21. Experimental Tests of Gravitational Theory

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Einstein's theory of General Relativity (GR), the current "standard" theory of gravitation, describes gravity as a universal deformation of the Minkowski metric:

$$g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda) \ , \ \text{where } \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \ . \tag{21.1}$$

GR is classically defined by two postulates. One postulate states that the Lagrangian density describing the propagation and self-interaction of the gravitational field is

$$\mathcal{L}_{\text{Ein}}[g_{\alpha\beta}] = \frac{c^4}{16\pi G_N}\sqrt{g} g^{\mu\nu} R_{\mu\nu}(g_{\alpha\beta}) \ , \tag{21.2}$$

where G_N is Newton's constant, $g = -\det(g_{\mu\nu})$, $g^{\mu\nu}$ is the matrix inverse of $g_{\mu\nu}$, and where the Ricci tensor $R_{\mu\nu} \equiv R^\alpha_{\ \mu\alpha\nu}$ is the only independent trace of the curvature tensor

$$R^\alpha_{\ \mu\beta\nu} = \partial_\beta \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\sigma\beta}\Gamma^\sigma_{\mu\nu} - \Gamma^\alpha_{\sigma\nu}\Gamma^\sigma_{\mu\beta} \ , \tag{21.3}$$

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}) \ , \tag{21.4}$$

A second postulate states that $g_{\mu\nu}$ couples universally, and minimally, to all the fields of the Standard Model by replacing everywhere the Minkowski metric $\eta_{\mu\nu}$. Schematically (suppressing matrix indices and labels for the various gauge fields and fermions and for the Higgs doublet),

$$\begin{aligned}\mathcal{L}_{\text{SM}}[\psi, A_\mu, H, g_{\mu\nu}] = &-\tfrac{1}{4}\sum \sqrt{g} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} - \sum \sqrt{g}\, \overline{\psi}\, \gamma^\mu D_\mu \psi \\ &- \tfrac{1}{2}\sqrt{g} g^{\mu\nu} \overline{D_\mu H} D_\nu H - \sqrt{g}\, V(H) - \sum \lambda \sqrt{g}\, \overline{\psi}\, H \psi \ ,\end{aligned} \tag{21.5}$$

where $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, and where the covariant derivative D_μ contains, besides the usual gauge field terms, a spin-dependent gravitational contribution. From the total action follow Einstein's field equations,

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} \ . \tag{21.6}$$

Here $R = g^{\mu\nu} R_{\mu\nu}$, $T_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} T^{\alpha\beta}$, and $T^{\mu\nu} = (2/\sqrt{g})\delta\mathcal{L}_{\text{SM}}/\delta g_{\mu\nu}$ is the (symmetric) energy-momentum tensor of the Standard Model matter. The theory is invariant under arbitrary coordinate transformations: $x'^\mu = f^\mu(x^\nu)$. To solve the field equations Eq. (21.6), one needs to fix this coordinate gauge freedom. *E.g.*, the "harmonic gauge" (which is the analogue of the Lorenz gauge, $\partial_\mu A^\mu = 0$, in electromagnetism) corresponds to imposing the condition $\partial_\nu(\sqrt{g} g^{\mu\nu}) = 0$.

In this *Review*, we only consider the classical limit of gravitation (*i.e.* classical matter and classical gravity). Quantum gravitational effects are expected (when considered at low energy) to correct the classical action Eq. (21.3) by additional terms involving quadratic and higher powers of the curvature tensor. This suggests that the validity of classical gravity extends (at most) down to length scales of order the Planck length $L_P = \sqrt{\hbar G_N / c^3} \simeq 1.62 \times 10^{-33}$ cm, i.e. up to energy scales of order the Planck energy $E_P = \sqrt{\hbar c^5 / G_N} \simeq 1.22 \times 10^{19}$ GeV. Considering quantum matter in a classical gravitational background also poses interesting challenges, notably the possibility that the zero-point fluctuations of the matter fields generate a nonvanishing vacuum energy density $\rho_{\rm vac}$, corresponding to a term $-\sqrt{g}\,\rho_{\rm vac}$ in $\mathcal{L}_{\rm SM}$ [1]. This is equivalent to adding a "cosmological constant" term $+\Lambda\, g_{\mu\nu}$ on the left-hand side of Einstein's equations Eq. (21.6), with $\Lambda = 8\pi G_N\, \rho_{\rm vac}/c^4$. Recent cosmological observations (see the following *Reviews*) suggest a positive value of Λ corresponding to $\rho_{\rm vac} \approx (2.3 \times 10^{-3}{\rm eV})^4$. Such a small value has a negligible effect on the non cosmological tests discussed below.

