Rescaling Errors from Constrained Fits and Averages

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PDG Collaboration Meeting

September 22, 2006

Overview

- Rationale for rescaling errors.
- Rescaling errors of constrained fits.
- Rescaling errors of averages.

Rationale for rescaling errors.

- A large value of χ^2 is likely to be due to underestimation of errors by some of the experiments.
- The normal calculation of errors from a constrained fit or an average does not take this into account.
- Thus if the value of χ^2 is large we should increase the quoted errors.
- These increased errors may provide a better estimate of the actual uncertainty of the result.

Rescaling Errors from Constrained Fits - Overview

- Use pull quantities for rescaling errors from constrained fits.
- Rescaling measurements with no input correlations.
- Previous method for rescaling measurements with correlated input.
- New method for rescaling measurements with correlated input.

Use pull quantities to rescale errors from constrained fits.

- For a constrained fit we do not know exactly what the partial χ^2 for one variable should be. Thus we cannot use its value for rescaling the measurement errors of a variable.
- Instead we use pull quantities for rescaling. The pull quantity is defined for one variable as:

$$p_i = (x_i - x_{avg}) / \{ < [x_i - x_{avg}]^2 > \}^{1/2}$$

Use pull quantities to rescale errors from constrained fits (continued).

- We may also form the RMS pull quantity which for N measurements is:

$$p_{rms} = \{\Sigma [\ p_i^2\ ] / N\ \}^{1/2}$$

  and the sum is over the N measurements.

- By construction this quantity p_{rms} will have an average value of one for errors that are Gaussian distributed.

Rescaling measurements with no input correlations.

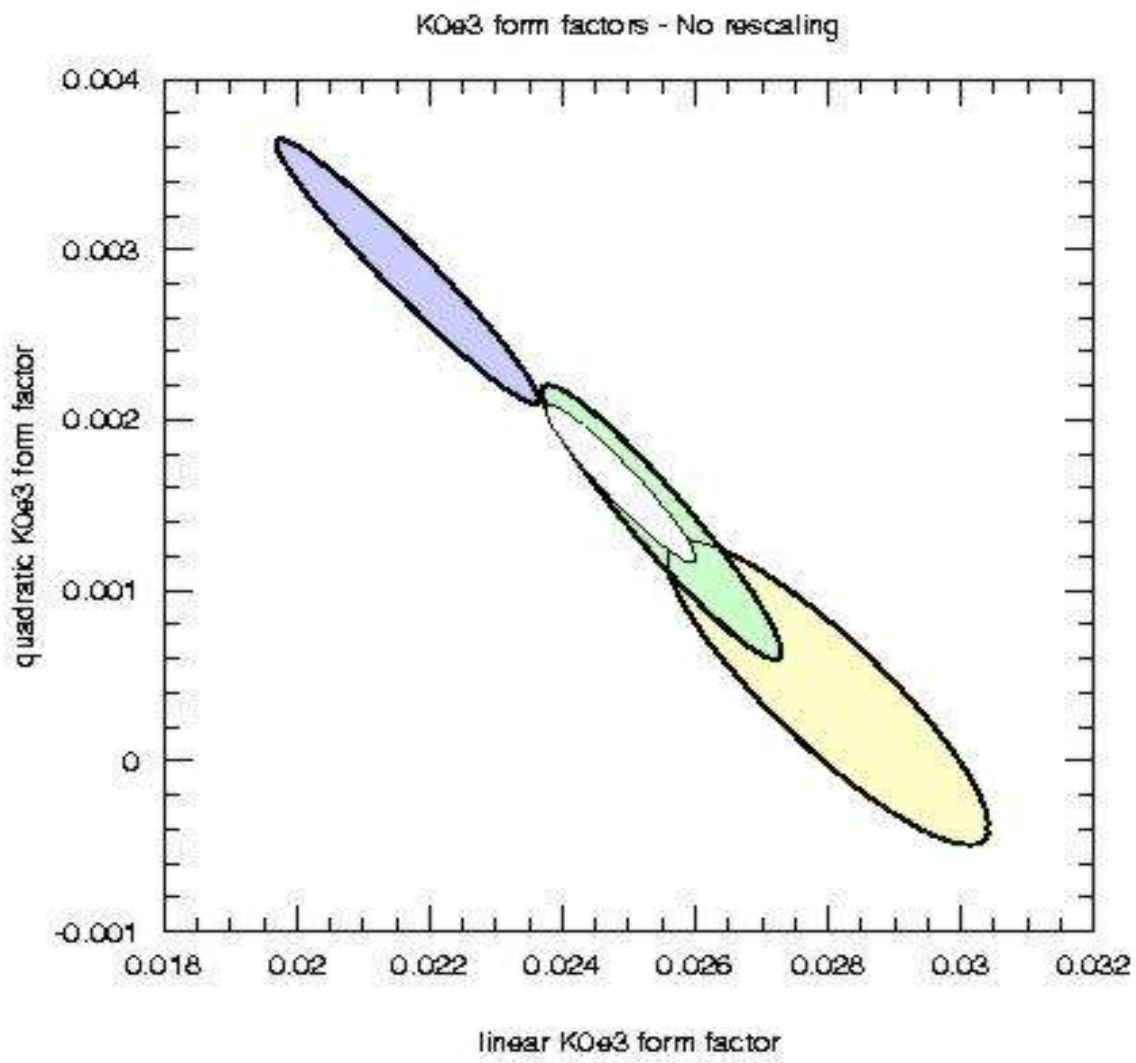
- With no input correlations we may use the pull quantity p_{rms} as defined on the previous page.
- We use the same scale factor for all uncorrelated measurements of a variable.
- If any variable has a value of p_{rms} that is greater then one, we rescale the measurement errors for that variable by the factor p_{rms} and refit to obtain new larger errors.
  - If $p_{rms} < 1$ do not change errors.
  - If $p_{rms} > 1$ increase errors by p_{rms}.

