107. Radiative Hyperon Decays

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The weak radiative decays of spin-1/2 hyperons, \( B_i \to B_f \gamma \), yield information about matrix elements (form factors) similar to that gained from weak hadronic decays. For a polarized spin-1/2 hyperon decaying radiatively via a \( \Delta Q = 0, \Delta S = 1 \) transition, the angular distribution of the direction \( \hat{p} \) of the final spin-1/2 baryon in the hyperon rest frame is

\[
\frac{dN}{d\Omega} = \frac{N}{4\pi} (1 + \alpha_\gamma P_i \cdot \hat{p}) .
\]

(107.1)

Here \( P_i \) is the polarization of the decaying hyperon, and \( \alpha_\gamma \) is the asymmetry parameter. In terms of the form factors \( F_1(q^2) \), \( F_2(q^2) \), and \( G(q^2) \) of the effective hadronic weak electromagnetic vertex,

\[
F_1(q^2)\gamma_\lambda + iF_2(q^2)\sigma_\lambda\mu q^\mu + G(q^2)\gamma_\lambda\gamma_5
\]

\( \alpha_\gamma \) is

\[
\alpha_\gamma = \frac{2 \text{Re}[G(0)F_M^*(0)]}{|G(0)|^2 + |F_M(0)|^2} ,
\]

(107.2)

where \( F_M = (m_i - m_f)[F_2 - F_1/(m_i + m_f)] \). If the decaying hyperon is unpolarized, the decay baryon has a longitudinal polarization given by \( P_f = -\alpha_\gamma [1] \).

The angular distribution for the weak hadronic decay, \( B_i \to B_f \pi \), has the same form as Eq. (107.1), but of course with a different asymmetry parameter, \( \alpha_\pi \). Now, however, if the decaying hyperon is unpolarized, the decay baryon has a longitudinal polarization given by \( P_f = +\alpha_\pi [2,3] \). The difference of sign is because the spins of the pion and photon are different.

107.1. \( \Xi^0 \to \Lambda \gamma \) decay

The radiative decay \( \Xi^0 \to \Lambda \gamma \) of an unpolarized \( \Xi^0 \) uses the hadronic decay \( \Lambda \to p\pi^- \) as the analyzer. As noted above, the longitudinal polarization of the \( \Lambda \) will be \( P_\Lambda = -\alpha_{\Xi\Lambda} \gamma \). Let \( \alpha_- \) be the \( \Lambda \to p\pi^- \) asymmetry parameter and \( \theta_\Lambda p \) be the angle, as seen in the \( \Lambda \) rest frame, between the \( \Lambda \) line of flight and the proton momentum. Then the hadronic version of Eq. (107.1) applied to the \( \Lambda \to p\pi^- \) decay gives

\[
\frac{dN}{d\cos \theta_\Lambda p} = \frac{N}{2} (1 - \alpha_{\Xi\Lambda} \gamma \alpha_- \cos \theta_\Lambda p)
\]

(107.3)

for the angular distribution of the proton in the \( \Lambda \) frame. Our current value, from the CERN NA48/1 experiment [4], is \( \alpha_{\Xi\Lambda} \gamma = -0.704 \pm 0.019 \pm 0.064 \).

107.2. \( \Xi^0 \to \Sigma^0 \gamma \) decay

The asymmetry parameter here, \( \alpha_{\Xi\Sigma} \gamma \), is measured by following the decay chain \( \Xi^0 \to \Sigma^0 \gamma, \Sigma^0 \to \Lambda \gamma, \Lambda \to p\pi^- \). Again, for an unpolarized \( \Xi^0 \), the longitudinal polarization of the \( \Sigma^0 \) will be \( P_\Sigma = -\alpha_{\Xi\Sigma} \). In the \( \Sigma^0 \to \Lambda \gamma \) decay, a parity-conserving magnetic-dipole transition, the polarization of the \( \Sigma^0 \) is transferred to the \( \Lambda \), as may be seen as follows. Let \( \theta_\Sigma \Lambda \) be the angle seen in the \( \Sigma^0 \) rest frame between the \( \Sigma^0 \) line of flight and the \( \Lambda \) momentum. For \( \Sigma^0 \) helicity +1/2, the probability amplitudes for positive...
and negative spin states of the $\Sigma^0$ along the $\Lambda$ momentum are $\cos(\theta_{\Sigma\Lambda}/2)$ and $\sin(\theta_{\Sigma\Lambda}/2)$. Then the amplitude for a negative helicity photon and a negative helicity $\Lambda$ is $\cos(\theta_{\Sigma\Lambda}/2)$, while the amplitude for positive helicities for the photon and $\Lambda$ is $\sin(\theta_{\Sigma\Lambda}/2)$. For $\Sigma^0$ helicity $-1/2$, the amplitudes are interchanged. If the $\Sigma^0$ has longitudinal polarization $P_\Sigma$, the probabilities for $\Lambda$ helicities $\pm 1/2$ are therefore

$$p(\pm 1/2) = \frac{1}{2}(1 \mp P_\Sigma \cos^2(\theta_{\Sigma\Lambda}/2) + \frac{1}{2}(1 \pm P_\Sigma \sin^2(\theta_{\Sigma\Lambda}/2), \quad (107.4)$$

and the longitudinal polarization of the $\Lambda$ is

$$P_\Lambda = -P_\Sigma \cos \theta_{\Sigma\Lambda} = +\alpha_{\Xi\Sigma\gamma} \cos \theta_{\Sigma\Lambda}. \quad (107.5)$$

Using Eq. (107.1) for the $\Lambda \rightarrow p\pi^-$ decay again, we get for the joint angular distribution of the $\Sigma^0 \rightarrow \Lambda\gamma$, $\Lambda \rightarrow p\pi^-$ chain,

$$\frac{d^2N}{d\cos \theta_{\Sigma\Lambda} d\cos \theta_{\Lambda p}} = \frac{N}{4} \left(1 + \alpha_{\Xi\Sigma\gamma} \cos \theta_{\Sigma\Lambda} \alpha_{\Xi} \cos \theta_{\Lambda p}\right). \quad (107.6)$$

Our current average for $\alpha_{\Xi\Sigma\gamma}$ is $-0.69 \pm 0.06$ [4,5].

References:
1. R.E. Behrends, Phys. Rev. 111, 1691 (1958); see Eq. (7) or (8).
2. In ancient times, the signs of the asymmetry term in the angular distributions of radiative and hadronic decays of polarized hyperons were sometimes opposite. For roughly 50 years, however, the overwhelming convention has been to make them the same. The aim, not always achieved, is to remove ambiguities.
3. For the definition of $\alpha_\pi$, see the note on “Baryon Decay Parameters” in the Neutron Listings.