CP Violation in $K_S \to 3\pi$

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The possible final states for the decay $K^0 \to \pi^+\pi^-\pi^0$ have isospin $I = 0$, 1, 2, and 3. The $I = 0$ and $I = 2$ states have $CP = +1$ and K_S can decay into them without violating CP symmetry, but they are expected to be strongly suppressed by centrifugal barrier effects. The $I = 1$ and $I = 3$ states, which have no centrifugal barrier, have $CP = -1$ so that the K_S decay to these requires CP violation.

In order to see CP violation in $K_S \to \pi^+\pi^-\pi^0$, it is necessary to observe the interference between K_S and K_L decay, which determines the amplitude ratio

$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)} \ . \tag{1}$$

If η_{+-0} is obtained from an integration over the whole Dalitz plot, there is no contribution from the $I = 0$ and $I = 2$ final states and a nonzero value of η_{+-0} is entirely due to CP violation.

Only $I = 1$ and $I = 3$ states, which are $CP = -1$, are allowed for $K^0 \to \pi^0\pi^0\pi^0$ decays and the decay of K_S into $3\pi^0$ is an unambiguous sign of CP violation. Similarly to η_{+-0}, η_{000} is defined as

$$\eta_{000} = \frac{A(K_S \to \pi^0\pi^0\pi^0)}{A(K_L \to \pi^0\pi^0\pi^0)} \ . \tag{2}$$

If one assumes that CPT invariance holds and that there are no transitions to $I = 3$ (or to nonsymmetric $I = 1$ states), it can be shown that

$$\begin{aligned} \eta_{+-0} &= \eta_{000} \\ &= \epsilon + i\frac{\text{Im } a_1}{\text{Re } a_1} \ . \end{aligned} \tag{3}$$

With the Wu-Yang phase convention, a_1 is the weak decay amplitude for K^0 into $I = 1$ final states; ϵ is determined from CP violation in $K_L \to 2\pi$ decays. The real parts of η_{+-0} and η_{000} are equal to $\text{Re}(\epsilon)$. Since currently-known upper limits on $|\eta_{+-0}|$ and $|\eta_{000}|$ are much larger than $|\epsilon|$, they can be interpreted as upper limits on $\text{Im}(\eta_{+-0})$ and $\text{Im}(\eta_{000})$ and so as limits on the CP-violating phase of the decay amplitude a_1.