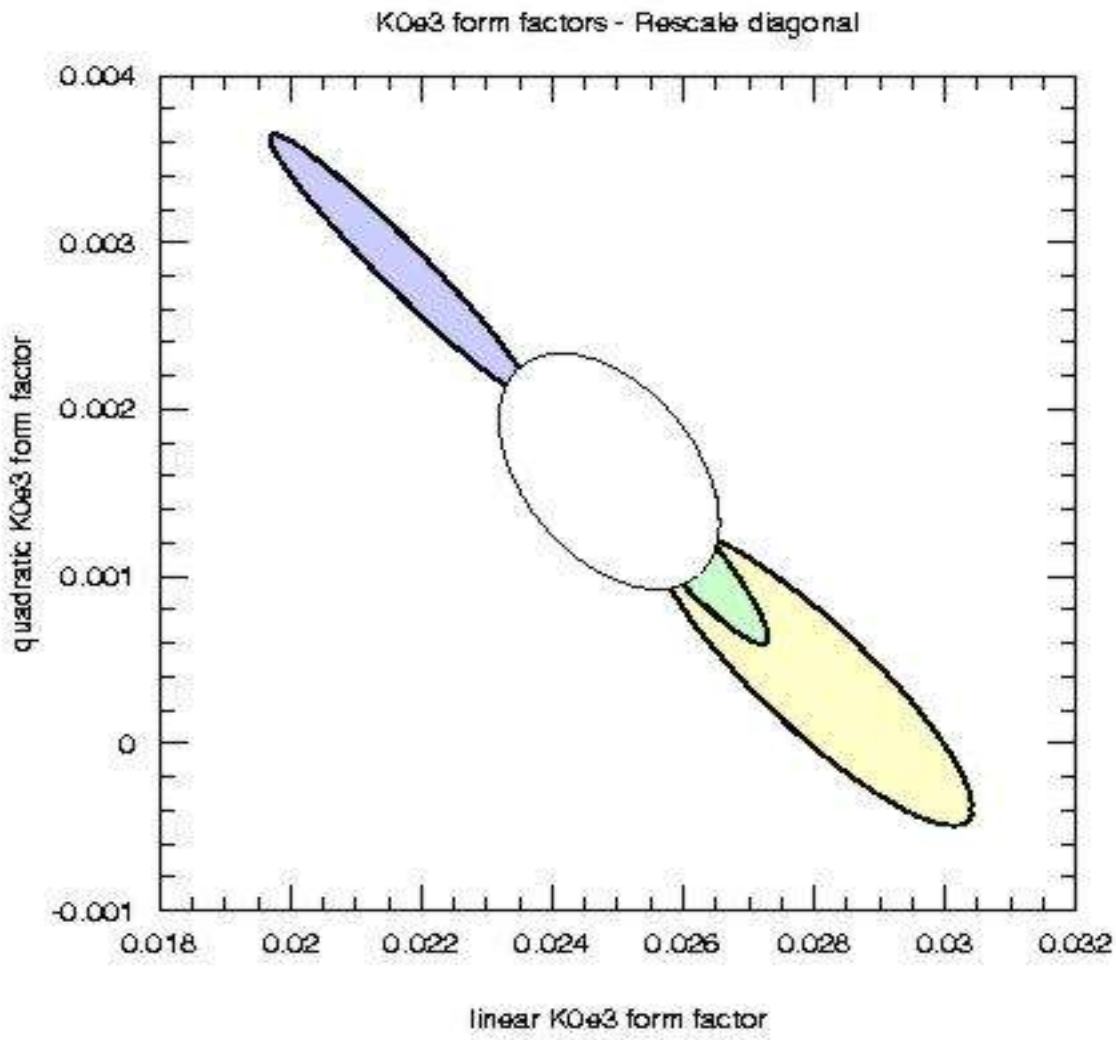
Rescaling measurements with no input correlation (continued).