21.1. Experimental tests of the matter-gravity coupling

The universality of the coupling between $g_{\mu\nu}$ and the Standard Model matter postulated in Eq. (21.5) ("Equivalence Principle") has many observable consequences. First, it predicts that the outcome of a local non-gravitational experiment, referred to local standards, does not depend on where, when, and in which locally inertial frame, the experiment is performed. This means, for instance, that local experiments should neither feel the cosmological evolution of the universe (constancy of the "constants"), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance). These predictions are consistent with many experiments and observations. Stringent limits on a possible time variation of the basic coupling constants have been obtained by analyzing a natural fission reactor phenomenon which took place at Oklo, Gabon, two billion years ago [2,3]. These limits are at the 1×10^{-8} level for the fractional variation of the fine-structure constant $\alpha_{\rm em}$ [3], and at the 4×10^{-9} level for the fractional variation of the ratio m_q/Λ_{QCD} between the light quark masses and Λ_{QCD} [4]. The determination of the lifetime of Rhenium 187 from isotopic measurements of some meteorites dating back to the formation of the solar system (about 4.6 Gyr ago) yields comparably strong limits [5]. Measurements of absorption lines in astronomical spectra also give stringent limits on the variability of both $\alpha_{\rm em}$ and $\mu = m_p/m_e$ at cosmological redshifts. E.g.

$$\Delta\alpha_{\rm em}/\alpha_{\rm em} = (1.2 \pm 1.7_{\rm stat} \pm 0.9_{\rm sys}) \times 10^{-6} \tag{21.7}$$

at redshifts $z = 1.0$–2.4 [6], and

$$|\Delta\mu/\mu| < 4 \times 10^{-7} (95\%\ C.L.)\ , \tag{21.8}$$

at a redshift $z = 0.88582$ [7]. There are also strong limits on the variation of $\alpha_{\rm em}$ and $\mu = m_p/m_e$ at redshift $z \sim 10^3$ from cosmic microwave background data, e.g. $\Delta\alpha_{\rm em}/\alpha_{\rm em} = (3.6 \pm 3.7) \times 10^{-3}$ [8]. Direct laboratory limits (based on monitoring the frequency ratio of several different atomic clocks) on the present time variation of $\alpha_{\rm em}$, $\mu = m_p/m_e$, and m_q/Λ_{QCD} have reached the levels [9]:

$$d\ln(\alpha_{\rm em})/dt = (-2.5 \pm 2.6) \times 10^{-17} {\rm yr}^{-1},$$

$$d\ln(\mu)/dt = (-1.5 \pm 3.0) \times 10^{-16}\mathrm{yr}^{-1},$$
$$d\ln(m_q/\Lambda_{QCD})/dt = (7.1 \pm 4.4) \times 10^{-15}\mathrm{yr}^{-1}. \tag{21.9}$$

There are also experimental limits on a possible dependence of coupling constants on the gravitational potential [9,10].

The highest precision tests of the isotropy of space have been performed by looking for possible quadrupolar shifts of nuclear energy levels [11]. The (null) results can be interpreted as testing the fact that the various pieces in the matter Lagrangian Eq. (21.5) are indeed coupled to one and the same external metric $g_{\mu\nu}$ to the 10^{-29} level. For astrophysical constraints on possible Planck-scale violations of Lorentz invariance, see Ref. 12.

The universal coupling to $g_{\mu\nu}$ postulated in Eq. (21.5) implies that two (electrically neutral) test bodies dropped at the same location and with the same velocity in an external gravitational field fall in the same way, independently of their masses and compositions. The universality of the acceleration of free fall has been verified, for laboratory bodies, both on the ground [13,14](at the 10^{-13} level) and, in space [15](at the 10^{-14} level):

$$(\Delta a/a)_{\mathrm{BeTi}} = (0.3 \pm 1.8) \times 10^{-13} \ ,$$
$$(\Delta a/a)_{\mathrm{BeAl}} = (-0.7 \pm 1.3) \times 10^{-13} \ ,$$
$$(\Delta a/a)_{\mathrm{TiPt}} = (-1 \pm 9_{\mathrm{stat}} \pm 9_{\mathrm{sys}}) \times 10^{-15} \ . \tag{21.10}$$

The universality of free fall has also been verified when comparing the fall of classical and quantum objects (6×10^{-9} level [16]) , or of two quantum objects (5×10^{-7} level [17]) . The gravitational accelerations of the Earth and the Moon toward the Sun have also been verified to agree [18],

$$(\Delta a/a)_{\mathrm{EarthMoon}} = (-0.8 \pm 1.3) \times 10^{-13} \ . \tag{21.11}$$

The latter result constrains not only how $g_{\mu\nu}$ couples to matter, but also how it couples to itself [19]("strong equivalence principle").

Finally, Eq. (21.5) also implies that two identically constructed clocks located at two different positions in a static external Newtonian potential $U(\boldsymbol{x}) = \sum G_N m/r$ exhibit, when intercompared by means of electromagnetic signals, the (apparent) difference in clock rate, $\tau_1/\tau_2 = \nu_2/\nu_1 = 1 + [U(\boldsymbol{x}_1) - U(\boldsymbol{x}_2)]/c^2 + O(1/c^4)$, independently of their nature and constitution. This universal gravitational redshift of clock rates has been verified at the 10^{-4} level by comparing a hydrogen-maser clock flying on a rocket up to an altitude $\sim 10,000$ km to a similar clock on the ground [20]. The redshift due to a height change of only 33 cm has been detected by comparing two optical clocks based on $^{27}\mathrm{Al}^{+}$ ions [21].

