $R_1(1) = 1.429 \pm 0.061 \pm 0.044$ , and  $R_2(1) = 0.827 \pm 0.038 \pm 0.022$ .

<sup>10</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.29 \pm 0.03 \pm 0.27$ .

<sup>11</sup> Combines with previous partial reconstructed  $D^*$  measurement with a  $\rho^2 = 1.39 \pm 0.10 \pm 0.33$ .

<sup>12</sup> Measured using exclusive  $B^0 \rightarrow D^*(892)^- e^+ \nu$  decays with  $\rho^2 = 1.35 \pm 0.17 \pm 0.19$  and a correlation of 0.91.

- <sup>13</sup> BRIERE 02 result is based on the same analysis and data sample reported in ADAM 03.
- <sup>14</sup> ABREU 96P: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples.
- <sup>15</sup> BARISH 95: measured using both exclusive reconstructed  $B^0 \rightarrow D^{*-} \ell^+ \nu$  and  $B^+ \rightarrow D^{*0} \ell^+ \nu$  samples. They report their experiment's uncertainties  $\pm 0.0019 \pm 0.0018 \pm 0.0008$ , where the first error is statistical, the second is systematic, and the third is the uncertainty in the lifetimes. We combine the last two in quadrature.

**$|V_{cb}| \times G(1)$  (from  $B \rightarrow D^- \ell^+ \nu$ )**

| VALUE | DOCUMENT ID | TECN | COMMENT |
|-------|-------------|------|---------|
|-------|-------------|------|---------|

**0.04265 ± 0.00153 OUR EVALUATION** with  $\rho^2 = 1.190 \pm 0.054$  and a correlation 0.83.

The fitted  $\chi^2$  is 0.5 for 8 degrees of freedom.

**0.0422 ± 0.0010 OUR AVERAGE**

|                          |                         |     |      |                                    |
|--------------------------|-------------------------|-----|------|------------------------------------|
| 0.04229 ± 0.00137        | <sup>16</sup> GLATTAUER | 16  | BELL | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0423 ± 0.0019 ± 0.0014 | <sup>17</sup> AUBERT    | 10  | BABR | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0431 ± 0.0008 ± 0.0023 | <sup>18</sup> AUBERT    | 09A | BABR | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0416 ± 0.0047 ± 0.0037 | <sup>19</sup> BARTELT   | 99  | CLE2 | $e^+ e^- \rightarrow \Upsilon(4S)$ |
| 0.0278 ± 0.0068 ± 0.0065 | <sup>20</sup> BUSKULIC  | 97  | ALEP | $e^+ e^- \rightarrow Z$            |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                                                                            |                       |     |      |                       |
|----------------------------------------------------------------------------|-----------------------|-----|------|-----------------------|
| 0.0411 ± 0.0044 ± 0.0052                                                   | <sup>21</sup> ABE     | 02E | BELL | Repl. by GLATTAUER 16 |
| 0.0337 ± 0.0044 $\begin{smallmatrix} +0.0072 \\ -0.0049 \end{smallmatrix}$ | <sup>22</sup> ATHANAS | 97  | CLE2 | Repl. by BARTELT 99   |

- <sup>16</sup> Obtained from a fit to the combined partially reconstructed  $B \rightarrow \bar{D} \ell \nu_\ell$  sample while tagged by the other fully reconstructed  $B$  meson in the event. Also reports fitted  $\rho^2 = 1.09 \pm 0.05$ .
- <sup>17</sup> Obtained from a fit to the combined  $B \rightarrow \bar{D} \ell^+ \nu_\ell$  sample in which a hadronic decay of the second  $B$  meson is fully reconstructed and  $\rho^2 = 1.20 \pm 0.09 \pm 0.04$ .
- <sup>18</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu_\ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.20 \pm 0.04 \pm 0.07$ .
- <sup>19</sup> BARTELT 99: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples.
- <sup>20</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^\pm$  with a  $a^2 = -0.05 \pm 0.53 \pm 0.38$ . The statistical correlation is 0.99.
- <sup>21</sup> Using the missing energy and momentum to extract kinematic information about the undetected neutrino in the  $B^0 \rightarrow D^- \ell^+ \nu$  decay.
- <sup>22</sup> ATHANAS 97: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples with a  $\rho^2 = 0.59 \pm 0.22 \pm 0.12 \begin{smallmatrix} +0.59 \\ -0 \end{smallmatrix}$ . They report their experiment's uncertainties  $\pm 0.0044 \pm 0.0048 \begin{smallmatrix} +0.0053 \\ -0.0012 \end{smallmatrix}$ , where the first error is statistical, the second is systematic, and the third is the uncertainty due to the form factor model variations. We combine the last two in quadrature.

