68. Form Factors for Radiative Pion and Kaon Decays

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The radiative decays, $\pi^{\pm} \to l^{\pm}\nu\gamma$ and $K^{\pm} \to l^{\pm}\nu\gamma$, with l standing for an e or a μ, and γ for a real or virtual photon (e^+e^- pair), provide a powerful tool to investigate the hadronic structure of pions and kaons. The structure-dependent part SD_i of the amplitude describes the emission of photons from virtual hadronic states, and is parametrized in terms of form factors V, A, (vector, axial vector), in the standard description [1,2,3,4]. Note that in the Listings below and some literature, equivalent nomenclature F_V and F_A for the vector and axial form factors is often used. Exotic, non-standard contributions like $i = T, S$ (tensor, scalar) have also been considered. Apart from the SD terms, there is also the Inner Bremsstrahlung amplitude, IB, corresponding to photon radiation from external charged particles and described by Low theorem in terms of the physical decay $\pi^{\pm}(K^{\pm}) \to l^{\pm}\nu$. Experiments try to optimize their kinematics so as to minimize the IB part of the amplitude.

The SD amplitude in its standard form is given as

$$M(\mathrm{SD}_V) = \frac{-eG_F U_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu} V^P \epsilon_{\mu\nu\sigma\tau} k^{\sigma} q^{\tau} \tag{68.1}$$

$$\begin{aligned} M(\mathrm{SD}_A) = {} & \frac{-ieG_F U_{qq'}}{\sqrt{2}m_P}\epsilon^{\mu} l^{\nu} \{A^P[(qk - k^2)g_{\mu\nu} - q_{\mu}k_{\nu}] \\ & + R^P k^2 g_{\mu\nu}\} \ , \end{aligned} \tag{68.2}$$

which contains an additional axial form factor R^P which only can be accessed if the photon remains virtual. $U_{qq'}$ is the Cabibbo-Kobayashi-Maskawa mixing-matrix element; ϵ^{μ} is the polarization vector of the photon (or the effective vertex, $\epsilon^{\mu} = (e/k^2)\overline{u}(p_-)\gamma^{\mu}v(p_+)$, of the e^+e^- pair); $\ell^{\nu} = \overline{u}(p_{\nu})\gamma^{\nu}(1-\gamma_5)v(p_{\ell})$ is the lepton-neutrino current; q and k are the meson and photon four-momenta ($k = p_+ + p_-$ for virtual photons); and P stands for π or K.

For decay processes where the photon is real, the partial decay width can be written in analytical form as a sum of IB, SD, and IB/SD interference terms INT [1,4]:

$$\begin{aligned} \frac{d^2\Gamma_{P\to\ell\nu\gamma}}{dxdy} &= \frac{d^2\left(\Gamma_{\mathrm{IB}} + \Gamma_{\mathrm{SD}} + \Gamma_{\mathrm{INT}}\right)}{dxdy} \\ &= \frac{\alpha}{2\pi}\Gamma_{P\to\ell\nu}\frac{1}{(1-r)^2}\Bigg\{\mathrm{IB}(x,y) \\ & + \frac{1}{r}\left(\frac{m_P}{2f_P}\right)^2\left[(V+A)^2\mathrm{SD}^+(x,y) + (V-A)^2\mathrm{SD}^-(x,y)\right] \\ & + \epsilon_P\frac{m_P}{f_P}\left[(V+A)\mathrm{S}^+_{\mathrm{INT}}(x,y) + (V-A)\mathrm{S}^-_{\mathrm{INT}}(x,y)\right]\Bigg\}. \end{aligned} \tag{68.3}$$

