31. Accelerator Physics of Colliders

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31.1. Luminosity

This article provides background for the High-Energy Collider Parameter Tables that follow. The number of events, N_{exp}, is the product of the cross section of interest, σ_{exp}, and the time integral over the instantaneous *luminosity,* $\mathscr{L}$:

$$N_{exp} = \sigma_{exp} \times \int \mathscr{L}(t) dt. \tag{31.1}$$

Today's colliders all employ bunched beams. If two bunches containing n_1 and n_2 particles collide head-on with frequency $f_{\rm coll}$, a basic expression for the luminosity is

$$\mathscr{L} = f_{\rm coll} \frac{n_1 n_2}{4\pi\sigma_x\sigma_y} \tag{31.2}$$

where σ_x and σ_y characterize the rms transverse beam sizes in the horizontal (bend) and vertical directions. In this form it is assumed that the bunches are identical in transverse profile, that the profiles are Gaussian and independent of position along the bunch, and the particle distributions are not altered during bunch crossing. Nonzero beam crossing angles and long bunches will reduce the luminosity from this value.

Whatever the distribution at the source, by the time the beam reaches high energy, the normal form is a useful approximation as suggested by the σ-notation. In the case of an electron storage ring, synchrotron radiation leads to a Gaussian distribution in equilibrium, but even in the absence of radiation the central limit theorem of probability and the diminished importance of space charge effects produce a similar result.

The luminosity may be obtained directly by measurement of the beam properties in Eq. (31.2). For continuous measurements, an expression similar to Eq. (31.1) with N_{ref} from a known reference cross section, σ_{ref}, may be used to determine σ_{exp} according to $\sigma_{exp} = (N_{exp}/N_{ref})\sigma_{ref}$.

In the Tables, luminosity is stated in units of cm^{-2}s^{-1}. Integrated luminosity, on the other hand is usually quoted as the inverse of the standard measures of cross section such as femtobarns and, recently, attobarns. Subsequent sections in this report briefly expand on the dynamics behind collider design, comment on the realization of collider performance in a selection of today's facilities, and end with some remarks on future possibilities.

31.2. Beam Dynamics

The first concern of beam dynamics is stability. While a reference particle proceeds along the design, or reference, trajectory other particles in the bunch are to remain close by. Assume that the reference particle carries a right-handed Cartesian coordinate system, with the z-coordinate pointed in the direction of motion along the reference trajectory. The independent variable is the distance s of the reference particle along this trajectory rather than time, and for simplicity this path is taken to be planar. The transverse coordinates are x and y, where $\{x, z\}$ defines the plane of the reference trajectory.

Several time scales are involved, and the approximations used in writing the equations of motion reflect that circumstance. All of today's high energy colliders are alternating-gradient synchrotrons or, respectively, storage rings [1,2], and the shortest time scale is that associated with transverse motion, that is described in terms of betatron oscillations, so called because of their analysis for the betatron accelerator species years ago. The linearized equations of motion of a particle displaced from the reference particle are

$$\begin{aligned} x'' + K_x x = 0, \qquad K_x &\equiv \frac{q}{p}\frac{\partial B}{\partial x} + \frac{1}{\rho^2} \\ y'' + K_y y = 0, \qquad K_y &\equiv -\frac{q}{p}\frac{\partial B}{\partial x} \\ z' = -x/\rho \end{aligned} \tag{31.3}$$

where the magnetic field $B(s)$ along the design trajectory is only in the y direction, contains only dipole and quadrupole terms, and is treated as static here. The radius of curvature due to the field on the reference orbit is ρ; z represents the longitudinal distance from the reference particle; p and q are the particle's momentum and charge, respectively. The prime denotes d/ds. The pair $(x,\ x')$ describes approximately-canonical variables. For more general cases (e.g. acceleration) one should use $(x,\ p_x)$ instead, where p_x denotes the transverse momentum in the x-direction.

The equations for x and y are those of harmonic oscillators but with a restoring force periodic in s; that is, they are instances of Hill's equation. The solution may be written in the form

$$\begin{aligned} x(s) &= A_x \sqrt{\beta_x}\ \cos\psi_x \\ x'(s) &= -\frac{A_x}{\sqrt{\beta_x}}\left[\alpha_x \cos\psi_x + \sin\psi_x\right] \end{aligned} \tag{31.4}$$

where A_x is a constant of integration, $\alpha_x \equiv -(1/2)d\beta_x(s)/ds$, and the envelope of the motion is modulated by the *amplitude function*, β_x. A solution of the same form describes the motion in y. The subscripts will be suppressed in the following discussion.

