69. Scalar Mesons below 2 GeV

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69.1 Introduction

In contrast to the vector and tensor mesons, the identification of the scalar mesons is a long-standing puzzle. Scalar resonances are difficult to resolve because some of them have large decay widths which cause a strong overlap between resonances and background. In addition, several decay channels sometimes open up within a short mass interval (*e.g.* at the $K\bar{K}$ and $\eta\eta$ thresholds), producing cusps in the line shapes of the near-by resonances. Furthermore, one expects non-$q\bar{q}$ scalar objects, such as glueballs and multiquark states in the mass range below 2 GeV (for reviews see, *e.g.*, Refs. [1–5] and the mini-review on *non–$\bar{q}q$ states* in this Review of Particle Physics (RPP)).

Light scalars are produced, for example, in πN scattering on polarized/unpolarized targets, $p\bar{p}$ annihilation, central hadronic production, J/Ψ, B-, D- and K-meson decays, $\gamma\gamma$ formation, and ϕ radiative decays. Especially for the lightest scalar mesons simple parameterizations fail and more advanced theory tools are necessary to extract the resonance parameters from data. In the analyses available in the literature fundamental properties of the amplitudes such as unitarity, analyticity, Lorentz invariance, chiral and flavor symmetry are implemented at different levels of rigor. Especially, chiral symmetry implies the appearance of zeros close to the threshold in elastic S-wave scattering amplitudes involving soft pions [6,7], which may be shifted or removed in associated production processes [8]. The methods employed are the K-matrix formalism, the N/D-method, the Dalitz–Tuan ansatz, unitarized quark models with coupled channels, effective chiral field theories and the linear sigma model, *etc.* Dynamics near the lowest two-body thresholds in some analyses are described by crossed channel (t, u) meson exchange or with an effective range parameterization instead of, or in addition to, resonant features in the s-channel. Dispersion theoretical approaches are applied to pin down the location of resonance poles for the low–lying states [9–12].

The mass and width of a resonance are found from the position of the nearest pole in the process amplitude (T-matrix or S-matrix) at an unphysical sheet of the complex energy plane, traditionally labeled as

$$\sqrt{s_{\text{Pole}}} = M - i\,\Gamma/2 \ . \tag{69.1}$$

It is important to note that the pole of a Breit-Wigner parameterization agrees with this pole position only for narrow and well–separated resonances, far away from the opening of decay channels. For a detailed discussion of this issue we refer to the review on *Resonances* in this RPP.

In this note, we discuss the light scalars below 2 GeV organized in the listings under the entries ($I = 1/2$) $K_0^*(700)$ (or κ), $K_0^*(1430)$, ($I = 1$) $a_0(980)$, $a_0(1450)$, and ($I = 0$) $f_0(500)$ (or σ), $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. This list is minimal and does not necessarily exhaust the list of actual resonances. The ($I = 2$) $\pi\pi$ and ($I = 3/2$) $K\pi$ phase shifts do not exhibit any resonant behavior.

69.2 The $I = 1/2$ States

The $\boldsymbol{K_0^*(1430)}$ [13] is perhaps the least controversial of the light scalar mesons. The $K\pi$ S-wave scattering has two possible isospin channels, I=1/2 and I=3/2. The I=3/2 wave is elastic and repulsive up to 1.7 GeV [14] and contains no known resonances. The I=1/2 $K\pi$ phase shift, measured from about 100 MeV above threshold in Kp production, rises smoothly, passes 90° at 1350 MeV, and continues to rise to about 170° at 1600 MeV. The first important inelastic threshold is $K\eta'(958)$. In the inelastic region the continuation of the amplitude is uncertain since the partial-wave decomposition has several solutions. The data are extrapolated towards the $K\pi$ threshold using effective range type formulas [13,15] or chiral perturbation predictions [16,17]. From analyses using unitarized amplitudes there is agreement on the presence of a resonance pole around 1410 MeV having a width of about 300 MeV. With reduced model dependence, Ref. [18] finds a larger width of 500 MeV.

Similar to the situation for the $f_0(500)$, discussed in the next section, the presence and properties of the light $\boldsymbol{K_0^*(700)}$ (or $\boldsymbol{\kappa}$) meson in the 700-900 MeV region are difficult to establish since it appears to have a very large width ($\Gamma \approx 500$ MeV) and resides close to the $K\pi$ threshold. Hadronic D- and B-meson decays provide additional data points in the vicinity of the $K\pi$ threshold and are discussed in detail in the *Review on Multibody Charm Analyses* in this RPP. Precision information from semileptonic D decays avoiding the theoretically more demanding final states with three strongly interacting particles is not available. BES II [19] (re-analyzed in [20]) finds a $K_0^*(700)$–like structure in J/ψ decays to $\bar{K}^{*0}(892)K^+\pi^-$ where $K_0^*(700)$ recoils against the $K^*(892)$. Also clean with respect to final-state interaction is the decay $\tau^- \to K_S^0\pi^-\nu_\tau$ studied by Belle [21], with $K_0^*(700)$ parameters fixed to those of Ref. [19].

