

## 2. Astrophysical Constants and Parameters

**Table 2.1:** Revised August 2019 by D.E. Groom (LBNL) and D. Scott (University of British Columbia). The figures in parentheses after some values give the  $1\text{-}\sigma$  uncertainties in the last digit(s). Physical constants are from Ref. [1]. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference. The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the derived parameters reported in this table have non-Gaussian likelihoods. Parameters may be highly correlated, so care must be taken in propagating errors. Unless otherwise specified, cosmological parameters are derived from a 6-parameter  $\Lambda$ CDM cosmology fit to *Planck* cosmic microwave background 2018 temperature (TT) + polarization (TE,EE+lowE) + lensing data [2]. For more information see Ref. [3] and the original papers.

Quantity	Symbol, equation.	Value	Reference, footnote
Newtonian constant of gravitation	$G_N$	$6.674\,30(15) \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$	[1]
Planck mass	$M_P = \sqrt{\hbar c/G_N}$	$1.220\,890(14) \times 10^{19} \text{GeV}/c^2 = 2.176\,434(24) \times 10^{-8} \text{kg}$	[1]
Planck length	$l_P = \sqrt{\hbar G_N/c^3}$	$1.616\,255(18) \times 10^{-35} \text{m}$	[1]
tropical year (equinox to equinox, 2020)	yr	$31\,556\,925.1 \text{ s} = 365.242\,189 \text{ days}$	[4]
sidereal year (period of Earth around Sun relative to stars)		$31\,558\,149.8 \text{ s} \approx \pi \times 10^7 \text{ s}$	[4]
mean sidereal day (Earth rotation period relative to stars)		$23^{\text{h}}\,56^{\text{m}}\,04^{\text{s}}.090\,53$	[4]
astronomical unit	au	$149\,597\,870\,700 \text{ m}$	exact [5]
parsec (1 au/1 arc sec)	pc	$3.085\,677\,581\,49 \times 10^{16} \text{ m} = 3.261\,56 \dots \text{ly}$	exact [6]
light year (deprecated unit)	ly	$0.306\,601 \dots \text{pc} = 0.946\,073 \dots \times 10^{16} \text{ m}$	[7]
solid angle	$\text{deg}^2$	$(\pi/180)^2 \text{sr} = 3.046\,17 \dots \times 10^{-4} \text{sr}$	[8]
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	$2.953\,250\,076\,100\,25 \text{ km}$	[9]
Solar mass	$M_\odot$	$1.988\,41(4) \times 10^{30} \text{ kg}$	[10]
nominal Solar equatorial radius	$\mathcal{R}_\odot$	$6.957 \times 10^8 \text{ m}$	exact [11]
nominal Solar constant	$\mathcal{S}_\odot$	$1361 \text{ W m}^{-2}$	exact [11, 12]
nominal Solar photosphere temperature	$\mathcal{T}_\odot$	$5772 \text{ K}$	exact [11]
nominal Solar luminosity	$\mathcal{L}_\odot$	$3.828 \times 10^{26} \text{ W}$	exact [11, 13]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	$8.870\,055\,940 \text{ mm}$	[9]
Earth mass	$M_\oplus$	$5.972\,17(13) \times 10^{24} \text{ kg}$	[10]
nominal Earth equatorial radius	$\mathcal{R}_\oplus$	$6.3781 \times 10^6 \text{ m}$	exact [11]
Chandrasekhar mass	$M_{\text{Ch}}$	$3.097\,972 \mu^{-2} M_P^3/m_H^2 = 1.433\,77(6) (\mu/2)^{-2} M_\odot$	[14, 15]
Eddington luminosity	$L_{\text{Ed}}$	$1.257\,065\,179\,8(12) \times 10^{31} (M/M_\odot) \text{ W}$ $= 3.283\,869\,330\,8(31) \times 10^4 (M/M_\odot) \mathcal{L}_\odot$	[16, 17]
jansky (flux density)	Jy	$10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$	definition