The amplitude function satisfies

$$2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4, \tag{31.5}$$

and in a region free of magnetic field it should be noted that the solution of Eq. (31.5) is a parabola. Expressing A in terms of $x,\ x'$ yields

$$\begin{aligned} A^2 &= \gamma x^2 + 2\alpha x x' + \beta x'^2 \\ &= \frac{1}{\beta}\left[x^2 + (\alpha x + \beta x')^2\right] \end{aligned} \tag{31.6}$$

with $\gamma \equiv (1+\alpha^2)/\beta$. In a single pass system such as a linac, the *Courant-Snyder parameters* α, β, γ may be selected to match the x, x' distribution of the input beam; in a recursive system, the parameters are usually defined by the structure rather than by the beam.

The relationships between the parameters and the structure may be seen by treatment of a simple *lattice* consisting of equally-spaced thin-lens quadrupoles whose magnetic-field gradients are equal in magnitude but alternating in sign. For this discussion, the weak focusing effects of the bending magnets may be neglected. The propagation of $X \equiv \{x,\, x'\}$ through a repetition period may be written $X_2 = MX_1$, with the matrix $M = FODO$ composed of the matrices

$$F = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \; D = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}, \; O = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix},$$

where f is the magnitude of the focal length and L the lens spacing. Then

$$M = \begin{pmatrix} 1 + \dfrac{L}{f} & 2L + \dfrac{L^2}{f} \\ -\dfrac{L}{f^2} & 1 - \dfrac{L}{f} - \dfrac{L^2}{f^2} \end{pmatrix}. \tag{31.7}$$

The matrix for y is identical in form differing only by a change in sign of the terms linear in $1/f$. An eigenvector-eigenvalue analysis of the matrix M shows that the motion is stable provided $f > L/2$. While that criterion is easily met, in practice instability may be caused by many other factors, including the beam-beam interaction itself.

Standard focus-drift-defocus-drift, or *FODO*, cells such as characterized in simple form by Eq. (31.7) occupy most of the layout of a large collider ring and may be used to set the scale of the amplitude function and related phase advance. Conversion of Eq. (31.4) to a matrix form equivalent to Eq. (31.7) (but more generally valid, i.e. for any stable periodic linear motion) gives

$$M = \begin{pmatrix} C + \alpha S & \beta S \\ -\gamma S & C - \alpha S \end{pmatrix} \tag{31.8}$$

where $C \equiv \cos\Delta\psi$, $S \equiv \sin\Delta\psi$, and the relation between structure and amplitude function is specified by setting the values of the latter to be the same at both ends of the cell. By comparison of Eq. (31.7) and Eq. (31.8) one finds $C = 1 - L^2/(2f^2)$, so that the choice $f = L/\sqrt{2}$ would give a phase advance $\Delta\psi$ of 90 degrees for the standard cell. The amplitude function would have a maximum at the focusing quadrupole of magnitude $\hat{\beta} = 2.7L$, illustrating the relationship of alternating gradient focusing amplitudes to relatively local aspects of the design. Other functionalities such as injection, extraction, and HEP experiments are included by lattice sections matched to the standard cell parameters $(\beta,\, \alpha)$ at the insertion points.

The phase advances according to $d\psi/ds = 1/\beta$; that is, β also plays the role of a local $\lambda/2\pi$, and the *tune*, ν, is the number of such oscillations per turn about the closed path. In the neighborhood of an interaction point (IP), the beam optics of the ring is configured so as to produce a narrow focus; the value of the amplitude function at this point is designated β^*.

The motion as it develops with s describes an ellipse in $\{x, x' \equiv dx/ds\}$ phase space, the area of which is πA^2, where A is the constant in Eq. (31.4). If the interior of that ellipse is populated by an ensemble of non-interacting particles, that area, given the name *emittance* and denoted by ε, would change only with energy. More precisely, for a beam with a Gaussian distribution in x, x', the area containing one standard deviation σ_x, divided by π, is used as the definition of emittance in the Tables:

$$\varepsilon_x \equiv \frac{\sigma_x^2}{\beta_x} , \tag{31.9}$$

with a corresponding expression in the other transverse direction, y. This definition includes 39% of the beam. For most of the entries in the Tables the standard deviation is used as the beam radius.

To complete the coordinates used to describe the motion, we take as the variable conjugate to z the fractional momentum deviation $\delta p/p$ from that of the reference particle. Radiofrequency electric fields in the s direction provide a means for longitudinal oscillations, and the frequency determines the bunch length. The frequency of this system appears in the Tables as does the rms value of $\delta p/p$ characterized as "energy spread" of the beam.

