

DECAY CONSTANTS OF CHARGED PSEUDO-SCALAR MESONS

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Introduction. Charged mesons formed from a quark and an anti-quark can decay to a charged lepton and a neutrino when these objects annihilate via a virtual W boson [1]. Fig. 1 illustrates this process for the purely leptonic decay of a D^+ meson.

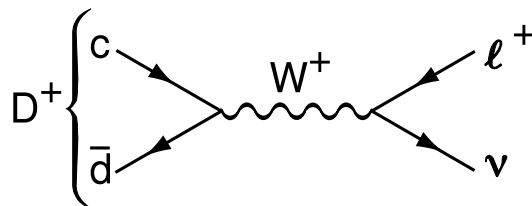


Figure 1: The annihilation process for pure D^+ leptonic decays in the Standard Model.

Similar quark-antiquark annihilations via a virtual W^+ to the $\ell^+\nu$ final states occur for the π^+ , K^+ , D_s^+ , and B^+ mesons. (Charge-conjugate particles and decays are implied.) Let P be any of these pseudoscalar mesons. To lowest order, the decay width is

$$\Gamma(P \rightarrow \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 m_\ell^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 |V_{q_1 q_2}|^2 . \quad (1)$$

Here m_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $q_1 \bar{q}_2$ in P , and G_F is the Fermi coupling constant. The parameter f_P is the decay constant, and is related to the wave-function overlap of the quark and antiquark.

The decay P^\pm starts with a spin-0 meson, and ends up with a left-handed neutrino or right-handed antineutrino. By angular momentum conservation, the ℓ^\pm must then also be left-handed or right-handed, respectively. In the $m_\ell = 0$ limit, the decay is forbidden, and can only occur as a result of the

finite ℓ mass. This helicity suppression is the origin of the m_ℓ^2 dependence of the decay width.

There is a complication in measuring purely leptonic decay rates. The process $P \rightarrow \ell\nu\gamma$ is not simply a radiative correction, although radiative corrections contribute. The P can make a transition to a virtual P^* , emitting a real photon, and the P^* decays into $\ell\nu$, avoiding helicity suppression. The importance of this amplitude depends on the decaying particle and the detection technique. The $\ell\nu\gamma$ rate for a heavy particle such as B decaying into a light particle such as a muon can be larger than the width without photon emission [2]. On the other hand, for decays into a τ^\pm , the helicity suppression is mostly broken and these effects appear to be small.

Measurements of purely leptonic decay branching fractions and lifetimes allow experimental determination of the product $|V_{q_1q_2}| f_P$. If the CKM element is well known from other measurements, then f_P can be well measured. If, on the other hand, the CKM element is not well measured, having theoretical input on f_P can allow a determination of the CKM element. The importance of measuring $\Gamma(P \rightarrow \ell\nu)$ depends on the particle being considered. For the B system, f_B is crucial for extracting information on the fundamental CKM parameters from measurements of B^0 - \bar{B}^0 mixing. Knowledge of f_{B_s} is also needed, but it cannot be directly measured as the B_s is neutral, so the violation of the SU(3) relation $f_{B_s} = f_B$ must be estimated theoretically. This difficulty does not occur for D mesons as both the D^+ and D_s^+ are charged, allowing direct measurement of SU(3) breaking and a direct comparison with theory.

For B^- and D_s^+ decays, the existence of a charged Higgs boson (or any other charged object beyond the Standard Model) would modify the decay rates; however, this would not necessarily be true for the D^+ [3,4]. More generally, the ratio of $\tau\nu$ to $\mu\nu$ decays can serve as one probe of lepton universality [3,5].

As $|V_{ud}|$ has been quite accurately measured in superallowed β decays [6], with a value of 0.97425(22) [7], measurements of $\Gamma(\pi^+ \rightarrow \mu^+\nu)$ yield a value for f_π . Similarly, $|V_{us}|$ has been well measured in semileptonic kaon decays, so a value for f_K from $\Gamma(K^- \rightarrow \mu^-\bar{\nu})$ can be compared to theoretical

calculations. Lattice gauge theory calculations, however, have been claimed to be very accurate in determining f_K , and these have been used to predict $|V_{us}|$ [8].