luminosity conversion	$f_0$	$3.0128 \times 10^{28} \times 10^{-0.4 M_{\text{Bol}}} \text{ W}$ ( $M_{\text{Bol}}$ = absolute bolometric magnitude = bolometric magnitude at 10 pc)	exact [18]
flux conversion	$\mathcal{F}$	$2.518\,021\,002 \times 10^{-8} \times 10^{-0.4 m_{\text{Bol}}} \text{ W m}^{-2}$ ( $m_{\text{Bol}}$ = apparent bolometric magnitude)	exact [18]
ABsolute monochromatic magnitude	AB	$-2.5 \log_{10} f_\nu - 56.10$ (for $f_\nu$ in $\text{W m}^{-2} \text{ Hz}^{-1}$ ) $= -2.5 \log_{10} f_\nu + 8.90$ (for $f_\nu$ in Jy)	[19]
Solar angular velocity around Galactic center	$\Theta_0/R_0$	$27.1(5) \text{ km s}^{-1} \text{ kpc}^{-1}$	[20]
Solar distance from Galactic center	$R_0$	$8.178 \pm 0.013(\text{stat.}) \pm 0.022(\text{sys.}) \text{ kpc}$	[21, 22]
circular velocity at $R_0$	$v_0$ or $\Theta_0$	$240(8) \text{ km s}^{-1}$	[22, 23]
escape velocity from the Galaxy	$v_{\text{esc}}$	$492 \text{ km s}^{-1} < v_{\text{esc}} < 587 \text{ km s}^{-1}$ (90%)	[24]
local disk density	$\rho_{\text{disk}}$	$6.6(9) \times 10^{-24} \text{ g cm}^{-3} = 3.7(5) \text{ GeV}/c^2 \text{ cm}^{-3}$	[25]
local dark matter density	$\rho_\chi$	canonical value $0.3 \text{ GeV}/c^2 \text{ cm}^{-3}$ within factor 2–3	[26]
present-day CMB temperature	$T_0$	$2.7255(6) \text{ K}$	[27, 28]
present-day CMB dipole amplitude	$d$	$3.3621(10) \text{ mK}$	[27, 29]
Solar velocity with respect to CMB	$v_\odot$	$369.82(11) \text{ km s}^{-1}$ towards $(l, b) = (264.021(11)^\circ, 48.253(5)^\circ)$	[29]
Local Group velocity with respect to CMB	$v_{\text{LG}}$	$620(15) \text{ km s}^{-1}$ towards $(l, b) = (271.9(20)^\circ, 29.6(14)^\circ)$	[29]
number density of CMB photons	$n_\gamma$	$410.7(3) (T/2.7255)^3 \text{ cm}^{-3}$	[30]
density of CMB photons	$\rho_\gamma$	$4.645(4) (T/2.7255)^4 \times 10^{-34} \text{ g cm}^{-3} \approx 0.260 \text{ eV cm}^{-3}$	[30]
entropy density/Boltzmann constant	$s/k$	$2.891.2 (T/2.7255)^3 \text{ cm}^{-3}$	[30]
present-day Hubble expansion rate	$H_0$	$100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = h \times (9.777\,752 \text{ Gyr})^{-1}$	[31]
scaling factor for Hubble expansion rate	$h$	$0.674(5)$	[2, 32]
Hubble length	$c/H_0$	$0.925\,0629 \times 10^{26} h^{-1} \text{ m} = 1.372(10) \times 10^{26} \text{ m}$	
scaling for cosmological constant	$c^2/3H_0^2$	$2.85247 \times 10^{51} h^{-2} \text{ m}^2 = 6.21(9) \times 10^{51} \text{ m}^2$	
critical density of the Universe	$\rho_{\text{crit}} = 3H_0^2/8\pi G_N$	$1.878\,34(4) \times 10^{-29} h^2 \text{ g cm}^{-3}$ $= 1.053\,672(24) \times 10^{-5} h^2 (\text{GeV}/c^2) \text{ cm}^{-3}$ $= 2.775\,36627 \times 10^{11} h^2 M_\odot \text{ Mpc}^{-3}$	
baryon-to-photon ratio (from BBN)	$\eta = n_b/n_\gamma$	$5.8 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}$ (95% CL)	[33]
number density of baryons	$n_b$	$2.515(17) \times 10^{-7} \text{ cm}^{-3}$ $(2.4 \times 10^{-7} < n_b < 2.7 \times 10^{-7}) \text{ cm}^{-3}$ (95% CL, $\eta \times n_\gamma$ )	[2, 3, 34, 35]
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_{\text{crit}}$	$2.473 \times 10^{-5} (T/2.7255)^4 h^{-2} = 5.38(15) \times 10^{-5}$	[30]