For HEP bunch length is a significant quantity for a variety of reasons, but in the present context if the bunch length, or (with nonzero crossing angle) the effective interaction length, becomes larger than β^* the luminosity is adversely affected. This is because β grows parabolically as one proceeds away from the interaction point and so the beam size increases thus lowering the contribution to the luminosity from such locations. This is often called the "hourglass" effect.

The other major external electromagnetic field interaction in the single particle context is the production of synchrotron radiation due to centripetal acceleration, given by the Larmor formula multiplied by a relativistic magnification factor of γ^4 [3]. In the case of electron rings this process determines the equilibrium emittance through a balance between radiation damping and excitation of oscillations, and further serves as a barrier to future higher energy versions in this variety of collider. A related phenomenon is beamstrahlung, i.e. the synchrotron radiation emitted during the collision in the field of the opposing beam, which is relevant for both linear colliders (where it degrades the luminosity spectrum) and future highest-energy circular colliders (where it limits the beam lifetime). For both types of colliders the beamstrahlung is mitigated by making the colliding beams as flat as possible ($\sigma_x^* \gg \sigma_y^*$).

A more comprehensive discussion of betatron oscillations, longitudinal motion, and synchrotron radiation is available in the 2008 version of the PDG review [4].

31.3. Road to High Luminosity

Eq. (31.2) can be recast in terms of emittances and amplitude functions as

$$\mathscr{L} = f \frac{n_1 n_2}{4\pi \sqrt{\epsilon_x \, \beta_x^* \, \epsilon_y \, \beta_y^*}} \, \mathcal{F}. \tag{31.10}$$

Here, $\mathcal{F} \leq 1$ is a factor that takes into account effects such as crossing angles, hour glass factors, pinch effects, and so on. So to achieve high luminosity, all one has to do is make high population bunches of low emittance collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible.

Expressions for the reductions due to crossing angle and other effects can be found elsewhere [5]. While there are no fundamental limits to producing luminosity, there are certainly challenges. Here we have space to mention only a few of these. The beam-beam tune shift appears in the Tables. A bunch in beam 1 presents a (nonlinear) lens to a particle in beam 2 resulting in changes to the particle's transverse tune with a range characterized by the parameter [5]

$$\xi_{y,2} = \left(\frac{\mu_0}{8\pi^2}\right) \frac{q_1 q_2 n_1 \beta_{y,2}^*}{m_{A,2} \gamma_2 \sigma_{y,1} (\sigma_{x,1} + \sigma_{y,1})} \tag{31.11}$$

where q_1 (q_2) denotes the particle charge of beam 1 (2) in units of the elementary charge, $m_{A,2}$ the mass of beam-2 particles, and μ_0 the vacuum permeability. The transverse oscillations are susceptible to resonant perturbations from a variety of sources such as imperfections in the magnetic guide field, so that certain values of the tune must be avoided. Accordingly, the tune spread arising from ξ is limited, but limited to a value difficult to predict. But a glance at the Tables shows that electrons are more forgiving than protons thanks to the damping effects of synchrotron radiation; the ξ-values for the former are about an order of magnitude larger than those for protons.

A subject of present intense interest is the *electron-cloud effect* [6,7]; actually a variety of related processes come under this heading. They typically involve a buildup of electron density in the vacuum chamber due to emission from the chamber walls stimulated by electrons or photons originating from the beam itself. For instance, there is a process closely resembling the multipacting effects familiar from radiofrequency system commissioning. Low energy electrons are ejected from the walls by photons from positron or proton beam-produced synchrotron radiation. These electrons are accelerated toward a beam bunch, but by the time they reach the center of the vacuum chamber the bunch has gone and so the now-energetic electrons strike the opposite wall to produce more secondaries. These secondaries are now accelerated by a subsequent bunch, and so on. Among the disturbances that this electron accumulation can produce is an enhancement of the tune spread within the bunch; the near-cancellation of bunch-induced electric and magnetic fields is no longer in effect.

If the luminosity of Eq. (31.10) is rewritten in terms of the beam-beam parameter, Eq. (31.11), the emittance itself disappears. However, the emittance must be sufficiently small to realize a desired magnitude of beam-beam parameter, but once ξ_y reaches this limit, further lowering the emittance does not lead to higher luminosity.