Charmed mesons. We review current measurements, starting with the charm system. The CLEO collaboration has performed the only measurement of the branching fraction for $D^+ \rightarrow \mu^+\nu$ [9]. CLEO uses e^+e^- collisions at the $\psi(3770)$ resonant energy where D^-D^+ pairs are copiously produced. They fully reconstruct one of the D mesons, find a candidate muon track of opposite sign to this tag, and then use kinematical constraints to infer the existence of a missing neutrino and hence the $\mu\nu$ decay of the other D . They find $\mathcal{B}(D^+ \rightarrow \mu^+\nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$. We use the well-measured D^+ lifetime of 1.040(7) ps, and assuming $|V_{cd}|$ equals $|V_{us}| = 0.2246(12)$ [7] minus higher order correction terms [10], we find $|V_{cd}| = 0.2245(12)$. (The errors on these quantities are included in the systematic error.) The CLEO branching fraction result then translates into a value of

$$f_{D^+} = (206.7 \pm 8.5 \pm 2.5) \text{ MeV} . \quad (2)$$

This result includes a 1% correction (reduction) of the rate due to the presence of the radiative $\mu^+\nu\gamma$ final state based on the estimate by Dobrescu and Kronfeld [11].

Before we compare this result with theoretical predictions, we discuss the D_s^+ . Measurements of $f_{D_s^+}$ have been made by several groups and are listed in Table 1 [12–16]. We exclude values [17–21] obtained by normalizing to D_s^+ decay modes (mentioned in the 2008 version of this Review) that are not well defined. Many measurements, for example, used the $\phi\pi^+$ mode. This decay is a subset of the $D_s^+ \rightarrow K^+K^-\pi^+$ channel which has interferences from other modes populating the K^+K^- mass region near the ϕ , the most prominent of which is the $f_0(980)$. Thus the extraction of effective $\phi\pi^+$ rate is sensitive to the mass resolution of the experiment and the cuts used to define the ϕ mass region [22,23].

The CLEO and Belle $\mu^+\nu$ results rely on fully reconstructing all the final state particles except for the neutrino and using a missing-mass technique to infer the existence of the neutrino.

CLEO uses $e^+e^- \rightarrow D_s D_s^*$ collisions at 4170 MeV, while Belle uses $e^+e^- \rightarrow DKn\pi D_s^*$ collisions at energies near the $\Upsilon(4S)$.

When selecting the $\tau^+ \rightarrow \pi^+\bar{\nu}$ and $\tau^+ \rightarrow \rho^+\bar{\nu}$ decay modes, CLEO uses both calculation of the missing-mass and the fact that there should be no extra energy in the event beyond that deposited by the measured tagged D_s^- and the τ^+ decay products. The $\tau^+ \rightarrow e^+\nu\bar{\nu}$ mode, however, uses only extra energy. BaBar measures $\Gamma(D_{s^+} \rightarrow \tau^+\nu)/\Gamma(D_{s^+} \rightarrow \bar{K}^0 K^+)$ using the $\tau^+ \rightarrow e^+\nu\bar{\nu}$ mode. Here the D_s^- tag is formed similarly to Belle by finding events with a D a K and pions opposite a single positron and little extra energy. Then the analysis is performed selecting modes with a $\bar{K}^0 K^+$ consistent with arising from the decay of a D_s^+ . (The fourth error in Table 1 on their measurement reflects the uncertainty due to the PDG value of $\mathcal{B}(D_{s^+} \rightarrow \bar{K}^0 K^+)$.)

Table 1: Experimental results for $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu)$, $\mathcal{B}(D_s^+ \rightarrow \tau^+\nu)$, and $f_{D_s^+}$. Numbers for $f_{D_s^+}$ have been extracted using updated values for masses and $|V_{cs}|$ (see text); radiative corrections have been included. Common systematic errors in the CLEO results have been taken into account.