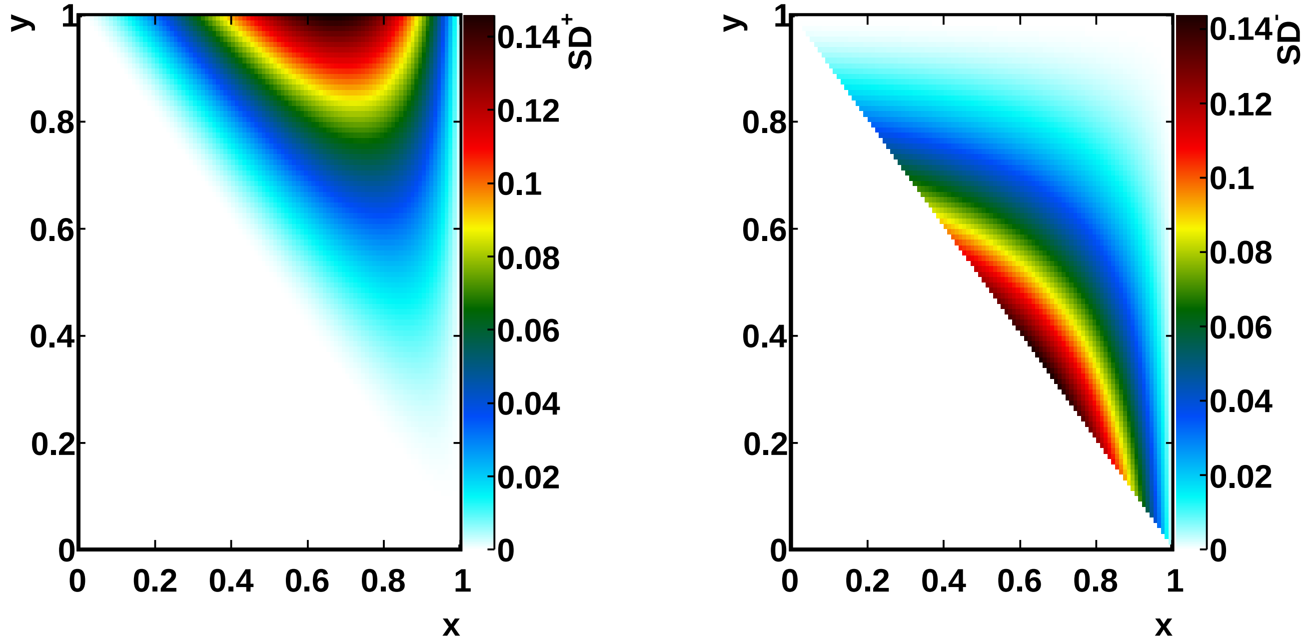

Figure 68.1: Components of the structure dependent terms of the decay width. Left: SD^+, right: SD^-

Here

$$\mathrm{IB}(x,y) = \left[\frac{1-y+r}{x^2(x+y-1-r)}\right]$$

$$\left[x^2 + 2(1-x)(1-r) - \frac{2xr(1-r)}{x+y-1-r}\right]$$

$$\mathrm{SD}^+(x,y) = (x+y-1-r)\Big[(x+y-1)(1-x)-r\Big]$$

$$\mathrm{SD}^-(x,y) = (1-y+r)\Big[(1-x)(1-y)+r\Big]$$

$$\mathrm{S}^+_{\mathrm{INT}}(x,y) = \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[(1-x)(1-x-y)+r\Big]$$

$$\mathrm{S}^-_{\mathrm{INT}}(x,y) = \left[\frac{1-y+r}{x(x+y-1-r)}\right]\Big[x^2-(1-x)(1-x-y)-r\Big] \tag{68.4}$$

where $x = 2E_\gamma/m_P$, $y = 2E_\ell/m_P$, $r = (m_\ell/m_P)^2$, f_P is the meson decay constant, and ϵ_P is +1 for pions and -1 for kaons. The structure dependent terms SD^+ and SD^- are shown in Fig. 1. The SD^- term is maximized in the same kinematic region where overwhelming IB term dominates (along $x + y = 1$ diagonal). Thus experimental yields with less background are dominated by SD^+ contribution and proportional to $A^P + V^P$ making simultaneous precise determination of the form factors difficult.

Recently, formulas (3) and (4) have been extended to describe polarized distributions in radiative meson and muon decays [7].

The "helicity" factor r is responsible for the enhancement of the SD over the IB amplitude in the decays $\pi^\pm \to e^\pm\nu\gamma$, while $\pi^\pm \to \mu^\pm\nu\gamma$ is dominated by IB. Interference terms are important for the decay $K^\pm \to \mu^\pm\nu\gamma$ [8], but contribute only a few percent correction to pion decays. However, they provide the basis for determining the signs of V and A. Radiative corrections to the decay $\pi^+ \to e^+\nu\gamma$ have to be taken into account in the analysis of the precision experiments. They make up to 4% corrections in the total decay rate [9]. In $\pi^\pm \to e^\pm\nu e^+e^-$ and $K^\pm \to \ell^\pm\nu e^+e^-$ decays, all three form factors, V^P, A^P, and R^P, can be determined [10,11].