Quantity	Symbol, equation.	Value	Reference, footnote
--- <i>Planck</i> 2018 6-parameter fit to flat $\Lambda$ CDM cosmology ---			
baryon density of the Universe	$\Omega_b = \rho_b/\rho_{\text{crit}}$	$^{\ddagger} 0.02237(15) h^{-2} = ^{\dagger} 0.0493(6)$	[2, 3, 27]
cold dark matter density of the Universe	$\Omega_c = \rho_c/\rho_{\text{crit}}$	$^{\ddagger} 0.1200(12) h^{-2} = ^{\dagger} 0.265(7)$	[2, 3, 27]
$100 \times$ approx to $r_*/D_\Lambda$	$100 \times \theta_{\text{MC}}$	$^{\ddagger} 1.04092(31)$	[2, 3, 27]
reionization optical depth	$\tau$	$^{\ddagger} 0.054(7)$	[2, 3, 27]
ln(power prim. curv. pert.) ( $k_0 = 0.05 \text{ Mpc}^{-1}$ )	$\ln(10^{10} \Delta_{\mathcal{R}}^2)$	$^{\ddagger} 3.044(14)$	[2, 3, 27]
scalar spectral index	$n_s$	$^{\ddagger} 0.965(4)$	[2, 3, 27]
pressureless matter parameter	$\Omega_m = \Omega_c + \Omega_b$	$^{\dagger} 0.315(7)$	[2, 3]
dark energy density parameter	$\Omega_\Lambda$	$^{\dagger} 0.685(7)$	[2, 3]
energy density of dark energy	$\rho_\Lambda$	$^{\dagger} 5.83(16) \times 10^{-30} \text{ g cm}^{-3}$	[2]
cosmological constant	$\Lambda$	$^{\dagger} 1.088(30) \times 10^{-56} \text{ cm}^{-2}$	[2]
fluctuation amplitude at $8 h^{-1} \text{ Mpc}$ scale	$\sigma_8$	$^{\dagger} 0.811(6)$	[2, 3]
redshift of matter-radiation equality	$z_{\text{eq}}$	$^{\dagger} 3402(26)$	[2, 36]
age at matter-radiation equality	$t_{\text{eq}}$	$^{\dagger} 51.1(8) \text{ kyr}$	[2, 37]
redshift at which optical depth equals unity	$z_*$	$^{\dagger} 1089.92(25)$	[2]
comoving size of sound horizon at $z_*$	$r_*$	$^{\dagger} 144.43(26) \text{ Mpc}$	[2, 38]
age when optical depth equals unity	$t_*$	$^{\dagger} 372.9(10) \text{ kyr}$	[2, 37]
redshift at half reionization	$z_i$	$^{\dagger} 7.7(7)$	[2, 39]
age at half reionization	$t_i$	$^{\dagger} 690(90) \text{ Myr}$	[2]
redshift when acceleration was zero	$z_q$	$^{\dagger} 0.636(18)$	[2, 37]
age when acceleration was zero	$t_q$	$^{\dagger} 7.70(10) \text{ Gyr}$	[2]
age of the Universe today	$t_0$	$^{\dagger} 13.797(23) \text{ Gyr}$	[2]
effective number of neutrinos	$N_{\text{eff}}$	$^{\#} 2.99(17)$	[2, 40, 41]
sum of neutrino masses	$\Sigma m_\nu$	$^{\#} < 0.12 \text{ eV}$ (95%, CMB + BAO); $\geq 0.06 \text{ eV}$ (mixing)	[2, 41–43]
neutrino density of the Universe	$\Omega_\nu = h^{-2} \Sigma m_{\nu_j} / 93.14 \text{ eV}$	$^{\#} < 0.003$ (95%, CMB + BAO); $\geq 0.0012$ (mixing)	[2, 42, 43]
curvature	$\Omega_K$	$^{\#} 0.0007(19)$	[2]
running spectral index, $k_0 = 0.05 \text{ Mpc}^{-1}$	$dn_s/d \ln k$	$^{\#} -0.004(7)$	[2]
tensor-to-scalar field perturbations ratio,	$r_{0.002} = T/S$	$^{\#} < 0.058$ (95% CL, $k_0 = 0.002 \text{ Mpc}^{-1}$ , no running)	[2, 44, 45]
dark energy equation of state parameter	$w$	$-1.028(31)$	[2, 46]
primordial helium fraction	$Y_p$	$0.245(4)$	[47]