Experiment	Mode	$\mathcal{B} \times 10^{-2}$	$f_{D_s^+}$ (MeV)
CLEO-c [12]	$\mu^+\nu$	$(0.565 \pm 0.045 \pm 0.017)$	$257.6 \pm 10.3 \pm 4.3$
Belle [13]	$\mu^+\nu$	$(0.638 \pm 0.076 \pm 0.057)$	$274 \pm 16 \pm 12$
Average	$\mu^+\nu$	(0.580 ± 0.043)	261.5 ± 9.7
CLEO-c [12]	$\tau^+\nu$ ($\pi^+\bar{\nu}$)	$(6.42 \pm 0.81 \pm 0.18)$	$278.0 \pm 17.5 \pm 3.8$
CLEO-c [14]	$\tau^+\nu$ ($\rho^+\bar{\nu}$)	$(5.52 \pm 0.57 \pm 0.21)$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [15]	$\tau^+\nu$ ($e^+\nu\bar{\nu}$)	$(5.30 \pm 0.47 \pm 0.22)$	$252.6 \pm 11.1 \pm 5.2$
BaBar [16]	$\tau^+\nu$ ($e^+\nu\bar{\nu}$)	$(4.54 \pm 0.53 \pm 0.40 \pm 0.28)$	$233.8 \pm 13.7 \pm 12.6$
Average	$\tau^+\nu$	(5.58 ± 0.35)	255.5 ± 7.5
Average	$\mu^+\nu + \tau^+\nu$		257.5 ± 6.1

We extract the decay constant from the measured branching ratios using the D_s^+ mass of 1.96849(34) GeV, the τ^+ mass of 1.77684(17) GeV, and a lifetime of 0.500(7) ps. We use the first order correction $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2$ [10]; taking $|V_{ud}| = 0.97425(22)$ [6], and $|V_{cb}| = 0.04$ from an average of exclusive and inclusive semileptonic B decay results as discussed in Ref. [24], we find $|V_{cs}| = 0.97345(22)$. Our experimental average,

$$f_{D_s^+} = (257.5 \pm 6.1) \text{ MeV}, \quad (3)$$

uses only those results that are included in Table 1. We have again included the radiative correction of 1% in the $\mu^+\nu$ rates listed in the table estimated by Dobrescu and Kronfeld [11] (the $\tau^+\nu$ rates need not be corrected). Other theoretical calculations show that the $\gamma\mu^+\nu$ rate is a factor of 40–100 below the $\mu^+\nu$ rate for charm [25].

Table 2: Theoretical predictions of $f_{D_s^+}$, f_{D^+} , and $f_{D_s^+}/f_{D^+}$. QL indicates a quenched-lattice calculation, while PQL indicates a partially-quenched lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$ (MeV)	f_{D^+} (MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	257.5 ± 6.1	206.7 ± 8.9	1.25 ± 0.06
Lattice [26]	241 ± 3	208 ± 4	1.162 ± 0.009
Lattice [27]	260 ± 10	217 ± 10	1.20 ± 0.02
PQL [28]	244 ± 8	197 ± 9	1.24 ± 0.03
QL (QCDSF) [29]	$220 \pm 6 \pm 5 \pm 11$	$206 \pm 6 \pm 3 \pm 22$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan) [30]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [31]	$236 \pm 8_{-14}^{+17}$	$210 \pm 10_{-16}^{+17}$	$1.13 \pm 0.02_{-0.02}^{+0.04}$
QL [32]	$231 \pm 12_{-1}^{+6}$	$211 \pm 14_{-12}^{+2}$	1.10 ± 0.02
QCD Sum Rules [33]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [34]	235 ± 24	203 ± 20	1.15 ± 0.04
Field Correlators [35]	260 ± 10	210 ± 10	1.24 ± 0.03
Light Front [36]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04

Two ratios are of particular interest. The ratio of decay constants for the $\tau^+\nu : \mu^+\nu$ modes is $f_{D_s^+}(\tau^+\nu)/f_{D_s^+}(\mu^+\nu) = 0.98 \pm 0.05$, and the ratio of D_s^+ to D^+ decay constants is $f_{D_s^+}/f_{D^+} = 1.25 \pm 0.06$. Table 2 compares the experimental $f_{D_s^+}$ with theoretical calculations [26–36]. While most theories give values lower than the $f_{D_s^+}$ measurement, the errors are sufficiently large, in most cases, to declare success. An unquenched lattice calculation [26], however, differs by 2.4 standard deviations [37]. Remarkably it agrees with f_{D^+} and consequently disagrees in the ratio $f_{D_s^+}/f_{D^+}$, with less significance as the error in f_{D^+} is substantial.

The Fermilab-MILC result has been updated; the preliminary values for f_{D^+} and $f_{D_s^+}$ were raised by 10 MeV and 11 MeV, respectively [27]. These changes bring the predictions for both numbers within errors of experiment.