21.2. Tests of the dynamics of the gravitational field in the weak field regime

The effect on matter of one-graviton exchange, *i.e.*, the interaction Lagrangian obtained when solving Einstein's field equations Eq. (21.6) written in, say, the harmonic gauge at first order in $h_{\mu\nu}$,

$$\Box h_{\mu\nu} = -\frac{16\pi G_N}{c^4}(T_{\mu\nu} - \tfrac{1}{2}T\eta_{\mu\nu}) + O(h^2) + O(hT) \ , \tag{21.12}$$

reads $-(8\pi G_N/c^4)T^{\mu\nu}\Box^{-1}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu})$. For a system of N moving point masses, with free Lagrangian $L^{(1)} = \sum_{A=1}^{N} - m_A c^2\sqrt{1-\boldsymbol{v}_A^2/c^2}$, this interaction, expanded to order v^2/c^2, reads (with $r_{AB} \equiv |\boldsymbol{x}_A - \boldsymbol{x}_B|$, $\boldsymbol{n}_{AB} \equiv (\boldsymbol{x}_A - \boldsymbol{x}_B)/r_{AB}$)

$$\begin{aligned} L^{(2)} =& \frac{1}{2}\sum_{A\neq B}\frac{G_N\, m_A\, m_B}{r_{AB}}\left[1 + \frac{3}{2c^2}(\boldsymbol{v}_A^2 + \boldsymbol{v}_B^2) - \frac{7}{2c^2}(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)\right. \\ &\left. -\frac{1}{2c^2}(\boldsymbol{n}_{AB}\cdot\boldsymbol{v}_A)(\boldsymbol{n}_{AB}\cdot\boldsymbol{v}_B) + O\left(\frac{1}{c^4}\right)\right] \ . \end{aligned} \tag{21.13}$$

The two-body interactions, Eq. (21.13), exhibit v^2/c^2 corrections to Newton's $1/r$ potential induced by spin-2 exchange ("gravito-magnetism"). Consistency at the "post-Newtonian" level $v^2/c^2 \sim G_N\, m/rc^2$ requires that one also considers the three-body interactions induced by some of the three-graviton vertices and other nonlinearities (terms $O(h^2)$ and $O(hT)$ in Eq. (21.12)),

$$L^{(3)} = -\frac{1}{2}\sum_{B\neq A\neq C}\frac{G_N^2\, m_A\, m_B\, m_C}{r_{AB}\, r_{AC}\, c^2} + O\left(\frac{1}{c^4}\right) \ . \tag{21.14}$$

All currently performed gravitational experiments in the solar system, including perihelion advances of planetary orbits, the bending and delay of electromagnetic signals passing near the Sun, and very accurate ranging data to the Moon obtained by laser echoes, are compatible with the post-Newtonian results Eqs. (21.12)–(21.14). The "gravito-magnetic" interactions $\propto v_A v_B$ contained in Eq. (21.13) are involved in many of these experimental tests. They have been particularly tested in lunar laser ranging data [18], in the combined LAGEOS-LARES satellite data [22,23], and in the dedicated Gravity Probe B mission [24].

Similar to what is done in discussions of precision electroweak experiments, it is useful to quantify the significance of precision gravitational experiments by parametrizing plausible deviations from GR. Here, we shall focus on the simplest, and most conservative deviations from Einstein's pure spin-2 theory defined by adding new, bosonic light or massless, macroscopically coupled fields. The possibility of new gravitational-strength couplings leading (on small, and possibly large, scales) to deviations from Einsteinian (and Newtonian) gravity is suggested by String Theory [25], and by Brane World ideas [26]. Experiments have set limits on non-Newtonian forces down to the micrometer range [27].

Here, we shall focus on the parametrization of long-range deviations from relativistic gravity obtained by adding a strictly massless (*i.e.* without self-interaction $V(\varphi) = 0$) scalar field φ coupled to the trace of the energy-momentum tensor $T = g_{\mu\nu}T^{\mu\nu}$ [28,29]. The most general such theory contains an arbitrary function $a(\varphi)$ of the scalar field, and can be defined by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm tot}[g_{\mu\nu}, \varphi, \psi, A_\mu, H] =& \frac{c^4}{16\pi G}\sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) \\ &+\mathcal{L}_{\rm SM}[\psi, A_\mu, H, \widetilde{g}_{\mu\nu}] \ ,\end{aligned} \tag{21.15}$$

where G is a "bare" Newton constant, and where the Standard Model matter is coupled not to the "Einstein" (pure spin-2) metric $g_{\mu\nu}$, but to the conformally related ("Jordan-Fierz") metric $\widetilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$. The scalar field equation $\Box_g\varphi = -(4\pi G/c^4)\alpha(\varphi)T$ displays $\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi$ as the basic (field-dependent) coupling between φ and matter [29,30]. The one-parameter (ω) Jordan-Fierz-Brans-Dicke theory [28] is the special case $a(\varphi) = \alpha_0\varphi$ leading to a field-independent coupling $\alpha(\varphi) = \alpha_0$ (with ${\alpha_0}^2 = 1/(2\omega + 3)$). The addition of a self-interaction term $V(\varphi)$ in Eq. (21.15) introduces new phenomenological possibilities; notably the "chameleon mechanism" [31].