## $V_{ub}$ MEASUREMENTS

For the discussion of  $V_{ub}$  measurements, which is not repeated here, see the review on "Determination of  $|V_{cb}|$  and  $|V_{ub}|$ ."

The CKM matrix element  $|V_{ub}|$  can be determined by studying the rate of the charmless semileptonic decay  $b \rightarrow u\ell\nu$ . The relevant branching ratio measurements based on exclusive and inclusive decays can be found in the *B* Listings, and are not repeated here.

## $V_{cb}$ and $V_{ub}$ CKM Matrix Elements REFERENCES

|            |      |                |                                     |                  |
|------------|------|----------------|-------------------------------------|------------------|
| GLATTAUER  | 16   | PR D93 032006  | R. Glattauer <i>et al.</i>          | (BELLE Collab.)  |
| AUBERT     | 10   | PRL 104 011802 | B. Aubert <i>et al.</i>             | (BABAR Collab.)  |
| DUNGEL     | 10   | PR D82 112007  | W. Dungen <i>et al.</i>             | (BELLE Collab.)  |
| AUBERT     | 09A  | PR D79 012002  | B. Aubert <i>et al.</i>             | (BABAR Collab.)  |
| AUBERT     | 08AT | PRL 100 231803 | B. Aubert <i>et al.</i>             | (BABAR Collab.)  |
| AUBERT     | 08R  | PR D77 032002  | B. Aubert <i>et al.</i>             | (BABAR Collab.)  |
| AUBERT     | 05E  | PR D71 051502  | B. Aubert <i>et al.</i>             | (BABAR Collab.)  |
| ABDALLAH   | 04D  | EPJ C33 213    | J. Abdallah <i>et al.</i>           | (DELPHI Collab.) |
| ADAM       | 03   | PR D67 032001  | N.E. Adam <i>et al.</i>             | (CLEO Collab.)   |
| ABE        | 02E  | PL B526 258    | K. Abe <i>et al.</i>                | (BELLE Collab.)  |
| ABE        | 02F  | PL B526 247    | K. Abe <i>et al.</i>                | (BELLE Collab.)  |
| BRIERE     | 02   | PRL 89 081803  | R. Briere <i>et al.</i>             | (CLEO Collab.)   |
| ABREU      | 01H  | PL B510 55     | P. Abreu <i>et al.</i>              | (DELPHI Collab.) |
| ABBIENDI   | 00Q  | PL B482 15     | G. Abbiendi <i>et al.</i>           | (OPAL Collab.)   |
| BARTELT    | 99   | PRL 82 3746    | J. Bartelt <i>et al.</i>            | (CLEO Collab.)   |
| CAPRINI    | 98   | NP B530 153    | I. Caprini, L. Lellouch, M. Neubert | (BCIP, CERN)     |
| ACKERSTAFF | 97G  | PL B395 128    | K. Ackerstaff <i>et al.</i>         | (OPAL Collab.)   |
| ATHANAS    | 97   | PRL 79 2208    | M. Athanas <i>et al.</i>            | (CLEO Collab.)   |
| BUSKULIC   | 97   | PL B395 373    | D. Buskulic <i>et al.</i>           | (ALEPH Collab.)  |
| ABREU      | 96P  | ZPHY C71 539   | P. Abreu <i>et al.</i>              | (DELPHI Collab.) |
| BARISH     | 95   | PR D51 1014    | B.C. Barish <i>et al.</i>           | (CLEO Collab.)   |
| BUSKULIC   | 95N  | PL B359 236    | D. Buskulic <i>et al.</i>           | (ALEPH Collab.)  |