Akeroyd and Chen [38] pointed out that leptonic decay widths are modified in two-Higgs-doublet models (2HDM). Specifically, for the D^+ and D_s^+ , Eq. (1) is modified by a factor r_q multiplying the right-hand side [39]:

$$r_q = \left[1 + \left(\frac{1}{m_c + m_q} \right) \left(\frac{M_{D_q}}{M_{H^+}} \right)^2 \left(m_c - \frac{m_q \tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right) \right]^2, \quad (4)$$

where M_{H^+} is the charged Higgs mass, M_{D_q} is the mass of the D meson (containing the light quark q), m_c is the charm quark mass, m_q is the light-quark mass, and $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. In models where the fermion mass arises from coupling to more than one vacuum expectation value ϵ_0 can be non-zero, perhaps as large as 0.01. For the D^+ , $m_d \ll m_c$, and the change due to the H^+ is very small. For the D_s^+ , however, the effect can be substantial.

A major concern is the need for the Standard Model (SM) value of $f_{D_s^+}$. We can take that from a theoretical model. Our most aggressive choice is that of the unquenched lattice calculation [26], because they claim the smallest error. Since the charged Higgs would lower the rate compared to the SM, in principle, experiment gives a lower limit on the charged Higgs

mass. However, the value for the predicted decay constant using this model is 2.4 standard deviations *below* the measurement, implying that (a) either the model of Ref. 26 is not representative; (b) no value of m_{H^+} in the two-Higgs doublet model will satisfy the constraint at 99% confidence level; or (c) there is new physics, different from the 2HDM, that interferes constructively with the SM amplitude such as in the R-parity-violating model of Akeroyd and Recksiegel [40]. New physics can affect the $\mu^+\nu$ and $\tau^+\nu$ final states differently and thus these should be studied separately [39].

To sum up, the situation is not clear. To set limits on new physics we need an accurate calculation of $f_{D_s^+}$ and more precise measurements would also be useful.

The B meson. The Belle and BaBar collaborations have found evidence for $B^- \rightarrow \tau^- \bar{\nu}$ decay in $e^+e^- \rightarrow B^-B^+$ collisions at the $\Upsilon(4S)$ energy. The analysis relies on reconstructing a hadronic or semi-leptonic B decay tag, finding a τ candidate in the remaining track and or photon candidates, and examining the extra energy in the event which should be close to zero for a real τ decay opposite a B meson tag. The results are listed in Table 3.

Table 3: Experimental results for $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu})$. We have computed an average for the two Belle measurements assuming that the systematic errors are fully correlated.

Experiment	Tag	$\mathcal{B} \times 10^{-4}$
Belle [41]	Hadronic	$(1.79^{+0.56}_{-0.49} \quad +0.46_{-0.51})$
Belle [42]	Semileptonic	$(1.65^{+0.38}_{-0.37} \quad +0.35_{-0.34})$
Belle	Our Average	$(1.70^{+0.47}_{-0.46})$
BaBar [43]	Hadronic	$(1.8^{+0.9}_{-0.8} \pm 0.4)$
BaBar [44]	Semileptonic	$(1.7 \pm 0.8 \pm 0.4)$
BaBar [44]	Average	$(1.8^{+1.0}_{-0.9})$
Our Average		$(1.72^{+0.43}_{-0.42})$

There are large backgrounds under the signals in all cases. The systematic errors are also quite large, on the order of 20%. Thus the significances are not that large. Belle quotes 3.5σ and 3.8σ for their hadronic and semileptonic tags, respectively, while BaBar quotes 2.8σ for their combined result. We note that the four central values are remarkably close to the average considering the large errors on all the measurements. More accuracy would be useful to investigate the effects of new physics. Here the effect of a charged Higgs is different as it can either increase or decrease the expected SM branching ratio. The factor r in the 2HDM that multiplies the right side of Eq. (1) is given in terms of the B meson mass, M_B , by [3,39]

$$r = \left(1 - \frac{\tan^2 \beta}{1 - \epsilon_0 \tan \beta} \frac{M_B^2}{m_{H^+}^2} \right)^2. \quad (5)$$