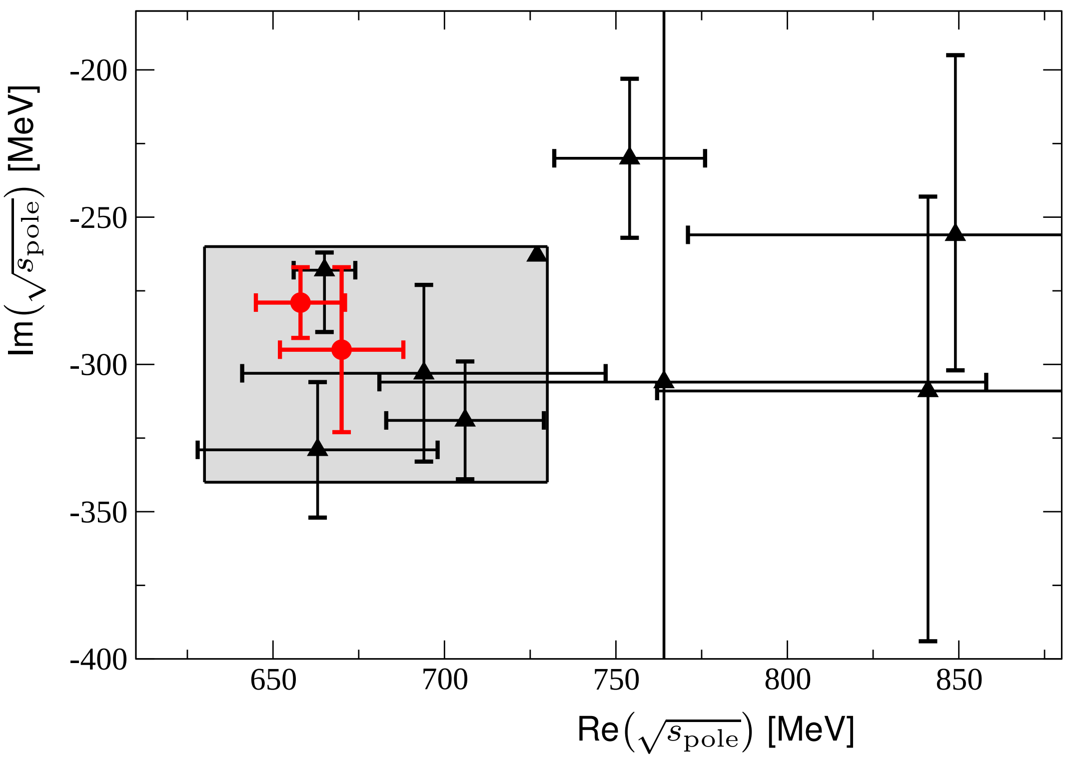

Figure 69.1: Location of the $K_0^*(700)$ (or κ) poles in the complex energy plane. Circles denote the results of the most refined analyses based on dispersion relations, while all other analyses quoted in the listings are denoted by triangles. The corresponding references are given in the listing.

Some authors find a $K_0^*(700)$ pole in their phenomenological analysis (see, *e.g.*, [22–33]), while others do not need to include it in their fits (see, *e.g.*, [17,34–37]). Similarly to the case of the $f_0(500)$ discussed below, all works including constraints from chiral symmetry at low energies naturally seem to find a light $K_0^*(700)$ below 800 MeV, see, *e.g.*, [38–42]. In these works the $K_0^*(700)$, $f_0(500)$, $f_0(980)$ and $a_0(980)$ appear to form a nonet [39, 40]. Additional evidence for this assignment is presented in Ref. [12], where the couplings of the nine states to $\bar{q}q$ sources were compared. The same low–lying scalar nonet was also found earlier in the unitarized quark model of Ref. [41]. The analysis of Ref. [43] is based on the Roy-Steiner equations, which include analyticity and crossing symmetry. Ref. [44] uses the Padé method to extract pole parameters after refitting scattering data constrained to satisfy forward dispersion relations. Both arrive at compatible pole positions for the $K_0^*(700)$ that are consistent with the pole parameters deduced either from other theoretical methods or Breit-Wigner fits. This is illustrated in Fig. 69.1. The compilation in this figure is used as justification for the range of pole parameters of the $K_0^*(700)$ we quote as "our estimate", namely

$$\sqrt{s^{\kappa}_{\rm Pole}} = (630 - 730) - i(260 - 340) \text{ MeV} . \tag{69.2}$$