In the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system, the addition of φ modifies Einstein's predictions only through the appearance of two "post-Einstein" dimensionless parameters: $\overline{\gamma} = -2\alpha_0^2/(1 + \alpha_0^2)$ and $\overline{\beta} = +\frac{1}{2}\beta_0\alpha_0^2/(1 + \alpha_0^2)^2$, where $\alpha_0 \equiv \alpha(\varphi_0)$, $\beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$, φ_0 denoting the vacuum expectation value of φ. These parameters show up also naturally (in the form $\gamma_{\rm PPN} = 1+\overline{\gamma}$, $\beta_{\rm PPN} = 1 + \overline{\beta}$) in phenomenological discussions of possible deviations from GR [32]. The parameter $\overline{\gamma}$ measures the admixture of spin 0 to Einstein's graviton, and contributes an extra term $+\overline{\gamma}(\boldsymbol{v}_A - \boldsymbol{v}_B)^2/c^2$ in the square brackets of the two-body Lagrangian Eq. (21.13). The parameter $\overline{\beta}$ modifies the three-body interaction Eq. (21.14) by an overall multiplicative factor $1 + 2\overline{\beta}$. Moreover, the combination $\eta \equiv 4\overline{\beta} - \overline{\gamma}$ parametrizes the lowest order effect of the self-gravity of orbiting masses by modifying the Newtonian interaction energy terms in Eq. (21.13) into $G_{AB}m_Am_B/r_{AB}$, with a body-dependent gravitational "constant" $G_{AB} = G_N[1+\eta(E_A^{\rm grav}/m_Ac^2 + E_B^{\rm grav}/m_Bc^2) + O(1/c^4)]$, where $G_N = G\exp[2a(\varphi_0)](1 + \alpha_0^2)$ and where $E_A^{\rm grav}$ denotes the gravitational binding energy of body A.

The best current limits on the post-Einstein parameters $\overline{\gamma}$ and $\overline{\beta}$ are (at the 68% confidence level):

$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5} \ , \tag{21.16}$$

deduced from the additional Doppler shift experienced by radio-wave beams connecting the Earth to the Cassini spacecraft when they passed near the Sun [33], and

$$|\overline{\beta}| < 7 \times 10^{-5} \ , \tag{21.17}$$

from a study of the global sensitivity of planetary ephemerides to post-Einstein parameters [34]. More stringent limits on $\overline{\gamma}$ are obtained in models (*e.g.*, string-inspired ones [25]) where scalar couplings violate the Equivalence Principle.

21.3. Tests of the dynamics of the gravitational field in the radiative and/or strong field regimes: pulsars

The discovery of pulsars (*i.e.*, rotating neutron stars emitting a beam of radio noise) in gravitationally bound orbits [35,36] has given us our first experimental handle on the regime of radiative and/or strong gravitational fields. In these systems, the finite velocity of propagation of the gravitational interaction between the pulsar and its companion generates damping-like terms at order $(v/c)^5$ in the equations of motion [37]. These damping forces are the local counterparts of the gravitational radiation emitted at infinity by the system ("gravitational radiation reaction"). They cause the binary orbit to shrink and its orbital period P_b to decrease. The remarkable stability of pulsar clocks has allowed one to measure the corresponding very small orbital period decay $\dot{P}_b \equiv dP_b/dt \sim -(v/c)^5 \sim -10^{-12}$ in several binary systems, thereby giving us a direct experimental confirmation of the propagation properties of the gravitational field, and, in particular, an experimental confirmation that the speed of propagation of gravity c_g is equal to the velocity of light c to better than a part in a thousand. In addition, the surface gravitational potential of a neutron star $h_{00}(R_{\rm NS}) \simeq 2Gm/c^2R_{\rm NS} \simeq 0.4$ being a factor $\sim 10^8$ higher than the surface potential of the Earth, and a mere factor 2.5 below the black hole limit ($h_{00}(R_{\rm BH}) = 1$), pulsar data have allowed one to obtain several accurate tests of the strong-gravitational-field regime, as we discuss next.

Binary pulsar timing data record the times of arrival of successive electromagnetic pulses emitted by a pulsar orbiting around the center of mass of a binary system. After correcting for the Earth motion around the Sun and for the dispersion due to propagation in the interstellar plasma, the time of arrival of the Nth pulse t_N can be described by a generic, parametrized "timing formula" [38] whose functional form is common to the whole class of tensor-scalar gravitation theories:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p)\,;\,\{p^K\}\,;\,\{p^{PK}\}]\;. \tag{21.18}$$

Here, T_N is the pulsar proper time corresponding to the Nth turn given by $N/2\pi = \nu_p T_N + \frac{1}{2}\dot{\nu}_p T_N^2 + \frac{1}{6}\ddot{\nu}_p T_N^3$ (with $\nu_p \equiv 1/P_p$ the spin frequency of the pulsar, *etc.*), $\{p^K\} = \{P_b, T_0, e, \omega_0, x\}$ is the set of "Keplerian" parameters (notably, orbital period P_b, eccentricity e, periastron longitude ω_0 and projected semi-major axis $x = a\sin i/c$), and $\{p^{PK}\} = \{k, \gamma_{\rm timing}, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$ denotes the set of (separately measurable) "post-Keplerian" parameters. Most important among these are: the fractional periastron advance per orbit $k \equiv \dot{\omega}P_b/2\pi$, a dimensionful time-dilation parameter $\gamma_{\rm timing}$, the orbital period derivative $\dot{P}_b$, and the "range" and "shape" parameters of the gravitational time delay caused by the companion, r and s.