For electron synchrotrons and storage rings, radiation damping provides an automatic route to achieve a small emittance. In fact, synchrotron radiation is of key importance in the design and optimization of e^+e^- colliders. While vacuum stability and electron clouds can be of concern in the positron rings, synchrotron radiation along with the restoration of longitudinal momentum by the RF system has the positive effect of generating very small transverse beam sizes and small momentum spread. Further reduction of beam size at the interaction points using standard beam optics techniques and successfully contending with high beam currents has led to record luminosities in these rings. To maximize integrated luminosity the beam can be "topped off" by injecting new particles without removing existing ones – a feature difficult to imitate in hadron colliders.

For hadrons, particularly antiprotons, two inventions have played a prominent role. Stochastic cooling [8] was employed first to prepare beams for the $S\bar{p}pS$ and subsequently in the Tevatron, and to cool the beams at full energy in RHIC [9,10,11]. Electron cooling [12] was also used in the Tevatron complex to great advantage. Further innovations are underway driven by the needs of potential future projects; these are noted in the final section.

31.4. Recent High Energy Colliders

Collider accelerator physics of course goes far beyond the elements of the preceding sections. In this and the following section elaboration is made on various issues associated with some of the recently operating colliders, particularly factors which impact integrated luminosity. The various colliders utilizing hadrons each have unique characteristics and are, therefore, discussed separately. As space is limited, general references are provided where much further information can be obtained. A more complete list of recent colliders and their parameters can be found in the High-Energy Collider Parameters tables.

31.4.1. ***Tevatron*** : [14] The first synchrotron in history using superconducting magnets, the Tevatron, was the highest energy collider for 25 years. Operation was terminated in September 2011, after delivering more than 10 fb^{-1} to the p-$\bar{\text{p}}$ collider experiments CDF and D0. The route to high integrated luminosity in the Tevatron was governed by the antiproton production rate, the turn-around time to produce another store, and the resulting optimization of store time. The proton and antiproton beams in the Tevatron circulated in a single vacuum pipe and thus were placed on separated orbits which wrapped around each other in a helical pattern outside of the interaction regions. Hence, long-range encounters played an important role here as well, with the 70 long-range encounters distributed about the synchrotron, and mitigation was limited by the available aperture. The Tevatron ultimately achieved luminosities a factor of 400 over its original design specification.

31.4.2. ***HERA*** : [15] HERA, operated between 1992 and 2007, delivered nearly 1 fb^{-1} of integrated luminosity to the electron-proton collider experiments H1 and ZEUS. HERA was the first high-energy lepton-hadron collider, and also the first facility to employ both applications of superconductivity: magnets and accelerating structures. The proton beams of HERA had a maximum energy of 920 GeV. The lepton beams (positrons or electrons) were provided by the existing DESY complex, and were accelerated to 27.5 GeV using conventional magnets. At collision a 4-times higher frequency RF system, compared with the injection RF, was used to generate shorter bunches, thus helping alleviate the hourglass effect at the collision points. The lepton beam naturally would become transversely polarized (within about 40 minutes) and "spin rotators" were implemented on either side of an IP to produce longitudinal polarization at the experiment.

31.4.3. ***LEP*** : [16] Installed in a tunnel of 27 km circumference, LEP was the largest circular $\mathrm{e}^+\mathrm{e}^-$ collider built so far. It was operated from 1989 to 2000 with beam energies ranging from 45.6 to 104.5 GeV and a maximum luminosity of 10^{32} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$, at 98 GeV, surpassing all relevant design parameters.

31.4.4. ***SLC*** : [17] Based on an existing 3-km long S-band linac, the SLC was the first and only linear collider. It was operated from 1987 to 1998 with a constant beam energy of 45.6 GeV, up to about 80% electron-beam polarization, quasi-flat beams, and, in its last year, a typical peak luminosity of 2×10^{30} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$, a third of the design value.

31.5. Present Collider Facilities

31.5.1. ***LHC*** : [18] The superconducting Large Hadron Collider is the world's highest energy collider. Early operations for HEP were at 4 TeV per proton [19], with the beam energy increased to 6.5 TeV in 2015. The current status is best checked at the Web site [20]. In 2017 peak luminosities above 2×10^{34} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ (more than twice the design value) have been achieved. To meet its luminosity goals the LHC operates with a high beam current of approximately 0.5 A, leading to stored energies of several hundred MJ per beam. Component protection, beam collimation, and controlled energy deposition were given very high priorities. Additionally, at energies of 5-7 TeV per particle, synchrotron radiation moves from being a curiosity to a challenge in a hadron accelerator for the first time. At design beam current the cryogenic system must remove roughly 7 kW due to synchrotron radiation, intercepted at a temperature of 4.5-20 K. As the photons are emitted their interactions with the vacuum chamber wall can generate free electrons, with consequent "electron cloud" development. Much care was taken to design a special beam screen for the chamber to mitigate this issue.