69.3 The $I = 1$ States

Two isovector scalar states are known below 2 GeV, the $\boldsymbol{a_0(980)}$ and the $\boldsymbol{a_0(1450)}$. Independent of any model, the $K\bar{K}$ component in the $a_0(980)$ wave function must be large: it lies just below the opening of the $K\bar{K}$ channel to which it strongly couples [15, 45]. This generates an important cusp-like behavior in the resonant amplitude. Hence, its mass and width parameters are strongly distorted. To reveal its true coupling constants, a coupled–channel model with energy-dependent widths and mass shift contributions is necessary. All listed $a_0(980)$ measurements agree on a mass position value near 980 MeV, but the width takes values between 50 and 100 MeV, mostly due to the different models. For example, the analysis of the $p\bar{p}$-annihilation data [15] using a unitary K-matrix description finds a width as determined from the T-matrix pole of 92 ± 8 MeV, while the observed width of the peak in the $\pi\eta$ mass spectrum is about 45 MeV.

The relative coupling $K\bar{K}/\pi\eta$ is determined indirectly from $f_1(1285)$ [46–48] or $\eta(1410)$ decays [49–51], from the line shape observed in the $\pi\eta$ decay mode [52–55], or from the coupled-channel analysis of the $\pi\pi\eta$ and $K\bar{K}\pi$ final states of $p\bar{p}$ annihilation at rest [15].

The $a_0(1450)$ is seen in $p\bar{p}$ annihilation experiments with stopped and higher momenta antiprotons, with a mass of about 1450 MeV or close to the $a_2(1320)$ meson which is typically a dominant feature. A contribution from $a_0(1450)$ is also found in the analysis of the $D^{\pm} \to K^+K^-\pi^{\pm}$ [56] and $D^0 \to K^0_S K^{\pm}\pi^{\mp}$ [57] decay.

69.4 The $I = 0$ States

The $I = 0$, $J^{PC} = 0^{++}$ sector is the most complex one, both experimentally and theoretically. The data have been obtained from the $\pi\pi$, $K\bar{K}$, $\eta\eta$, 4π, and $\eta\eta'(958)$ systems produced in S-wave. Analyses based on several different production processes conclude that probably four poles are needed in the mass range from $\pi\pi$ threshold to about 1600 MeV. The claimed isoscalar resonances are found under separate entries $f_0(500)$ (or σ), $f_0(980)$, $f_0(1370)$, and $f_0(1500)$.

For discussions of the $\pi\pi$ S wave below the $K\bar{K}$ threshold and on the long history of the $f_0(500)$, which was suggested in linear sigma models more than 50 years ago, see our reviews in previous editions and the review [5].

Information on the $\pi\pi$ S-wave phase shift $\delta^I_J = \delta^0_0$ was already extracted many years ago from πN scattering [58–60], and near threshold from the K_{e4}-decay [61]. The kaon decays were later revisited leading to consistent data, however, with very much improved statistics [62, 63]. The reported $\pi\pi \to K\bar{K}$ cross sections [64–67] have large uncertainties. The πN data have been analyzed in combination with high-statistics data (see entries labeled as RVUE for re-analyses of the data). The $2\pi^0$ invariant mass spectra of the $p\bar{p}$ annihilation at rest [68–70] and the central collision [71] do not show a distinct resonance structure below 900 MeV, but these data are consistently described with the standard solution for πN data [59, 72], which allows for the existence of the broad $f_0(500)$. An enhancement is observed in the $\pi^+\pi^-$ invariant mass near threshold in the decays $D^+ \to \pi^+\pi^-\pi^+$ [73–75] and $J/\psi \to \omega\pi^+\pi^-$ [76, 77], and in $\psi(2S) \to J/\psi\pi^+\pi^-$ with very limited phase space [78, 79].

The precise $\boldsymbol{f_0(500)}$ (or $\boldsymbol{\sigma}$) pole is difficult to establish because of its large width, and because it can certainly not be modeled by a naive Breit-Wigner resonance. The $\pi\pi$ scattering amplitude shows an unusual energy dependence due to the presence of a zero in the unphysical regime close to the threshold [6, 7], required by chiral symmetry, and possibly due to crossed channel exchanges, the $f_0(1370)$, and other dynamical features. However, most of the analyses listed under $f_0(500)$ agree on a pole position near $(500 - i\,250$ MeV). In particular, analyses of $\pi\pi$ data that include unitarity, $\pi\pi$ threshold behavior, strongly constrained by the K_{e4} data, and the chiral symmetry constraints from Adler zeroes and/or scattering lengths find a light $f_0(500)$, see, *e.g.*, [80, 81].