Without assuming any specific theory of gravity, one can phenomenologically analyze the data from any binary pulsar by least-squares fitting the observed sequence of pulse arrival times to the timing formula Eq. (21.18). This fit yields the "measured" values of the parameters $\{\nu_p, \dot{\nu}_p, \ddot{\nu}_p\}$, $\{p^K\}$, $\{p^{PK}\}$. Now, each specific relativistic theory of gravity predicts that, for instance, k, $\gamma_{\rm timing}$, $\dot{P}_b$, r and s (to quote parameters that have been successfully measured from some binary pulsar data) are some theory-dependent functions of the Keplerian parameters and of the (unknown) masses m_1, m_2 of the pulsar and its companion. For instance, in GR, one finds (with $M \equiv m_1 + m_2$, $n \equiv 2\pi/P_b$)

$$\begin{aligned}
k^{\rm GR}(m_1, m_2) =& 3(1-e^2)^{-1}(G_N M n/c^3)^{2/3} \ , \\
\gamma^{\rm GR}_{\rm timing}(m_1, m_2) =& e n^{-1}(G_N M n/c^3)^{2/3} m_2 (m_1 + 2m_2)/M^2 \ , \\
\dot{P}^{\rm GR}_b(m_1, m_2) =& -(192\pi/5)(1-e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \\
& \times (G_N M n/c^3)^{5/3} m_1 m_2/M^2 \ , \\
r(m_1, m_2) =& G_N m_2/c^3 \ , \\
s(m_1, m_2) =& n x (G_N M n/c^3)^{-1/3} M/m_2 \ .
\end{aligned} \tag{21.19}$$

In tensor-scalar theories, each of the functions $k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm theory}_b(m_1, m_2)$, *etc.*, is modified by quasi-static strong field effects (associated with the self-gravities of the pulsar and its companion), while the particular function $\dot{P}^{\rm theory}_b(m_1, m_2)$ is further modified by radiative effects (associated with the spin 0 propagator) [30,39,40].

Let us give some highlights of the current experimental situation. In the first discovered binary pulsar PSR 1913+16 [35,36], it has been (recently [41]) possible to measure *five* post-Keplerian parameters: k, $\gamma_{\rm timing}$, $\dot{P}_b$, and (with less accuracy) r and s. [Even more post-Keplerian parameters have been recently measured [41], but they cannot be currently used to test gravity theories.] The five equations $k^{\rm measured} = k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm measured}_{\rm timing} = \gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm measured}_b = \dot{P}^{\rm theory}_b(m_1, m_2)$, $r^{\rm measured} = r^{\rm theory}(m_1, m_2)$, $s^{\rm measured} = s^{\rm theory}(m_1, m_2)$, determine, for each given theory, five curves in the two-dimensional mass plane. [The less accurate measurements of r and s determine strips rather than thin curves.] This yields *three* tests of the specified theory, according to whether the five curves (or strips) have one point in common, as they should. After subtracting a small ($\sim 10^{-14}$ level in $\dot{P}^{\rm obs}_b = (-2.423 \pm 0.001) \times 10^{-12}$), but significant, "galactic" perturbing effect (linked to galactic accelerations and to the pulsar proper motion) [42], one finds that GR passes these three (combined radiative/strong-field) tests with flying colors. The most accurate of these three tests involves the three quantities $(k - \gamma_{\rm timing} - \dot{P}_b)_{1913+16}$, and is passed with complete success at the 10^{-3} level [36,43,41]

$$\left[\frac{\dot{P}^{\rm obs}_b - \dot{P}^{\rm gal}_b}{\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1913+16} = 0.9983 \pm 0.0016 \ . \tag{21.20}$$

Here $\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]$ is the result of inserting in $\dot{P}^{\rm GR}_b(m_1, m_2)$ the values of the masses predicted by the two equations $k^{\rm obs} = k^{\rm GR}(m_1, m_2)$, $\gamma^{\rm obs}_{\rm timing} = \gamma^{\rm GR}_{\rm timing}(m_1, m_2)$. This yields experimental evidence for the reality of gravitational radiation damping forces at the $(-1.7 \pm 1.6) \times 10^{-3}$ level.

In the binary pulsar PSR 1534+12 [44] one has measured *five* post-Keplerian parameters: k, $\gamma_{\rm timing}$, r, s, and (with less accuracy)$\dot{P}_b$ [45,46]. This yields *three* more tests of relativistic gravity. Two among these tests accurately probe strong field gravity, without mixing of radiative effects [45]. General Relativity passes all these tests within the measurement accuracy. The most precise of the pure strong-field tests is the one obtained by combining the measurements of k, $\gamma_{\rm timing}$, and s. Using the most recent data [46], one finds agreement at the $(2 \pm 2) \times 10^{-3}$ level:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1534+12} = 1.002 \pm 0.002 \ . \tag{21.21}$$

In the binary pulsar PSR J1141−6545 [47](whose companion is probably a white dwarf) one has measured *four* observable parameters: $k, \gamma_{\rm timing}, \dot{P}_b$ [48,49], and the parameter s [50,49]. The latter parameter (which is equal to the sine of the inclination angle, $s = \sin i$) was consistently measured in two ways: from a scintillation analysis [50], and from timing measurements [49]. GR passes all the corresponding tests within measurement accuracy. See Fig. 21.1 which uses the (more precise) scintillation measurement of $s = \sin i$.