- For the refitted errors, this rescaling will in principle produce a value of p_{rms} equal to one (ignoring effects from the other variables).
- We then quote the fitted values from the original fit and the errors from this revised fit. The scale factor that we quote is the ratio of the errors from the revised fit and the original fit.

Previous method for rescaling measurements with correlated input

- Use the same pull quantity p_{rms} that we calculated for measurements with no input correlation.
- Rescale the error matrix by adding to it a positive semi-definite diagonal rescaling matrix.
- Form the values of the rescaling matrix so that the diagonal elements rescale the errors in the same way that is done for measurements with no input correlations.
- Do not change the off diagonal correlation elements.


K0e3 form factors - No rescaling
0.004
0.003
0.002
0.001
0
-0.001
quadratic K0e3 form factor
0.018
0.02
0.022
0.024
0.026
0.028
0.03
0.032
linear K0e3 form factor


K0e3 form factors - Rescale diagonal
0.004
0.003
0.002
0.001
0
-0.001
quadratic K0e3 form factor
0.018
0.02
0.022
0.024
0.026
0.028
0.03
0.032
linear K0e3 form factor

New method for rescaling measurements with correlated input.

- Separately rescale each measurement with input correlations.
- Define an overall pull quantity P that includes the correlations

$$P^2 = \Sigma \{ \Sigma [(x_i - x_{avg}) \times C_{ij} \times (x_j - x_{avg})] \}$$

where C is the inverse of the expected changes in values. That is

$$C = [< (x_i - x_{avg}) \times (x_j - x_{avg}) >]^{-1}$$

New method for rescaling measurements (continued).