The two proton beams are contained in separate pipes throughout most of the circumference, and are brought together into a single pipe at the interaction points. The large number of bunches, and subsequent short bunch spacing, would lead to approximately 30 head-on collisions through 120 m of common beam pipe at each IP. Thus, a small crossing angle is employed, which reduces the luminosity by about 15%. Still, the bunches moving in one direction will have long-range encounters with the counter-rotating bunches and the resulting perturbations of the particle motion constitute a continued course of study. The luminosity scale is absolutely calibrated by the "van der Meer method" as was invented for the ISR [21], and followed by multiple, redundant luminosity monitors (see for example [22] and references therein). The Tables also show the 2015 LHC luminosity performance in Pb-Pb collisions, which for the ATLAS and CMS experiments well exceeded the design value, while for the ALICE [23] experiment, the luminosity was "levelled" at the Pb-Pb design value of 10^{27} cm−2s^{-1}. The LHC can also provide Pb-p collisions as it did in 2013 and 2016, and other ion-ion or ion-proton collisions, at different energies.

In the coming years, an ambitious upgrade program, HL-LHC [24,25], has as its target an order-of-magnitude increase in integrated luminosity through the utilization of Nb_3Sn superconducting magnets, superconducting compact "crab" cavities and luminosity leveling also for ATLAS and CMS as its key ingredients.

31.5.2. ***e^+e^- Rings*** : Asymmetric energies of the two beams have allowed for the enhancement of B-physics research and for interesting interaction region designs. As the bunch spacing can be quite short, the lepton beams sometimes pass through each other at an angle and hence have reduced luminosity. Recently, however, the use of high frequency "crab crossing" schemes has produced full restoration of the luminous region. KEK-B attained over 1 fb^{-1} of integrated luminosity in a single day, and its upgrade, SuperKEKB, is aiming for luminosities of 8×10^{35} $cm^{-2}s^{-1}$ [26]. A different collision approach, called "crab waist", which relies on special sextupoles together with a large crossing angle, has been successfully implemented at DAΦNE [27] and has become a key ingredient for proposed future e^+e^- circular colliders. Other e^+e^- ring colliders in operation are BEPC-II, VEPP-2000 and VEPP-4M [26].

31.5.3. ***RHIC*** : [28] The Relativistic Heavy Ion Collider employs superconducting magnets, and collides combinations of fully-stripped ions such as H-H (p-p), p-Al, p-Au, d-Au, h-Au, Cu-Cu, Cu-Au, Au-Au, and U-U over a wide energy range [29]. The high charge per particle (+79 for gold, for instance) makes intra-beam scattering of particles within the bunch a special concern, even for seemingly moderate bunch intensities. In 2012, 3-D stochastic cooling was successfully implemented in RHIC [11] and is now routinely used. With stochastic cooling, steady increases in the bunch intensity, and numerous other upgrades, RHIC now operates at 44 times the Au-Au design average luminosity. Another special feature of accelerating heavy ions in RHIC is that the beams cross the "transition energy" during acceleration – a point where the derivative with respect to momentum of the revolution period is zero. This is more typical of low-energy accelerators, where the necessary phase jump required of the RF system is implemented rapidly and little time is spent near this condition. In the case of RHIC with heavy ions, the superconducting magnets do not ramp very quickly and the period of time spent crossing transition is long and must be dealt with carefully. For p-p operation the beams are always above their transition energy and so this condition is completely avoided.

RHIC is also unique in its ability to accelerate and collide polarized proton beams. As proton beam polarization must be maintained from its low-energy source, successful acceleration through the myriad of depolarizing resonance conditions in high energy circular accelerators has taken years to accomplish. An energy of 255 GeV per proton with

55% final polarization per beam has been realized. As part of a scheme to compensate the head-on beam-beam effect, electron lenses operated routinely during the polarized proton operation at 100 GeV in 2015 [30].