We can derive limits in the 2HDM $\tan\beta$ - m_{H^+} plane. Again, we need to know the SM prediction of this decay rate. We ascertain this value using Eq. (1). Here theory provides a value of $f_B = (193 \pm 11)$ MeV [45]. We also need a value for $|V_{ub}|$. Here significant differences arise between using inclusive charmless semileptonic decays and the exclusive decay $B \rightarrow \pi\ell^+\nu$ [46]. We find that the inclusive decays give rise to a value of $|V_{ub}| = (4.21 \pm 0.25) \times 10^{-3}$, while the $\pi\ell^+\nu$ measurements yield $|V_{ub}| = (3.50 \pm 0.35) \times 10^{-3}$. Taking an average over inclusive and exclusive determinations, and enlarging the error using the PDG prescription because the results differ, we find $|V_{ub}| = (3.97 \pm 0.55) \times 10^{-3}$, where the error is dominantly theoretical. We thus arrive at the SM prediction for the $\tau^-\bar{\nu}$ branching fraction of $(1.04 \pm 0.31) \times 10^{-4}$.

Taking the ratio of the experimental value to the predicted branching ratio at its 90% c.l. *upper* limit and using Eq. (5) with ϵ_0 set to zero, we find that we can limit $M_{H^+} / \tan \beta > 3.3$ GeV. The 90% c.l. *lower* limit also permits us to exclude the region 3.8 GeV $< M_{H^+} / \tan \beta < 18.0$ GeV [47]. Considering the large uncertainties on V_{ub} and the branching ratio measurements, this should be taken more as indication of what the data can eventually tell us when and if the situation improves.

Charged pions and kaons. We now discuss the determination of charged pion and kaon decay constants. The sum of branching fractions for $\pi^- \rightarrow \mu^- \bar{\nu}$ and $\pi^- \rightarrow \mu^- \bar{\nu} \gamma$ is 99.98770(4)%. The two modes are difficult to separate experimentally, so we use this sum, with Eq. (1) modified to include photon emission and radiative corrections [48]. The branching fraction together with the lifetime 26.033(5) ns gives

$$f_{\pi^-} = (130.41 \pm 0.03 \pm 0.20) \text{ MeV} . \quad (6)$$

The first error is due to the error on $|V_{ud}|$, 0.97425(22) [6]; the second is due to the higher-order corrections, and is much larger.

Similarly, the sum of branching fractions for $K^- \rightarrow \mu^- \bar{\nu}$ and $K^- \rightarrow \mu^- \bar{\nu} \gamma$ is 63.55(11)%, and the lifetime is 12.3840(193) ns [49]. Measurements of semileptonic kaon decays provide a value for the product $f_+(0)|V_{us}|$, where $f_+(0)$ is the form-factor at zero four-momentum transfer between the initial state kaon and the final state pion. We use a value for $f_+(0)|V_{us}|$ of 0.21664(48) [49]. The $f_+(0)$ must be determined theoretically. We follow Blucher and Marciano [7] in using the lattice calculation $f_+(0) = 0.9644 \pm 0.0049$ [50], since it appears to be more precise than the classic Leutwyler-Roos calculation $f_+(0) = 0.961 \pm 0.008$ [51]. The result is $|V_{us}| = 0.2246 \pm 0.0012$, which is consistent with the hyperon decay value of 0.2250 ± 0.0027 [52]. We derive

$$f_{K^-} = (156.1 \pm 0.2 \pm 0.8 \pm 0.2) \text{ MeV} . \quad (7)$$

The first error is due to the error on Γ ; the second is due to the CKM factor $|V_{us}|$, and the third is due to the higher-order corrections. The largest source of error in these corrections depends on the QCD part, which is based on one calculation in the large N_c framework. We have doubled the quoted error here; this would probably be unnecessary if other calculations were to come to similar conclusions. A large part of the additional uncertainty vanishes in the ratio of the K^- and π^- decay constants, which is

$$f_{K^-}/f_{\pi^-} = 1.197 \pm 0.002 \pm 0.006 \pm 0.001 . \quad (8)$$

The first error is due to the measured decay rates; the second is due to the uncertainties on the CKM factors; the third is due to the uncertainties in the radiative correction ratio.

These measurements have been used in conjunction with calculations of f_K/f_π in order to find a value for $|V_{us}|/|V_{ud}|$. Two recent lattice predictions of f_K/f_π are 1.189 ± 0.007 [26] and $1.192 \pm 0.007 \pm 0.006$ [53]. Together with the precisely measured $|V_{ud}|$, this gives an independent measure of $|V_{us}|$ [8,49].