$^{\ddagger}$  Parameter in 6-parameter  $\Lambda$ CDM fit;  $^{\dagger}$  Derived parameter in 6-parameter  $\Lambda$ CDM fit;  $^{\#}$  Extended model parameter, *Planck* + BAO data [2].

## References

- [1] CODATA recommended 2018 values of the fundamental physical constants: <https://physics.nist.gov/cuu/Constants/index.html>.
- [2] Planck Collab. 2018 Results VI (2018), [arXiv:1807.06209].
- [3] O. Lahav & A.R. Liddle, “The Cosmological Parameters,” Sec. 24 in this *Review*.
- [4] *The Astronomical Almanac for the year 2020*.
- [5] The astronomical unit of length (au) in meters is re-defined (IAU XXVIII General Assembly 2012, Resolution B2) to be a conventional unit of length in agreement with the value adopted in IAU XXVII 2009 Resolution B2. It is to be used with all time scales.
- [6] The distance at which 1 au subtends 1 arc sec: 1 au divided by  $\pi/648\,000$ .
- [7] IAU XVI GA 1976, Recommendations.
- [8] The number of square degrees on a sphere is  $360^2/\pi = 41\,259.9\dots$
- [9] Observationally determined mass parameter  $G_N M \times 2/c^2$  [1] for either the Sun or the Earth, where  $\mathcal{G}M_\odot = 1.327\,124\,4 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$  and  $\mathcal{G}M_\oplus = 3.986\,004 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$  [48].
- [10]  $G_N M \div G_N$  [1].
- [11] IAU XXIX GA, 2015, Resolution B3, “on recommended nominal conversion constants ...” Calligraphic symbol indicates recommended nominal value.
- [12] See also G. Kopp & J.L. Lean, *Geophys. Res. Lett.* **38**, L01706 (2011), who give  $(1360.8 \pm 0.6) \text{ W m}^{-2}$ ; see paper for caveats and other measurements.
- [13]  $4\pi (1 \text{ au})^2 \times \mathcal{S}_\odot$ , assuming isotropic irradiance.
- [14] S. Chandrasekhar, *Astrophys. J.* **74**, 81 (1931).