Theoretically, the first non-trivial χPT contributions to A^P and V^P appear at $\mathcal{O}(p^4)$ [4], respectively from Gasser-Leutwyler coefficients, L_i's, and the anomalous lagrangian:

$$A^P = \frac{4\sqrt{2}M_P}{F_\pi}(L_9^r + L_{10}^r), \quad V^P = \frac{\sqrt{2}M_P}{8\pi^2 F_\pi}. \tag{68.5}$$

In case of the kaon $A^K = 0.042$ and $V^K = 0.096$. $\mathcal{O}(p^6)$ contributions to A^K can be predicted accurately: they are flat in the momentum dependence and shift the $\mathcal{O}(p^4)$ value to 0.034. $\mathcal{O}(p^6)$ contributions to V^K are model dependent and can be approximated by a form factor linearly dependent on momentum. For example, when looking at the spread of results obtained within two different models, the constant piece of this linear form factor is shifted to 0.078 ± 0.005 [1,2,4].

We give the experimental $\pi^\pm$ form factors V^π, A^π, and R^π in the Listings below. In the $K^\pm$ Listings, we give the extracted sum $A^K + V^K$ and difference $A^K - V^K$, as well as V^K, A^K and R^K. In particular KLOE has measured for the constant piece of the form factor $A^K + V^K = 0.125 \pm 0.007 \pm 0.001$ [13] while ISTRA+, $V^K - A^K = 0.21 \pm 0.04 \pm 0.04$ [14].

The pion vector form factor, V^π, is related via CVC (Conserved Vector Current) to the $\pi^0 \to \gamma\gamma$ decay width. The constant term is given by $|V^\pi(0)| = (1/\alpha)\sqrt{2\Gamma_{\pi^0\to\gamma\gamma}/\pi m_{\pi^0}}$ [3]. The resulting value, $V^\pi(0) = 0.0259(9)$, has been confirmed by calculations based on chiral perturbation theory (χPT) [4], and by two experiments given in the Listings below. A recent experiment by the PIBETA collaboration [5] obtained a $V^\pi(0)$ that is in excellent agreement with the CVC hypothesis. It also measured the slope parameter a in $V^\pi(s) = V^\pi(0)(1 + a\cdot s)$, where $s = (1 - 2E_\gamma/m_\pi)$, and E_γ is the gamma energy in the pion rest frame: $a = 0.095 \pm 0.058$. A functional dependence on s is expected for all form factors. It becomes non-negligible in the case of $V^{\pi}(s)$ when a wide range of photon momenta is recorded; proper treatment in the analysis of K decays is mandatory.

The form factor, R^P, can be related to the electromagnetic radius, r_P, of the meson [2]: $R^P = \frac{1}{3} m_P f_P \langle r_P^2 \rangle$ using PCAC (Partial Conserved Axial vector Current).

In lowest order χPT, the ratio A^{π}/V^{π} is related to the pion electric polarizability $\alpha_E = [\alpha/(8\pi^2 m_\pi f_\pi^2)] \times A^{\pi}/V^{\pi}$ [6]. Direct experimental and theoretical status of pion polarizability studies currently is not settled. Most recent theoretical predictions from χPT [15] and experimental results from COMPASS collaboration [16] favor a small value of pion polarizability $\alpha_\pi \sim (2 \div 3) \times 10^{-4}\,\mathrm{fm}^3$. Dispersive analysis of $\gamma\gamma \to \pi^+\pi^-$ crossection [17] and experimental results from MAMI collaboration [18] report a much larger value of $\alpha_\pi \sim 6 \times 10^{-4}\,\mathrm{fm}^3$. Precise measurement of the pion form factors by PIBETA collaboration favors smaller values of polarizability $\alpha_\pi = 2.7^{+0.6}_{-0.5} \times 10^{-4}\,\mathrm{fm}^3$.

Several searches for the exotic form factors F_T^{π}, F_T^{K} (tensor), and F_S^{K} (scalar) have been pursued in the past. In particular, F_T^{π} has been brought into focus by experimental as well as theoretical work [12]. New high-statistics data from the PIBETA collaboration have been re-analyzed together with an additional data set optimized for low backgrounds in the radiative pion decay. In particular, lower beam rates have been used in order to reduce the accidental background, thereby making the treatment of systematic uncertainties easier and more reliable. The PIBETA analysis now restricts F_T^{π} to the range $-5.2 \times 10^{-4} < F_T^{\pi} < 4.0 \times 10^{-4}$ at a 90% confidence limit [5]. This result is in excellent agreement with the most recent theoretical work [4].

Precision measurements of radiative pion and kaon decays are effective tools to study QCD in the non-perturbative region and are of interest beyond the scope of radiative decays. Meanwhile other processes such as $\pi^+ \to e^+\nu$ that seem to be better suited to search for new physics at the precision frontier are currently studied. The advantages of such process are the very accurate and reliable theoretical predictions and the more straightforward experimental analysis.

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