Precise pole positions with an uncertainty of less than 20 MeV (see our table for the T-matrix pole) were extracted by use of Roy equations, which are twice subtracted dispersion relations derived from crossing symmetry and analyticity. In Ref. [10] the subtraction constants were fixed to the S-wave scattering lengths a_0^0 and a_0^2 derived from matching Roy equations and two-loop chiral perturbation theory [9]. The only additional relevant input to fix the $f_0(500)$ pole turned out to be the $\pi\pi$-wave phase shifts at 800 MeV. The analysis was improved further in Ref. [12]. Alternatively, in Ref. [11] only data were used as input inside Roy equations. In that reference also once-subtracted Roy–like equations, called GKPY equations, were used, since the extrapolation into the complex plane based on the twice subtracted equations leads to larger uncertainties mainly due to the limited experimental information on the isospin–2 $\pi\pi$ scattering length. Ref. [82] uses Padé approximants for the analytic continuation. All these extractions find consistent results. Using analyticity and unitarity only to describe data from $K_{2\pi}$ and K_{e4} decays, Ref. [83] finds consistent values for the pole position and the scattering length a_0^0. The importance of the $\pi\pi$ scattering data for fixing the $f_0(500)$ pole is nicely illustrated by comparing analyses of $\bar{p}p \to 3\pi^0$ omitting [68, 84] or including [69, 85] information on $\pi\pi$ scattering: while the former analyses find an extremely broad structure above 1 GeV, the latter find $f_0(500)$ masses of the order of 400 MeV.

As a result of the sensitivity of the extracted $f_0(500)$ pole position on the high accuracy low energy $\pi\pi$ scattering data [62, 63], the currently quoted range of pole positions for the $f_0(500)$, namely

$$\sqrt{s^{\sigma}_{\rm Pole}} = (400 - 550) - i(200 - 350)\ \text{MeV}\ , \tag{69.3}$$

in the listing was fixed including only those analyses consistent with these data, Refs. [26] [29] [39] [41] [42] [54] [69] [78–81, 83] [75, 86–99] as well as the advanced dispersion analyses [9–12, 82]. The pole positions from those references are compared to the range of pole positions quoted above in Fig. 69.2. Note that this range is labeled as 'our estimate' — it is not an average over the quoted analyses but is chosen to include the bulk of the analyses consistent with the mentioned criteria. An averaging procedure is not justified, since the analyses use overlapping or identical data sets.

If one uses just the most advanced dispersive analyses of Refs. [9–12] shown as solid dots in Fig. 69.2 to determine the pole location of the $f_0(500)$ the range narrows down to [5]

$$\sqrt{s^{\sigma}_{\rm Pole}} = (449^{+22}_{-16}) - i(275 \pm 12)\ \text{MeV}\ , \tag{69.4}$$

which is labeled as 'conservative dispersive estimate' in this reference.

Due to the large strong width of the $f_0(500)$ an extraction of its two–photon width directly from data is not possible. Thus, the values for $\Gamma(\gamma\gamma)$ quoted in the literature as well as the listing are based on the expression in the narrow width approximation [100] $\Gamma(\gamma\gamma) \simeq \alpha^2|g_\gamma|^2/(4\mathrm{Re}(\sqrt{s^{\sigma}_{\rm Pole}}))$ where g_γ is derived from the residue at the $f_0(500)$ pole to two photons and α denotes the electromagnetic fine structure constant. The explicit form of the expression may vary between different authors due to different definitions of the coupling constant, however, the expression given for $\Gamma(\gamma\gamma)$ is free of ambiguities. According to Refs. [101, 102], the data for $f_0(500) \to \gamma\gamma$ are consistent with what is expected for a two–step process of $\gamma\gamma \to \pi^+\pi^-$ via pion exchange in the t- and u-channel, followed by a final state interaction $\pi^+\pi^- \to \pi^0\pi^0$. The same conclusion is drawn in Ref. [103]