The discovery of the remarkable *double* binary pulsar PSR J0737−3039 A and B [51,52] has led to the measurement of *seven* independent parameters [53,54,55]: five of them are the post-Keplerian parameters k, $\gamma_{\rm timing}$, r, s and $\dot{P}_b$ entering the relativistic timing formula of the fast-spinning pulsar PSR J0737−3039 A, a sixth is the ratio $R = x_B/x_A$ between the projected semi-major axis of the more slowly spinning companion pulsar PSR J0737−3039 B, and that of PSR J0737−3039 A. [The theoretical prediction for the ratio $R = x_B/x_A$, considered as a function of the (inertial) masses $m_1 = m_A$ and $m_2 = m_B$, is $R^{\rm theory} = m_1/m_2 + O((v/c)^4)$ [38], independently of the gravitational theory considered.] Finally, the seventh parameter $\Omega_{\rm SO,B}$ is the angular rate of (spin-orbit) precession of PSR J0737−3039 B around the total angular momentum [54,55]. These seven measurements give us *five* tests of relativistic gravity [53,56,57]. GR passes all those tests with flying colors (see Fig. 21.1). Let us highlight here two of them (from [57]) .

One test is a new confirmation of the reality of gravitational radiation at the 10^{-3} level

$$\left[\frac{\dot{P}_b^{\rm obs}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, R^{\rm obs}]}\right]_{0737-3039} = 1.000 \pm 0.001 \ . \tag{21.22}$$

Another one is a new, 5×10^{-4} level, strong-field confirmation of GR:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs}, R^{\rm obs}]}\right]_{0737-3039} = 1.0000 \pm 0.0005 \ . \tag{21.23}$$