31.6. Future High Energy Colliders and Prospects

Recent accomplishments of particle physics have been obtained through high-energy and high-intensity experiments using hadron-hadron, lepton-lepton, and lepton-proton colliders. Following the discovery of the Higgs particle at the LHC and in view of ongoing searches for "new physics" and rare phenomena, various options are under discussions and development to pursue future particle-physics research at higher energy and with appropriate luminosity. This is the basis for several new projects, ideas, and R&D activities, which can only briefly be summarized here. Specifically, the following projects are noted: an energy upgrade of the LHC based on 16 T dipole magnets (HE-LHC), two approaches to an electron-positron linear collider, larger 100-km circular tunnels supporting e^+e^- collisions up to either 240 or 360 GeV in the centre of mass along with a 100-TeV proton-proton collider or 70-140 TeV muon ring collider, and potential use of plasma acceleration and other advanced schemes. Complementary studies are ongoing of a high-energy lepton-hadron collider bringing into collision a 60-GeV electron beam from an energy-recovery linac with the 7-TeV protons circulating in the LHC (LHeC) [31,32] or with the 50(35) TeV protons of the 100(70) TeV collider (FCC-he,SPPC), and of $\gamma\gamma$ collider Higgs factories based on recirculating electron linacs (e.g. SAPPHiRE [33]) . Tentative parameters of some of the colliders discussed, or mentioned, in this section are summarized in Table 31.1.

31.6.1. ***Electron-Positron Linear Colliders*** : For three decades efforts have been devoted to develop high-gradient technology e^+e^- colliders in order to overcome the synchrotron radiation limitations of circular e^+e^- machines in the TeV energy range.

The primary challenge confronting a high energy, high luminosity single pass collider design is the power requirement, so that measures must be taken to keep the demand within bounds as illustrated in a transformed Eq. (31.2) [34]:

$$\mathscr{L} \approx \frac{137}{8\pi r_e} \frac{P_{\rm wall}}{E_{cm}} \frac{\eta}{\sigma_y^*} N_\gamma \, H_D \, . \tag{31.12}$$

Here, $P_{\rm wall}$ is the total wall-plug power of the collider, $\eta \equiv P_b/P_{\rm wall}$ the efficiency of converting wall-plug power into beam power $P_b = f_{\rm coll} n E_{cm}$, E_{cm} the cms energy, n ($= n_1 = n_2$) the bunch population, and σ_y^* the vertical rms beam size at the collision point. In formulating Eq. (31.12) the number of beamstrahlung photons emitted per $e^\pm$, was approximated as $N_\gamma \approx 2\alpha r_e n/\sigma_x^*$, where α denotes the fine-structure constant. The management of $P_{\rm wall}$ leads to an upward push on the bunch population n with an attendant rise in the energy radiated due to the electromagnetic field of one bunch acting on the particles of the other. Keeping a significant fraction of the luminosity close to the nominal energy represents a design goal, which is met if N_γ does not exceed a value of about 1. A consequence is the use of flat beams, where N_γ is managed by the beam

Table 31.1: Tentative parameters of selected future high-energy colliders. Parameters of HL-LHC, ILC and CLIC can be found in the High-Energy Collider Parameters tables.

	LHeC	FCC-ee	CEPC	HE-LHC	FCC-hh	SPPC	μ collider
Species	ep	e^+e^-	e^+e^-	pp	pp	pp	$\mu^+\mu^-$
Beam Energy (TeV)	0.06(e), 7 (p)	0.046, 0.120, 0.175	0.120	13.5	50	37.5 (75)	0.063
Circumference (km)	9(e), 27 (p)	97.75	100	26.7	97.75	100	0.3
Interaction regions	1	2	2	2 (4)	4	2	1
Estimated integrated luminosity per exp. (ab^{-1}/year)	0.1	26, 0.9, 0.17	0.25	1.0	0.2–1.0	0.4	0.001
Peak luminosity (10^{34} cm^{-2} s^{-1})	0.1	230, 8, 2	2	25	5–30	10	0.008
Time between collisions (μs)	0.025	0.015, 0.5, 6.0	0.8	0.025	0.025	0.025	1
Energy spread (rms, 10^{-3})	0.03 (e), 0.1(p)	1.3, 1.5, 1.9	1.0	0.1	0.1	0.2	0.04
Bunch length (rms, mm)	0.06 (e), 75.5(p)	12.1, 4.9, 3.3	2.9	75.5	80	75.5	63
IP beam size (μm)	4.3 (round)	H: 6.3, 17, 45 V: 0.03, 0.04, 0.08	15(H), 0.09(V)	6.6	6.8 (inj.)	6.8 (inj.)	75
Injection energy (GeV)	1(e), 450(p)	on energy (topping off)	on energy (topping off)	450	3300	2100	on energy (topping off)
Transverse emittance (rms, nm)	0.45(e), 0.27(p)	H: 0.27, 0.63, 1.34 V: 0.001, 0.001, 0.003	1.31(H), 0.004(V)	0.17	0.04 (inj.)	0.06 (inj.)	335
β^*, amplitude function at interaction point (cm)	5.0(e), 7.0(p)	H: 15, 30, 100 V: 0.08, 0.1, 0.2	17(H), 0.2(V)	25	110–30	75	1.7
Beam-beam tune shift per crossing (10^{-3})	–(e), 0.4(p)	133, 108, 157	83	12	5–15	7.5	20
RF frequency (MHz)	800(e), 400(p)	400	650	400	400	400/200	805
Particles per bunch (10^{10})	0.23(e), 22(p)	17, 15, 27	9.7	10	10	15	400
Bunches per beam	–(e), 2808	16640, 393, 48	412	2808	10600	10080	1
Average beam current (mA)	15(e), 883(p)	1390, 29, 6.4	19.2	1120	500	730	640
Length of standard cell (m)	52.4(e arc), 107(p)	56	72	137	213	148	N/A
Phase advance per cell (deg)	310/90(eH/V) 90(p)	60, 90, 90	60	90	90	90	N/A
Peak magnetic field (T)	0.264(e), 8.33(p)	0.01, 0.03, 0.05	0.03	16	16	12(24)	10
Polarization (%)	90(e), 0(p)	≥10, 0, 0	0	0	0	0	0
SR power loss/beam (MW)	30(e), 0.01(p)	50	32	0.1	2.4	1.1	3×10^{-5}
Novel technology	high-energy ERL	—	—	16 T Nb_3Sn magnets	16 T Nb_3Sn magnets	HTS magnets	muon prod.