- [15] This value assumes an ideal Fermi gas, using a numerical constant from the Lane-Emden equation [49], and with  $\mu$  the average molecular weight per electron, defined relative to the mass of the single-proton hydrogen atom.
- [16] A. S. Eddington, *Mon. Not. R. Astron. Soc* **77**, 16 (1916).
- [17] The maximum luminosity assuming pure electron scattering for the outward force arising from radiation pressure:  $4\pi G_N M m_p c / \sigma_T$ .
- [18] IAU XXIX GA, 2015, Resolution B2, “on recommended zero points for the absolute and apparent bolometric magnitude scales”.
- [19] J. Oke and J. Gunn, *Astrophys. J.* **266**, 713 (1983).
- [20] J. Bovy, *Mon. Not. R. Astron. Soc* **468**, 1, L63 (2017).
- [21] R. Abuter *et al.* (2019), [arXiv:1904.05721].
- [22] IAU XIX GA (1985) suggested that “in cases where standardization on a common set of galactic parameters is desirable” that the values  $R_0 = (8.5 \pm 1.0)$  kpc and  $\theta_0 = (220 \pm 20)$  km s<sup>-1</sup> should be used.
- [23] M. Reid *et al.*, *Astrophys. J.* **783**, 2, 130 (2014).
- [24] T. Piffl *et al.*, *Astron. Astrophys.* **562**, A91 (2014), [arXiv:1309.4293].
- [25] C. F. McKee, A. Parravano and D. J. Hollenbach, *Astrophys. J.* **814**, 1, 13 (2015); This is representative of other published estimates.
- [26] J. Read, *J. Phys.* **G41**, 063101 (2014); A. M. Green, *J. Phys.* **G44**, 8, 084001 (2017); The conclusion is  $\rho_{\text{DM}}^{\text{local}} = 0.39 \pm 0.03$  GeV cm<sup>-3</sup>.
- [27] D. Scott & G.F. Smoot, “Cosmic Microwave Background,” Sec. 28 in this *Review*.
- [28] D. J. Fixsen, *Astrophys. J.* **707**, 916 (2009).
- [29] Planck Collab. 2018 Results I (2018), [arXiv:1807.06205].
- [30]  $n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3$ ;  $\rho_\gamma = \frac{\pi^2 kT}{15 c^2} \left(\frac{kT}{\hbar c}\right)^3$ ;  $s/k = \frac{2.43 \cdot \pi^2}{11.45} \left(\frac{kT}{\hbar c}\right)^3$ ;  $kT/\hbar c = 11.90 235(T/2.7255)/\text{cm}$ .
- [31] Conversion using length of sidereal year.
- [32] Distance-ladder estimates of  $H_0$  tend to give higher values than derived from the CMB, *e.g.* Riess *et al.*, *Astrophys. J.* **826**, 56 (2016) give  $h = 0.732 \pm 0.017$ ; for discussion see O. Lahav & A.R. Liddle, “The Cosmological Parameters,” Sec. 24 in this *Review*.
- [33] B.D. Fields, P. Molaro, & S. Sarkar, “Big-Bang Nucleosynthesis,” Sec. 23 in this *Review*.
- [34]  $n_b$  depends only upon the measured  $\Omega_b h^2$ , the average baryon mass at the present epoch [35], and  $G_N$ :  $n_b = (\Omega_b h^2)(h^{-2} \rho_{\text{crit}})/(0.93711 \text{ GeV}/c^2 \text{ per baryon})$ .
- [35] G. Steigman, *JCAP* **0610**, 016 (2006).
- [36] Here ‘radiation’ includes three species of light neutrinos as well as photons.
- [37] D. Scott, A. Narimani and D. N. Page, *Phys. Canada* **70**, 258 (2014).
- [38] D.H. Weinberg, M. White, “Dark Energy,” Sec. 27 in this *Review*.
- [39] Planck Collab. Interm. Results XLVI, *Astron. & Astrophys.* **596**, A108 (2016) extend the range by  $\Delta z \approx 1$ , depending on the reionization model.
- [40] Summary Tables in this *Review* list  $N_\nu = 2.984(8)$  (Standard Model fits to LEP-SLC data). Because neutrinos are not completely decoupled at  $e^\pm$  annihilation, the effective number of massless neutrino species is 3.045, rather than 3.
- [41] J. Lesgourgues & L. Verde, “Neutrinos in Cosmology,” Sec. 25 in this *Review*.

- [42] The sum is over all neutrino mass eigenstates, the lower limit following from neutrino mixing results reported in this *Review* combined with the assumptions that there are three light neutrinos and that the lightest neutrino is substantially less massive than the others.
- [43] Astrophysical determinations of  $\sum m_{\nu_j}$ , reported in the Full Listings of this *Review* under “Sum of the neutrino masses,” range from  $< 0.17$  eV to  $< 2.3$  eV in papers published since 2003.
- [44] P. A. R. Ade *et al.* (BICEP2, Keck Array), *Phys. Rev. Lett.* **121**, 221301 (2018).
- [45] *Planck* data alone give  $r < 0.10$ ; adding the BICEP/Keck data tightens the constraint.
- [46] This constraint uses BAO and SNe data, as described in Ref. [2]; see discussion in D.H. Weinberg, M. White, “Dark Energy,” Sec. 27 in this *Review*.
- [47] E. Aver, K. A. Olive and E. D. Skillman, *JCAP* **1507**, 07, 011 (2015).
- [48] IAU XXIX GA 2015, Resolution B2.
- [49] G. P. Horedt, *Astrophys. Space Sci.* **126**, 2, 357 (1986).