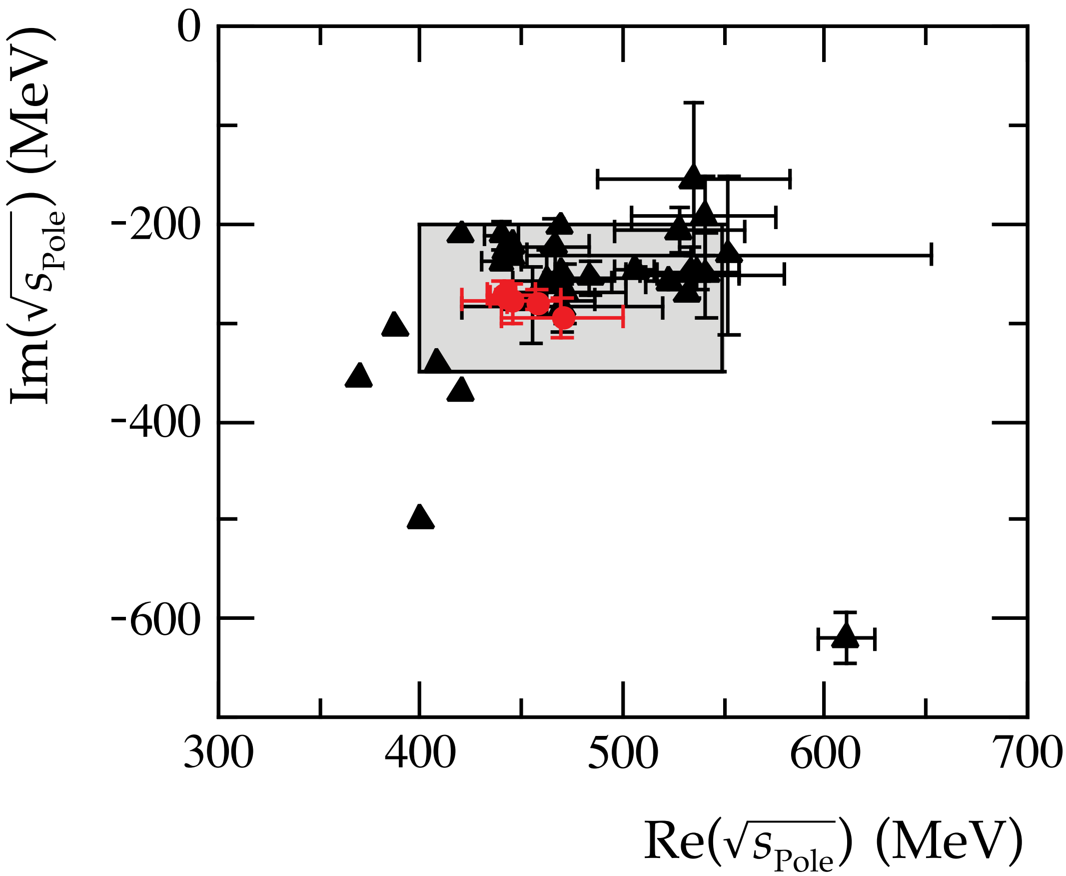

Figure 69.2: Location of the $f_0(500)$ (or σ) poles in the complex energy plane. Circles denote the recent analyses based on Roy(-like) dispersion relations, while all other analyses are denoted by triangles. The corresponding references are given in the listing.

where the bulk part of the $f_0(500) \to \gamma\gamma$ decay width is dominated by re–scattering. Therefore, it might be difficult to learn anything new about the nature of the $f_0(500)$ from its $\gamma\gamma$ coupling. For the most recent work on $\gamma\gamma \to \pi\pi$, see [83–106]. There are theoretical indications (*e.g.*, [107–137]) that the $f_0(500)$ pole behaves differently from a $q\bar{q}$-state – see next section and the mini-review on *non* $q\bar{q}$*-states* in this RPP for details.

The $\boldsymbol{f_0(980)}$ overlaps strongly with the background represented mainly by the $f_0(500)$ and the $f_0(1370)$. This can lead to a dip in the $\pi\pi$ spectrum at the $K\bar{K}$ threshold. It changes from a dip into a peak structure in the $\pi^0\pi^0$ invariant mass spectrum of the reaction $\pi^- p \to \pi^0\pi^0 n$ [111], with increasing four-momentum transfer to the $\pi^0\pi^0$ system, which means increasing the a_1-exchange contribution in the amplitude, while the π-exchange decreases. The $f_0(500)$ and the $f_0(980)$ are also observed in data for radiative decays ($\phi \to f_0\gamma$) from SND [112, 113], CMD2 [114], and KLOE [115, 116]. A dispersive analysis was used to simultaneously pin down the pole parameters of both the $f_0(500)$ and the $f_0(980)$ [11]; the uncertainty in the pole position quoted for the latter state is of the order of 10 MeV, only. We now quote for the mass

$$M_{f_0(980)} = 990 \pm 20 \text{ MeV} . \tag{69.5}$$

which is a range not an average, but is labeled as 'our estimate'.