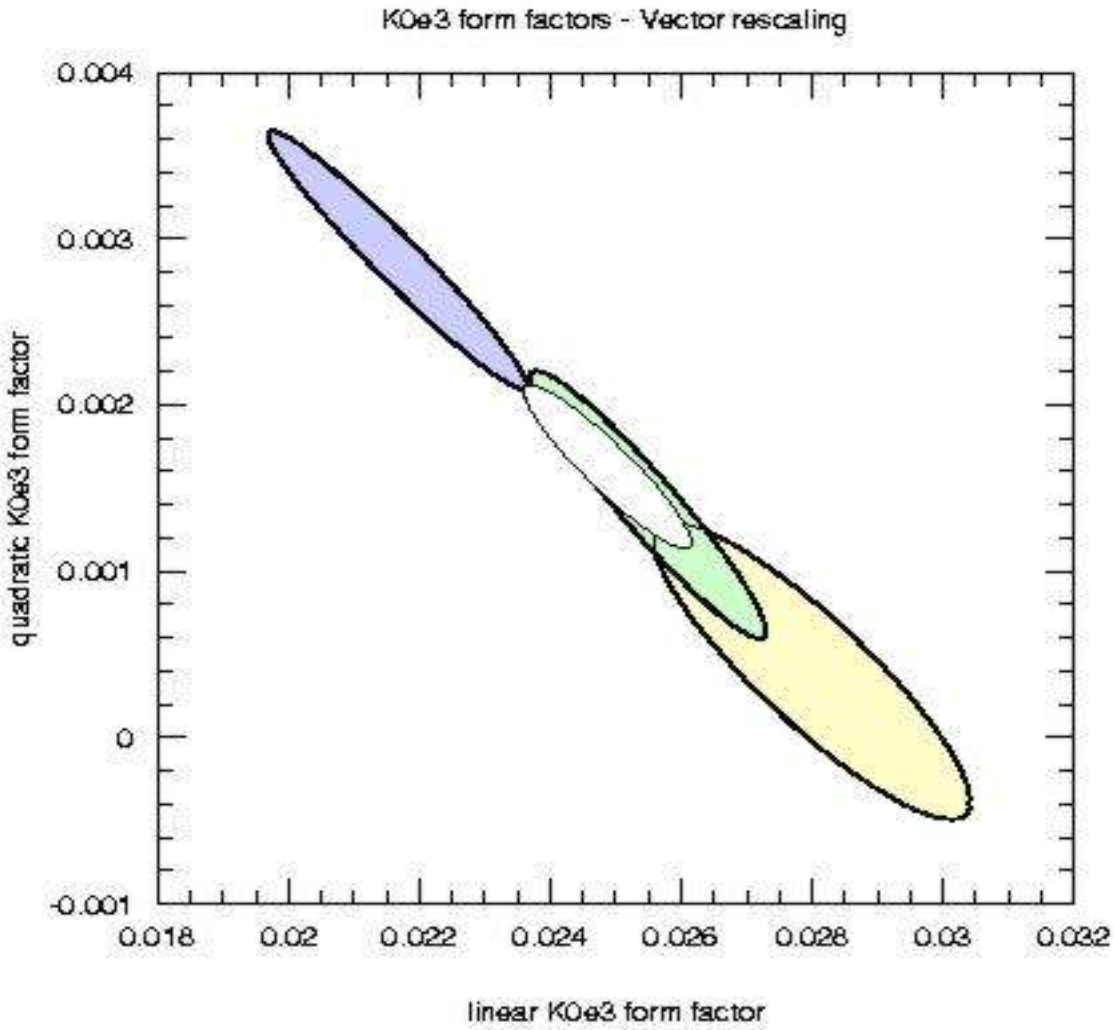
- Define a pull vector V which points in the direction where the correction is large

$$V_i = \Sigma \, [\, Q_{ij} \times (x_j - x_{avg}) \,]$$

with $Q = C^{1/2}$

- As before rescale the error matrix by adding to it a positive semi-definite rescaling matrix.

- However, form the values of the rescaling matrix so that it is diagonal in the direction of the pull vector.


K0e3 form factors - Vector rescaling
0.004
0.003
0.002
0.001
0
-0.001
quadratic K0e3 form factor
0.018
0.02
0.022
0.024
0.026
0.028
0.03
0.032
linear K0e3 form factor

Rescaling averages - Overview.