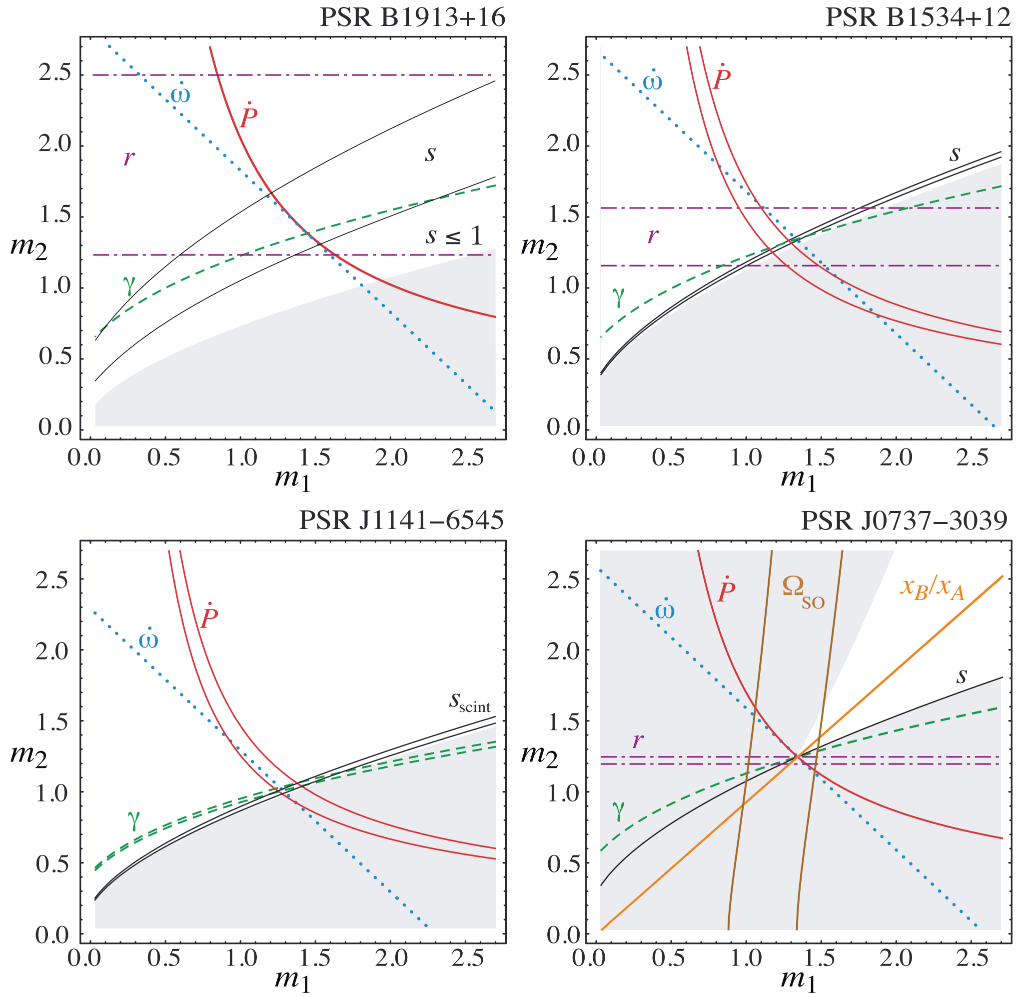

Figure 21.1: Illustration of the *thirteen* tests of relativistic gravity obtained in the four different binary pulsar systems PSR1913+16 (3 tests), PSR1534+12 (3 tests), PSR J1141−6545 (2 tests), and PSR J0737−3039 A,B (5 tests). Each curve (or strip) in the mass plane corresponds to the interpretation, within GR, of some observable parameter among: $\dot{P}_b$, $k \equiv \dot{\omega} P_b / 2\pi$, $\gamma_{\rm timing}$, r, $s = \sin i$, $\Omega_{\rm SO,B}$ and R. The shaded regions are excluded because they would correspond (in GR) to $s = |\sin i| > 1$. (Figure updated from [71]; courtesy of G. Esposito-Farèse.)

Fig. 21.1 illustrates the thirteen tests of strong-field and radiative gravity derived from the above-mentioned binary pulsars: $(5-2=)$ 3 tests from PSR1913+16, $(5-2=)$ 3 tests from PSR1534+12, $(4-2=)$ 2 tests from PSR J1141−6545, and $(7-2=)$ 5 tests from PSR J0737−3039. [See, also, [58] for additional, less accurate, and partially discrepant, tests of relativistic gravity.]

Data from several nearly circular binary systems (made of a neutron star and a white dwarf) have also led to strong-field confirmations (at the 4.6×10^{-3} level) of the 'strong equivalence principle,' *i.e.*, the fact that neutron stars and white dwarfs fall with the same acceleration in the gravitational field of the Galaxy [59,60,61]. The measurements of $\dot{P}_b$ in some pulsar-white dwarf systems lead to strong constraints on the variation of Newton's G_N, and on the existence of gravitational dipole radiation [62,63,64,66,67]. In addition, arrays of millisecond pulsars are sensitive detectors of ultra low frequency gravitational waves ($f \sim 10^{-9} - 10^{-8}$ Hz) [68,69]. Such waves might be generated by supermassive black-hole binary systems, by cosmic strings and/or during the inflationary era. Pulsar timing arrays have recently put strong constraints on current models of supermassive black-hole binaries by finding no evidence for a background of gravitational waves with periods between ~ 1 and ~ 10 years [70].

The constraints on tensor-scalar theories provided by the various binary-pulsar "experiments" have been analyzed in [45,40,64,65,71,72] and shown to exclude a large portion of the parameter space allowed by solar-system tests. Some of the most stringent tests follow from the measurement of the orbital period decay $\dot{P}_b$ of low-eccentricity pulsar-white dwarf systems (notably PSR J1738+0333 [64]) . Indeed, asymmetric binary systems are strong emitters of dipolar gravitational radiation in tensor-scalar theories, with $\dot{P}_b$ scaling (modulo matter-scalar couplings) like $m_1 m_2/(m_1+m_2)^2 (v/c)^3$, instead of the parametrically smaller quadrupolar radiation $\dot{P}_b \sim (v/c)^5$ [32,30]. As a result the basic matter-scalar coupling α_0^2 is more strongly constrained, over most of the parameter space, than the best current solar-system limits Eq. (21.16), Eq. (21.17) (namely below the 10^{-5} level) [64,65].

Measurements over several years of the pulse profiles of various pulsars have detected secular profile changes compatible with the prediction [73] that the general relativistic spin-orbit coupling should cause a secular change in the orientation of the pulsar beam with respect to the line of sight ("geodetic precession"). Such confirmations of general-relativistic spin-orbit effects were obtained in PSR 1913+16 [74], PSR B1534+12 [46], PSR J1141−6545 [75], PSR J0737−3039 [54,55] and PSR J1906+0746 [76]. In some cases (notably PSR 1913+16 and PSR J1906+0746) the secular change in the orientation of the pulsar beam is expected to lead to the disappearance of the beam (as seen on the Earth) on a human time scale (the second pulsar in the double system PSR J0737−3039 has already disappeared in March 2008 and is expected to reappear around 2035 [55]) .

21.4. Tests of the dynamics of the gravitational field in the radiative and strong field regimes: gravitational waves

The observation, by the US-based Laser Interferometer Gravitational-wave Observatory (LIGO), later joined by the Europe-based Virgo detector, of gravitational-wave (GW) signals [77,78,79,80,81], has opened up a novel testing ground for relativistic gravity. The four transient signals GW150914, GW151226, GW170104 and GW170814, are most readily interpreted as the GW signals emitted ($\gtrsim 400$ Mpc away) by the last inspiralling orbits and the merger of binary black holes. The longer (~ 100 s) and louder (signal-to-noise ratio [SNR] ~ 32) signal GW170817 is most readily interpreted as coming from a binary neutron star inspiral (~ 40 Mpc away), and was associated with a subsequent γ-ray burst, followed by transient counterparts across the electromagnetic spectrum [82]. Thanks to the rather high SNRs, respectively, ~ 24, ~ 13, ~ 13, ~ 18, ~ 32, of the LIGO-Virgo observations, one could test consistency with GR in several ways.