Analyses of $\gamma\gamma \to \pi\pi$ data [117–119] underline the importance of the $K\bar{K}$ coupling of $f_0(980)$, while the resulting two-photon width of the $f_0(980)$ cannot be determined precisely [120]. The prominent appearance of the $f_0(980)$ in the semileptonic D_s decays and decays of B and B_s-mesons implies a dominant $(\bar{s}s)$ component: those decays occur via weak transitions that alternatively result in $\phi(1020)$ production. Ratios of decay rates of B and/or B_s mesons into J/ψ plus $f_0(980)$ or $f_0(500)$ were proposed to allow for an extraction of the flavor mixing angle and to probe the tetraquark nature of those mesons within a certain model [121] [122]. The phenomenological fits of the LHCb collaboration using the isobar model do neither allow for a contribution of the $f_0(980)$ in the $B \to J/\psi\pi\pi$ [123] nor for an $f_0(500)$ in $B_s \to J/\psi\pi\pi$ decays [124]. From the former analysis the authors conclude that their data is incompatible with a model where $f_0(500)$ and $f_0(980)$ are formed from two quarks and two antiquarks (tetraquarks) at the eight standard deviation level. In addition, they extract an upper limit for the mixing angle of 17^o at 90% C.L. between the $f_0(980)$ and the $f_0(500)$ that would correspond to a substantial $(\bar{s}s)$ content in $f_0(980)$ [123]. However, in a dispersive analysis of the same data that allows for a model–independent inclusion of the hadronic final state interactions in Ref. [125] a substantial $f_0(980)$ contribution is also found in the B–decays putting into question the conclusions of Ref. [123].

Let us now deal with the f_0's above 1 GeV. A meson resonance that is very well studied experimentally, is the $\boldsymbol{f_0(1500)}$ seen by the Crystal Barrel experiment in five decay modes: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'(958)$, and 4π [15, 69, 70]. Due to its interference with the $f_0(1370)$ (and $\boldsymbol{f_0(1710)}$), the peak attributed to the $f_0(1500)$ can appear shifted in invariant mass spectra. Therefore, the application of simple Breit-Wigner forms arrives at slightly different resonance masses for $f_0(1500)$. Analyses of central-production data of the likewise five decay modes Refs. [126, 127] agree on the description of the S-wave with the one above. The $p\bar{p}$, $p\bar{n}/n\bar{p}$ measurements [70, 128–130] show a single enhancement at 1400 MeV in the invariant 4π mass spectra, which is resolved into $f_0(1370)$ and $f_0(1500)$ [131] [132]. The data on 4π from central production [133] require both resonances, too, but disagree on the relative content of $\rho\rho$ and $f_0(500)f_0(500)$ in 4π. All investigations agree that the 4π decay mode represents about half of the $f_0(1500)$ decay width and is dominant for $f_0(1370)$.

The determination of the $\pi\pi$ coupling of $\boldsymbol{f_0(1370)}$ is aggravated by the strong overlap with the broad $f_0(500)$ and $f_0(1500)$. Since it does not show up prominently in the 2π spectra, its mass and width are difficult to determine. Multichannel analyses of hadronically produced two- and three-body final states agree on a mass between 1300 MeV and 1400 MeV and a narrow $f_0(1500)$, but arrive at a somewhat smaller width for $f_0(1370)$.

The existence of the $f_0(1370)$ is questioned in the analysis of the $\pi^-p \to \pi^-\pi^-\pi^+p$ data from COMPASS [138]. However, $D^0 \to \pi^+\pi^-\pi^+\pi^-$ data from CLEO-c require a contribution from $f_0(500)f_0(1370) \to 4\pi$ [139].

69.5 Interpretation of the scalars below 1 GeV

In the literature, many suggestions are discussed, such as conventional $q\bar{q}$ mesons, compact $(qq)(\bar{q}\bar{q})$ structures (tetraquarks) or meson-meson bound states. In addition, one expects a scalar glueball in this mass range. In reality, there can be superpositions of these components, and one often depends on models to determine the dominant one. Although we have seen progress in recent years, this question remains open. Here, we mention some of the present conclusions.

The $f_0(980)$ and $a_0(980)$ are often interpreted as compact tetraquark states states [134–137, 140] or $K\bar{K}$ bound states [141]. The insight into their internal structure using two-photon widths [113] [142–148] is not conclusive. The $f_0(980)$ appears as a peak structure in $J/\psi \to \phi\pi^+\pi^-$ and in D_s decays without $f_0(500)$ background, while being nearly invisible in $J/\psi \to \omega\pi^+\pi^-$. Based on that observation it is suggested that $f_0(980)$ has a large $s\bar{s}$ component, which according to Ref. [149] is surrounded by a virtual $K\bar{K}$ cloud (see also Ref. [150]). Data on radiative decays ($\phi \to f_0\gamma$ and $\phi \to a_0\gamma$) from SND, CMD2, and KLOE (see above) are consistent with a prominent role of kaon loops. This observation is interpreted as evidence for a compact four-quark [151] or a molecular [152, 153] nature of these states. Details of this controversy are given in the comments [154, 155]; see also Ref. [156]. It remains quite possible that the states $f_0(980)$ and $a_0(980)$, together with the $f_0(500)$ and the $K_0^*(700)$, form a new low-mass state nonet of predominantly four-quark states, where at larger distances the quarks recombine into a pair of pseudoscalar mesons creating a meson cloud (see, *e.g.,* Ref. [157]). Different QCD sum rule studies [158–162] do not agree on a tetraquark configuration for the same particle group.