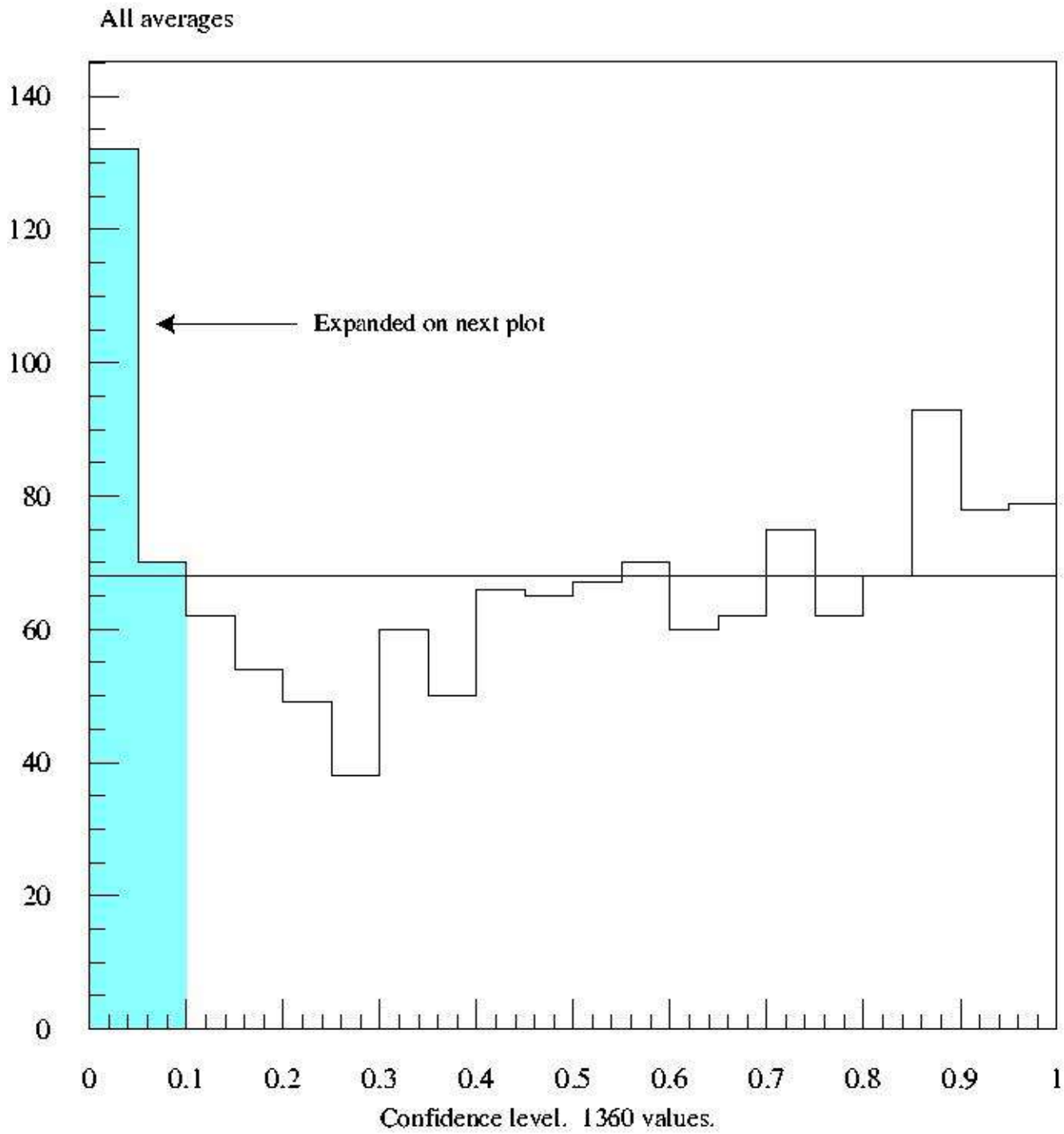
- Current method of rescaling.
- χ^2 (actually confidence level) distributions.
- Observations from the distributions.
- Possible revisions to the current method of rescaling.