For the binary black hole events, a first level of consistency check follows from the good global agreement between the full observed signal and the signal predicted by both analytical [83] and numerical [84] calculations of the gravitational waveform emitted by coalescing black holes. In particular, the noise-weighted correlation between the observed strain signal GW150914 and the best-fit GR-predicted waveform was found to be $\geq 96\%$ [85]. In other words, GR-violation effects that cannot be reabsorbed in a redefinition of physical parameters are limited (in a noise-weighted sense) to less than 4%. A perturbed black hole has characteristic ringing GW modes [86], whose frequencies and decay times are functions of the mass and spin of the black hole. The final black hole (with mass M_f and dimensionless spin parameter $a_f = J_f/(G_N M_f^2)$) formed by the coalescence of the two initial black holes emits, just after merger, a superposition of such (rapidly decaying) ringing GW modes. Currently, only GW150914 has allowed one to test the consistency between the observed signal and the (separately considered) GR predictions for the inspiral signal (up to the GW frequency $f^{\text{end inspiral}} = 132$ Hz), the post-inspiral one ($f_{\text{GW}} \geq f^{\text{end inspiral}}$), and also, to some extent, the post-merger signal (merger being defined as the moment where the GW amplitude $h_{\mu\nu}$ reaches a maximum). First, the joint posterior distribution for M_f and a_f, obtained by separately best fitting to the corresponding GR predictions either the inspiral signal or the post-inspiral one, have been found to be consistent (see Fig. 4 in [85]) . A second, less accurate, check has found consistency between the measurement of the frequency and decay time of the first (least-damped) ringing mode from the latish post-merger signal, and the values inferred (using GR predictions) from fitting the entire signal (see Fig. 5 in [85]) .

Quantitatively more precise tests have been obtained from GW151226, which features a much longer signal (~ 55 GW cycles). The most accurate test has consisted in phenomenologically allowing the numerical coefficient φ_3 (parametrizing the contribution proportional to $f_{\text{GW}}^{-2/3}$ in the frequency evolution of the Fourier-domain phase of the GW signal during the early inspiral) to vary [87,88]. [This contribution is physically related to "tail" effects in the curved spacetime propagation of the GW signal.] The 90 % credible limit on the fractional variation of φ_3 obtained from GW151226 data is $\Delta\varphi_3/\varphi_3 \leq 0.1$ [88]. The three-detector observation of GW170814 has allowed one to probe the polarization content of the GW signal: the data were found to strongly favor the GR-predicted pure tensor polarization of GWs [80].

GR predicts that GWs are non dispersive, and propagate at the same velocity as light. One can phenomenologically modify the GR-predicted GW phase evolution by adding the putative effect of an anomalous dispersion relation of the form $E^2 = p^2c^2 + Ap^\alpha c^\alpha$. GW data have been used to set bounds on the anomalous coefficient A for various values of the exponent α (see Fig. 5 in [79]) . The case $\alpha = 0$ is equivalent to assuming that gravitons disperse as a massive particle [89]. Combined GW data lead to the following phenomenological limit on the graviton mass: $m_g \leq 7.7 \times 10^{-23} eV/c^2$ [79]. [See [90] for other graviton mass bounds.] Finally, the observed time delay of ~ 1.7 s between GW170817 and the associated γ-ray burst constrains the fractional difference between the speed of GWs and the speed of light to be between -3×10^{-15} and $+7 \times 10^{-16}$ [91].

Contrary to the solar-system, and binary-pulsar, tests, the phenomenological GW emission tests deduced from black hole merger signals do not directly constrain the most conservative class of theoretical deviations from GR obtained by adding a light scalar field φ, as in Eq. (21.15). Indeed, the no-hair properties of (4-dimensional) black holes mean that φ does not couple to black holes, so that, when neglecting large-scale external gradients (or fine-tuned initial data), light scalar fields have no effect on either the dynamics or the GW emission of black hole binaries.

21.5. Conclusions

All present experimental tests are compatible with the predictions of the current "standard" theory of gravitation: Einstein's General Relativity. The universality of the coupling between matter and gravity (Equivalence Principle) has been verified around the 10^{-14} level. Solar system experiments have tested the weak-field predictions of Einstein's theory at the few 10^{-5} level. The propagation properties (in the near zone) of relativistic gravity, as well as several of its static strong-field aspects, have been verified at the 10^{-3} level (or better) in several binary pulsar experiments. Interferometric detectors of gravitational radiation have given direct observational proofs of the existence, and properties, of gravitational waves (in the wave zone), and of the existence of coalescing black holes, and they have started to explore several dynamic aspects of strong-field gravity. Recent laboratory experiments have set strong constraints on sub-millimeter modifications of Newtonian gravity. Quantitative confirmations of GR have also been obtained on astrophysical scales. The GR action on light and matter of an external gravitational field has been verified in many gravitational lensing systems [92]. Some tests on cosmological scales are also available [93]. Beyond the quantitative limits on various parametrized theoretical models discussed in the latter reference, one should remember the striking (strong-field-type) qualitative verification of GR embodied in the fact that relativistic cosmological models give an accurate picture of the Universe over a period during which the spatial metric has been blown up by a gigantic factor, say $(1+z)^2 \sim 10^{19}$ between Big Bang nucleosynthesis and now (though a skeptic might wish to keep in mind the two "dark clouds" of current cosmology, namely the need to assume dark matter and a cosmological constant).

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