Models that start directly from chiral Lagrangians, either in non-linear [25, 42, 80, 152] or in linear [163–169] realization, predict the existence of the $f_0(500)$ meson near 500 MeV. Here the $f_0(500)$, $a_0(980)$, $f_0(980)$, and $K_0^*(700)$ (in some models the $K_0^*(1430)$) would form a nonet (not necessarily $q\bar{q}$). In the linear sigma models the lightest pseudoscalars appear as their chiral partners. In these models the light $f_0(500)$ is often referred to as the "Higgs boson of strong interactions", since here the $f_0(500)$ plays a role similar to the Higgs particle in electro-weak symmetry breaking: within the linear sigma models it is important for the mechanism of chiral symmetry breaking, which generates most of the proton mass, and what is referred to as the constituent quark mass.

In the non–linear approaches of Refs. [25] and [80] the above resonances together with the low lying vector states are generated starting from chiral perturbation theory predictions near the first open channel, and then by extending the predictions to the resonance regions using unitarity and analyticity.

Ref. [163] uses a framework with explicit resonances that are unitarized and coupled to the light pseudoscalars in a chirally invariant way. Evidence for a non-$\bar{q}q$ nature of the lightest scalar resonances is derived from their mixing scheme. In Ref. [164] the scheme is extended and applied to the decay $\eta' \to \eta\pi\pi$, which lead to the same conclusions. To identify the nature of the resonances generated from scattering equations, in Ref. [170] the large N_c behavior of the poles was studied, with the conclusion that, while the light vector states behave consistent with what is predicted for $\bar{q}q$ states, the light scalars behave very differently. This finding provides strong support for a non-$\bar{q}q$ nature of the light scalar resonances. Note, the more refined study of Ref. [107] found, in case of the $f_0(500)$, in addition to a dominant non-$\bar{q}q$ nature, indications for a subdominant $\bar{q}q$ component located around 1 GeV. Additional support for the non-$q\bar{q}$ nature of the $f_0(500)$ is given in Ref. [171], where the connection between the pole of resonances and their Regge trajectories is analyzed.

A model–independent method to identify hadronic molecules goes back to a proposal by Weinberg [172], shown to be equivalent to the pole counting arguments of [173–182] in Ref. [176]. The formalism allows one to extract the amount of molecular component in the wave function from the effective coupling constant of a physical state to a nearby continuum channel. It can be applied to near threshold states only and provided strong evidence that the $f_0(980)$ is a $\bar{K}K$ molecule, while the situation turned out to be less clear for the $a_0(980)$ (see also Refs. [146, 148]) Further insights into $a_0(980)$ and $f_0(980)$ are expected from their mixing [177]. The corresponding signal predicted in Refs. [178, 179] was recently observed at BES III [180]. It turned out that in order to get a quantitative understanding of those data in addition to the mixing mechanism itself, some detailed understanding of the production mechanism seems necessary [181].

In the unitarized quark model with coupled $q\bar{q}$ and meson-meson channels, the light scalars can be understood as additional manifestations of bare $q\bar{q}$ confinement states, strongly mass shifted from the 1.3 - 1.5 GeV region and very distorted due to the strong $^3P_0$ coupling to S-wave two-meson decay channels [182, 183]. Thus, in these models the light scalar nonet comprising the $f_0(500)$, $f_0(980)$, $K_0^*(700)$, and $a_0(980)$, as well as the nonet consisting of the $f_0(1370)$, $f_0(1500)$ (or $f_0(1710)$), $K_0^*(1430)$, and $a_0(1450)$, respectively, are two manifestations of the same bare input states (see also Ref. [184]).

Other models with different groupings of the observed resonances exist and may, *e.g.*, be found in earlier versions of this review.

69.6 Interpretation of the f_0's above 1 GeV

The $f_0(1370)$ and $f_0(1500)$ decay mostly into pions (2π and 4π) while the $f_0(1710)$ decays mainly into the $K\bar{K}$ final states. The $K\bar{K}$ decay branching ratio of the $f_0(1500)$ is small [126] [185].