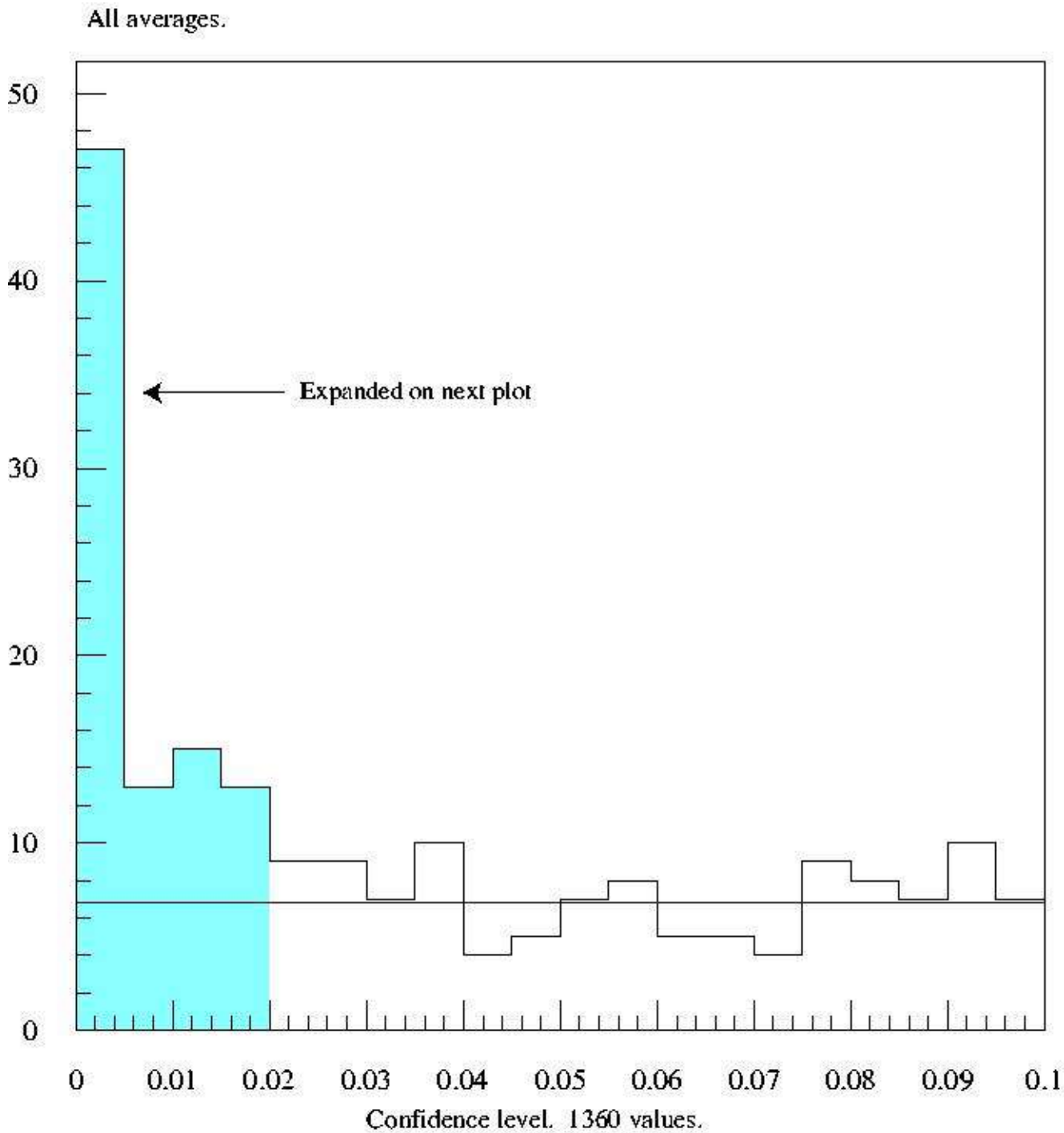
Current method of rescaling(slightly simplified).