width, and luminosity adjusted by the beam height, thus the explicit appearance of the vertical beam size σ_y^*. The final factor in Eq. (31.12), H_D, represents the enhancement of luminosity due to the pinch effect during bunch crossing (the effect of which has been neglected in the expression for N_γ).

The approach designated by the International Linear Collider (ILC) is presented in the Tables, and the contrast with the collision-point parameters of the circular colliders is striking, though reminiscent in direction of those of the SLAC Linear Collider. The ILC *Technical Design Report* [35] has a baseline cms energy of 500 GeV with upgrade provision for 1 TeV, and luminosity comparable to the LHC; recent tendancies have been toward a baseline of 250 GeV. The ILC is based on superconducting accelerating structures of the 1.3 GHz TESLA variety. Progress toward higher field gradients and Q values continues to be made, with nitrogen-doping techniques being a recent example [36].

At CERN, a design effort is underway on the Compact Linear Collider (CLIC), each linac of which is itself a two-beam accelerator, in that a high energy, low current beam is fed by a low energy, high current driver [37]. The CLIC design employs normal conducting 12 GHz accelerating structures at a gradient of 100 MeV/m, some three times the current capability of the superconducting ILC cavities. The design cms energy is 3 TeV, though recent staging options – 0.38, 1.5, and 3 TeV – have been developed [38].

31.6.2. ***Future Circular Colliders*** : The discovery, in 2012, of the Higgs boson at the LHC has stimulated interest in constructing a large circular tunnel which could host a variety of energy-frontier machines, including high-energy electron-positron, proton-proton, and lepton-hadron colliders. Such projects are under study by a global collaboration hosted at CERN (FCC) [39] and another one centered in China (CEPC/SPPC) [40], following earlier proposals for a Very Large Hadron Collider (VLHC) [41] and a Very Large Lepton Collider (VLLC) in the US, which would have been housed in the same 230-km long tunnel.

The maximum beam energy of a hadron collider is directly proportional to the magnetic field and to the ring circumference. The LHC magnets, based on Nb-Ti superconductor, achieve a maximum operational field of 8.33 T. The HL-LHC project develops the technology of higher field Nb_3Sn magnets as well as cables made from high-temperature superconductor (HTS). Nb_3Sn dipoles could ultimately reach an operational field around 16 T, and HTS inserts, requiring new engineering materials and substantial dedicated R&D, could boost this further. More cost-effective hybrid magnet designs incorporating Nb-Ti, two types of Nb_3Sn, and an inner layer of HTS providing fields of about 20 T have been examined [42]. However present project efforts are not utilizing this hybrid approach as of yet.

Aside from the magnets, the cryogenic beam vacuum system is another key component of any future hadron collider. A beam screen inside the cold bore of the magnets can intercept the synchrotron radiation at an elevated temperature, allowing a more efficient extraction of the synchrotron-radiation heat load. While the LHC beam screen has a temperature of 5–20 K, future, higher-energy machines are likely to raise this temperature to 50 K or 100 K.