References

1. For an expanded version of this review see J. L. Rosner and S. Stone, “Leptonic Decays of Charged Pseudoscalar Mesons,” arXiv:1002.1655.
2. Most predictions for the rate for $B^- \rightarrow \mu^- \bar{\nu} \gamma$ are in the range of 1 to 20 times the rate for $B^- \rightarrow \mu^- \bar{\nu}$. See G. Burdman, T. Goldman, and D. Wyler, Phys. Rev. **D51**, 111 (1995); P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. **B372**, 331 (1996); *ibid.*, **386**, 328 (1996); A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. **B358**, 129 (1995); G. Eilam, I. Halperin, and R. Mendel, Phys. Lett. **B361**, 137 (1995); D. Atwood, G. Eilam, and A. Soni, Mod. Phys. Lett. **A11**, 1061 (1996); C.Q. Geng and C.C. Lih, Phys. Rev. **D57**, 5697 (1998) and Mod. Phys. Lett. **A15**, 2087 (2000); G.P. Korchemsky, D. Pirjol, and T.M. Yang, Phys. Rev. **D61**, 114510 (2000); C.W. Hwang, Eur. Phys. J. **C46**, 379 (2006).
3. W.-S. Hou, Phys. Rev. **D48**, 2342 (1993).
4. See, for example, A.G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003); A.G. Akeroyd, Prog. Theor. Phys. **111**, 295 (2004).
5. J.L. Hewett, hep-ph/9505246, presented at *Lafex International School on High Energy Physics (LISHEP95)*, Rio de Janeiro, Brazil, Feb. 6-22, 1995.
6. I. S. Towner and J. C. Hardy, Phys. Rev. **C79**, 055502 (2009); I. S. Towner and J. C. Hardy, Phys. Rev. **C77**, 025501 (2008).
7. E. Blucher and W. J. Marciano, “ V_{ud} , V_{us} , The Cabibbo Angle, and CKM Unitarity” in PDG 2010.
8. W.J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004); A. Jüttner, [arXiv:0711.1239].

9. B. I. Eisenstein *et al.*, (CLEO Collab.), Phys. Rev. **D78**, 052003 (2008). See also M. Artuso *et al.*, (CLEO Collab.), Phys. Rev. Lett. **95**, 251801 (2005).
10. J. Charles *et al.*, Eur. Phys. J. **C41**, 1 (2005).
11. B. A. Dobrescu and A. S. Kronfeld, Phys. Rev. Lett. **100**, 241802 (2008).
12. J. P. Alexander *et al.*, (CLEO Collab.), Phys. Rev. **D79**, 052001 (2009); M. Artuso *et al.*, (CLEO Collab.), Phys. Rev. Lett. **99**, 071802 (2007).
13. L. Widhalm *et al.*, (Belle Collab.), Phys. Rev. Lett. **100**, 241801 (2008).
14. P. Naik *et al.*, (CLEO Collab.), Phys. Rev. **D80**, 112004 (2009).
15. P.U.E. Onyisi *et al.*, (CLEO Collab.), Phys. Rev. **D79**, 052002 (2009); K.M. Ecklund *et al.*, (CLEO Collab.), Phys. Rev. Lett. **100**, 161801 (2008).
16. J. Dingfelder, “Hot topics from Babar: Leptonic and Semileptonic B and D decays,” presented at Moriond QCD and High Energy Interactions, La Thuile, Italy March 2010; J. P. Lee *et al.* (BaBar Collab.) arXiv:1003.3063..
17. M. Chadha *et al.*, (CLEO Collab.), Phys. Rev. **D58**, 032002 (1998).
18. Y. Alexandrov *et al.*, (BEATRICE Collab.), Phys. Lett. **B478**, 31 (2000).
19. A. Heister *et al.*, (ALEPH Collab.), Phys. Lett. **B528**, 1 (2002).
20. M. Acciarri *et al.*, (L3 Collab.), Phys. Lett. **B396**, 327 (1997).
21. G. Abbiendi *et al.*, (OPAL Collab.), Phys. Lett. **B516**, 236 (2001).
22. See J. Alexander *et al.* (CLEO Collab.), Phys. Rev. Lett. **100**, 161804 (2008).
23. We have not included a BaBar result: $\mathcal{B}(D_s^+ \rightarrow \mu^+\nu) = (6.67 \pm 0.83 \pm 0.26 \pm 0.66) \times 10^{-3}$ and $f_{D_s^+} = (281 \pm 17 \pm 7 \pm 14)$ MeV based on $\mathcal{B}(D_s^+ \rightarrow \phi\pi^+) = (4.71 \pm 0.46)\%$. These measurements determined the ratio of the leptonic decay to a hadronic decay, usually $\Gamma(D_s^+ \rightarrow \ell^+\nu)/\Gamma(D_s^+ \rightarrow \phi\pi^+)$. B. Aubert *et al.*, (BABAR Collab.), Phys. Rev. Lett. **98**, 141801 (2007).
24. M. Artuso, E. Barberio and S. Stone, PMC Physics A, 3:3 (2009), arXiv:0902.3743 [hep-ph].
25. See most papers in Ref. [2] and C. D. Lü and G. L. Song, Phys. Lett. **B562**, 75 (2003).