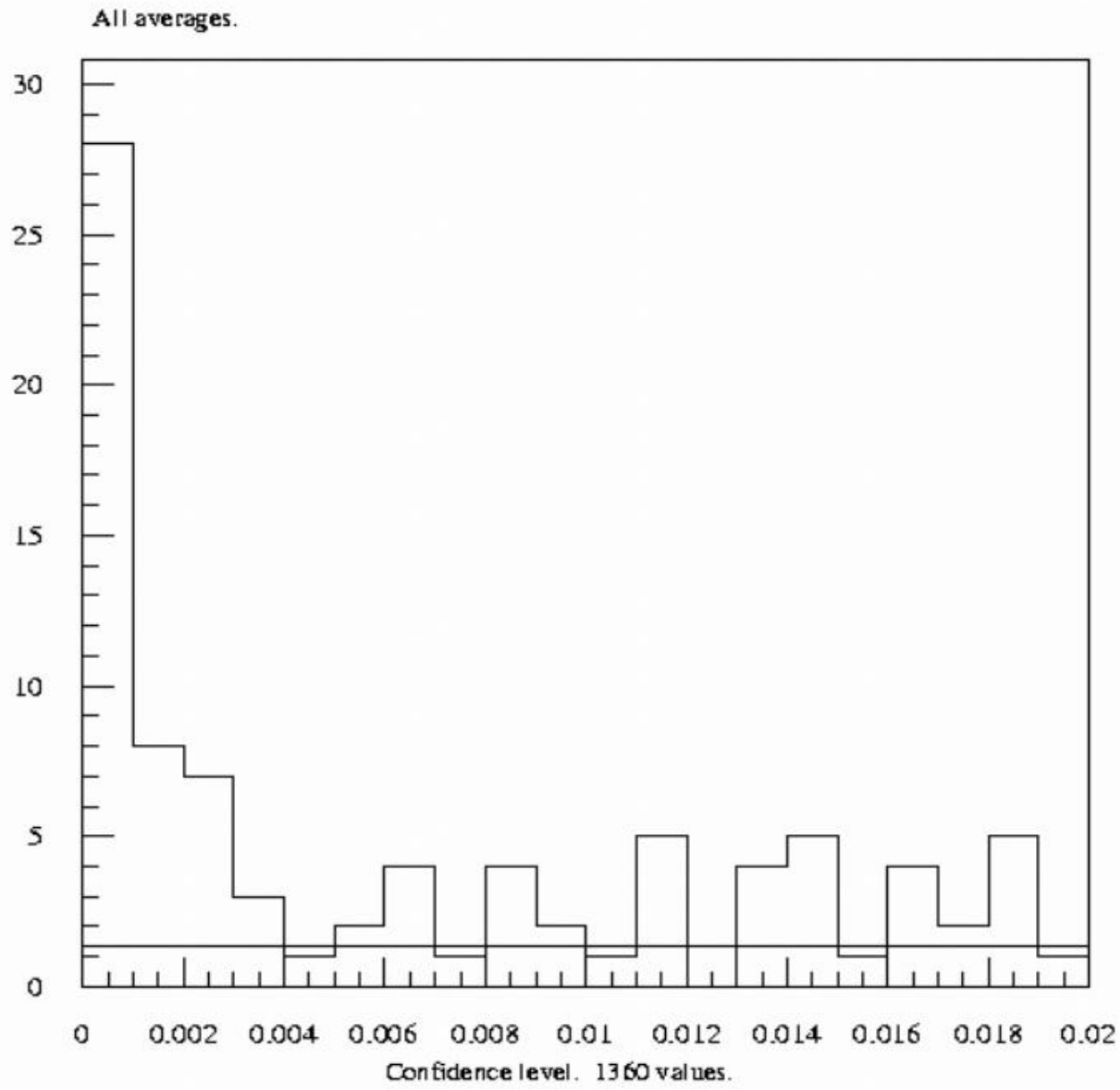
- Form $\chi^2 = \Sigma (x_i - x_{avg})^2 / \sigma_i^2$ where the sum is over the N experiments.
- If the measurements are Gaussian the expectation value of χ^2 will be N – 1.
- If $\chi^2 < N - 1$ accept the results.
- If $\chi^2 > N - 1$ form the scale factor $S = [\chi^2 / (N - 1)]^{1/2}$ and increase all errors by S. This forces χ^2 to be N – 1.
- This is the Lake Wobegon theory. All of our results are better then the expected value.