Further substantial increases in collision energy are possible only with a larger tunnel. The FCC hadron collider (FCC-hh) [43], formerly called VHE-LHC [44,44], is based on a new tunnel of about 100 km circumference, which would allow exploring energies up to 100 TeV in the centre of mass with proton-proton collisions, using 16 T magnets. This new tunnel could also accommodate a high-luminosity circular e^+e^- Higgs factory (FCC-ee) as well as a lepton-hadron collider (FCC-he). The SPPC is a 100 km hadron collider based on 12 T (later 24 T) iron-based high-temperature superconducting magnets, which could be installed in the same tunnel as the e^+e^- collider CEPC.

In order to serve as a Higgs factory a new circular e^+e^- collider needs to achieve a cms energy of at least 240 GeV. FCC-ee (formerly TLEP [46]) , installed in the ∼100 km tunnel of the FCC-hh, could reach even higher energies, e.g. 350 GeV cms for $t\bar{t}$ production. At these energies, the luminosity, limited by the synchrotron radiation power, would still be above 10^{34} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at each of two or four collision points. At lower energies (Z pole and WW threshold) FCC-ee could deliver up to three orders of magnitude higher luminosities, and also profit from radiative self polarization for precise energy calibration. The short beam lifetime at the high target luminosity, due to radiative Bhabha scattering, requires FCC-ee to be constructed as a double ring, where the collider ring operating at constant energy is complemented by a second injector ring installed in the same tunnel to "top off" the collider current. Beamstrahlung, i.e. synchrotron radiation emitted during the collision in the field of the opposing beam, introduces an additional beam lifetime limitation depending on momentum acceptance (so that achieving sufficient off-momentum dynamic aperture becomes one of the design challenges), as well as some bunch lengthening.

31.6.3. ***Muon Collider*** : The muon to electron mass ratio of 210 implies less concern about synchrotron radiation by a factor of about 2×10^9 and its 2.2 μs lifetime means that it will last for some $150B$ turns in a ring about half of which is occupied by bend magnets with average field B (Tesla). Design effort became serious in the mid 1990s and a collider outline emerged quickly.

Removal of the synchrotron radiation barrier reduces the scale of a muon collider facility to a level compatible with on-site placement at existing accelerator laboratories. The Higgs production cross section in the s-channel is enhanced by a factor of $(m_\mu/m_e)^2$ compared to that in e^+e^- collisions. And a neutrino factory could potentially be realized in the course of construction [47].

The challenges to luminosity achievement are clear and amenable to immediate study: targeting, collection, and emittance reduction are paramount, as well as the bunch manipulation required to produce $> 10^{12}$ muons per bunch without emittance degradation. The proton source needs to deliver a beam power of several MW, collection would be aided by magnetic fields common on neutron stars (though scaled back for application on earth), and the emittance requirements have inspired fascinating investigations into phase space manipulations that are finding applications in other facilities. The status was summarized in a White Paper submitted to "Snowmass 2013" [48]. More recently, direct production of a low-emittance muon beam by positron annihilation [49](or alternatively laser-hadron collisions [50,51]) has been proposed as a possible path towards a simpler and cheaper muon collider.

31.6.4. ***Plasma Acceleration and Other Advanced Concepts*** : At the 1956 CERN Symposium, a paper by Veksler, in which he suggested acceleration of protons to the TeV scale using a bunch of electrons, anticipated current interest in plasma acceleration [52]. A half-century later this became more than a suggestion, with the demonstration, as a striking example, of electron energy doubling from 42 to 84 GeV over 85 cm at SLAC [53] and the creation of a 1 GeV electron bunch with relatively small energy spread accelerated through a cm-scale plasma [54].

Whether plasma acceleration will find application in an HEP facility is not yet clear, given the necessity of staging and phase-locking acceleration in multiple plasma chambers. However, strides continue to be made, as multi-stage coupling of independent laser plasma accelerators have been demonstrated recently [55].

Maintaining beam quality and beam position as well as the acceleration of high-repetition bunch trains are also primary feasibility issues, addressed by active R&D. For recent discussions of parameters for a laser-plasma based electron positron collider, see, for example, relevant papers from the proceedings of the 2016 Advanced Accelerator Concepts Workshop [56] and the ICFA-ICUIL White Paper from 2011 [57].

Additional approaches aiming at accelerating gradients higher, or much higher, than those achievable with conventional metal cavities include the use of dielectric materials and, for the long-term future, crystals. Combining several innovative ideas, even a linear crystal muon collider driven by X-ray lasers has been proposed [58], as well as "accelerators on a chip" [59,60]. Not only the achievable accelerating gradient, but also the overall power efficiency, e.g. the attainable luminosity as a function of electrical input power, will determine the suitability of any novel technology for use in future high-energy accelerators.

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