If one uses the naive quark model, it is natural to assume that the $f_0(1370)$, $a_0(1450)$, and the $K_0^*(1430)$ are in the same SU(3) flavor nonet, being the $(u\bar{u}+d\bar{d})$, $u\bar{d}$ and $u\bar{s}$ states, probably mixing with the light scalars [186], while the $f_0(1710)$ is the $s\bar{s}$ state. Indeed, the production of $f_0(1710)$ (and $f_2'(1525)$) is observed in $p\bar{p}$ annihilation [187] but the rate is suppressed compared to $f_0(1500)$ (respectively, $f_2(1270)$), as would be expected from the OZI rule for $s\bar{s}$ states. The $f_0(1500)$ would also qualify as a $(u\bar{u}+d\bar{d})$ state, although it is very narrow compared to the other states and too light to be the first radial excitation.

However, in $\gamma\gamma$ collisions leading to $K_S^0K_S^0$ [188] a spin–0 signal is observed at the $f_0(1710)$ mass (together with a dominant spin–2 component), while the $f_0(1500)$ is not observed in $\gamma\gamma\to K\bar{K}$ nor $\pi^+\pi^-$ [189]. In $\gamma\gamma$ collisions leading to $\pi^0\pi^0$ Ref. [190] reports the observation of a scalar around 1470 MeV albeit with large uncertainties on the mass and $\gamma\gamma$ couplings. This state could be the $f_0(1370)$ or the $f_0(1500)$. The upper limit from $\pi^+\pi^-$ [189] excludes a large $n\bar{n}$ (here n stands for the two lightest quarks) content for the $f_0(1500)$ and hence points to a mainly $s\bar{s}$ state [191]. This appears to contradict the small $K\bar{K}$ decay branching ratio of the $f_0(1500)$ and makes a $q\bar{q}$ assignment difficult for this state. Hence the $f_0(1500)$ could be mainly glue due the absence of a 2γ-coupling, while the $f_0(1710)$ coupling to 2γ would be compatible with an $s\bar{s}$ state. This is in accord with the recent high–statistics Belle data in $\gamma\gamma\to K_S^0K_S^0$ [192] in which the $f_0(1500)$ is absent, while a prominent peak at 1710 MeV is observed with quantum numbers 0^{++}, compatible with the formation of an $s\bar{s}$ state. However, the 2γ-couplings are sensitive to glue mixing with $q\bar{q}$ [193].

Note that an isovector scalar, possibly the $a_0(1450)$ (albeit at a lower mass of 1317 MeV) is observed in $\gamma\gamma$ collisions leading to $\eta\pi^0$ [194]. The state interferes destructively with the non-resonant background, but its $\gamma\gamma$ coupling is comparable to that of the $a_2(1320)$, in accord with simple predictions (see, *e.g.*, Ref. [191]).

The small width of $f_0(1500)$, and its enhanced production at low transverse momentum transfer in central collisions [195–197] also favor $f_0(1500)$ to be non-$q\bar{q}$. In the mixing scheme of Ref. [193], which uses central production data from WA102 and the recent hadronic J/ψ decay data from BES [198, 199], glue is shared between $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The $f_0(1370)$ is mainly $n\bar{n}$, the $f_0(1500)$ mainly glue and the $f_0(1710)$ dominantly $s\bar{s}$. This agrees with previous analyses [200, 201].

However, alternative schemes have been proposed (*e.g.*, in [202–208], for detailed reviews see, *e.g.*, Ref. [1] and the mini-review on *non–$\bar{q}q$ states* in this Review of Particle Physics (RPP)). In Ref. [208], a large K^+K^- scalar signal reported by Belle in B decays into $KK\bar{K}$ [209], compatible with the $f_0(1500)$, is explained as due to constructive interference with a broad glueball background. However, the Belle data are inconsistent with the BaBar measurements which show instead a broad scalar at this mass for B decays into both $K^{\pm}K^{\pm}K^{\mp}$ [210] and $K^+K^-\pi^0$ [211].

The $f_0(1500)$ has also been proposed as a tetraquarks state [212]. Whether the $f_0(1500)$ is observed in 'gluon rich' radiative J/ψ decays is debatable [213] because of the limited amount of data - more data for this and the $\gamma\gamma$ mode are needed. In Ref. [214], further refined in Ref. [215], $f_0(1370)$ and $f_0(1710)$ (together with $f_2(1270)$ and $f_2'(1525)$) were interpreted as bound systems of two vector mesons. This picture could be tested in radiative J/ψ decays [216] as well as radiative decays of the states themselves [217]. The vector-vector component of the $f_0(1710)$ might also be the origin of the enhancement seen in $J/\psi \to \gamma\phi\omega$ near threshold [218] observed at BES [219]. Note that the results of Refs. [214] [215] were challenged in Ref. [220] where in a covariant formalism, e.g., the $f_2(1270)$ did not not emerge as a $\rho\rho$-bound state.

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