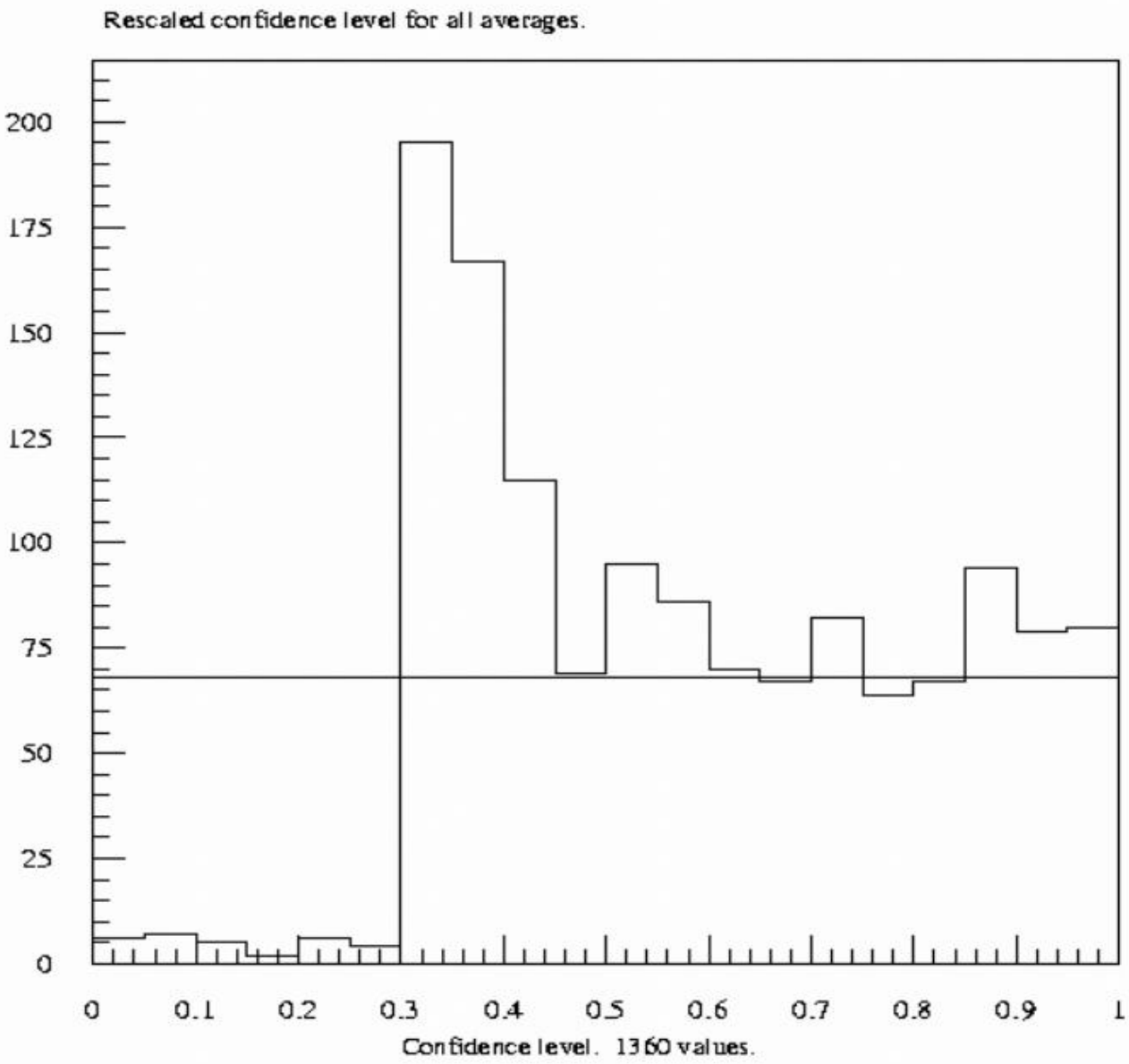
χ^2 (confidence level) distributions.

- For the 50th anniversary of PDG we looked at the χ^2 distributions. (For several years we had thought about doing this.)
- Actually we looked at the confidence level – the probability that a χ^2 would be greater then the observed χ^2. This allows us to combine values of χ^2 for different degrees of freedom.
- The results are interesting.


All averages
140
120
100
80
60
40
20
0
Expanded on next plot
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Confidence level. 1360 values.


All averages.
50
40
30
20
10
0
Expanded on next plot
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Confidence level. 1360 values.


All averages.
30
25
20
15
10
5
0
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
Confidence level. 1360 values.


Rescaled confidence level for all averages.
200
175
150
125
100
75
50
25
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Confidence level. 1360 values.

Observations from the distributions.

- The confidence level distribution is more or less as expected if the confidence level is greater then 0.02
- There is an excess of about 5% of the values with a confidence level less them 0.02
- We rescale way too many averages.

Possible revisions to our rescaling.

- Any rescaling scheme should be continuous. A small change in the original value of χ^2 should produce a small change in the rescaled χ^2.
- Rescaling does not need to produce a flat confidence level distribution.
- Actual values are hard to model with a Monte Carlo since they depend upon how experiments underestimate their errors.
- We do need to test any scheme on our data.

Possible revisions to our rescaling (continued)

- We probably should not rescale values with a confidence level greater then 0.02
- From other plots it appears that this effect is worse for averages with larger number of degrees of freedom. We should investigate this further.
- One possible method is to rescale linearly in the confidence level for values below 0.02