26. E. Follana *et al.*, (HPQCD and UKQCD Collabs.), Phys. Rev. Lett. **100**, 062002 (2008) The statistical error is given as 0.6%, with the other systematic errors added in quadrature yielding the 3 MeV total error.
27. We use the updated numbers from A. Bazavov *et al.* (Fermilab Lattice and MILC Collaborations), PoS LATTICE 2009 249, (2009), an update that increases both f_{D^+} and $f_{D_s^+}$ by 10 MeV and 11 MeV, respectively, from the values originally given in C. Bernard *et al.*, [arXiv:0904.1895] (2009); see also C. Aubin *et al.*, Phys. Rev. Lett. **95**, 122002 (2005).
28. B. Blossier *et al.*, [arXiv:0904.0954] (2009).
29. A. Ali Khan *et al.*, (QCDSF Collaboration), Phys. Lett. **B652**, 150 (2007).
30. T.W. Chiu *et al.*, Phys. Lett. **B624**, 31 (2005) [hep-ph/0506266].
31. L. Lellouch and C.-J. Lin (UKQCD Collaboration), Phys. Rev. **D64**, 094501 (2001).
32. D. Becirevic *et al.*, Phys. Rev. **D60**, 074501 (1999).
33. J. Bordes, J. Peñarrocha, and K. Schilcher, JHEP **0511**, 014 (2005).
34. S. Narison, [hep-ph/0202200] (2002).
35. A.M. Badalian *et al.*, Phys. Rev. **D75**, 116001 (2007); see also A.M. Badalian and B.L.G. Bakker, [hep-ph/0702229].
36. C.-W. Hwang, arXiv:0910.0145 [hep-ph] (2009).
37. The small errors quoted in Follana *et al.* [26] are still being discussed in the lattice community. See, for example, M. Della Morte [arXiv:0711.3160], page 6 (2007). See also A. S. Kronfeld [arXiv:0912.0543] (2009).
38. A.G. Akeroyd and C.H. Chen, Phys. Rev. **D75**, 075004 (2007) [hep-ph/0701078].
39. A.G. Akeroyd and F. Mahmoudi, JHEP **0904**, 121 (2009).
40. A. G. Akeroyd and S. Recksiegel, Phys. Lett. **B554**, 38 (2003) [hep-ph/0210376].
41. K. Ikado *et al.*, (Belle Collab.), Phys. Rev. Lett. **97**, 251802 (2006).
42. I. Adachi *et al.*, (Belle Collab.), [arXiv:0809.3834].
43. B. Aubert *et al.*, (BaBar Collab.), Phys. Rev. **D77**, 011107R (2008).
44. B. Aubert *et al.*, (BaBar Collab.) [arXiv:0912.2453].
45. C. Bernard *et al.* (Fermilab, MILC Collabs.), PoS LATTICE 2008 278, (2008) calculate $f_B = 195 \pm 11$ MeV,

while E. Gamiz *et al.* (HPQCD Collab.) Phys. Rev. **D80**, 014503 (2009) find $f_B = 190 \pm 13$ MeV. We average the two results assuming the errors are fully correlated.

46. See the discussion in M. Artuso, E. Barberio and S. Stone, PMC Physics A **3:3** (2009) [[arXiv:0902.3743](#)].
47. In supersymmetric models there may be corrections which weaken our bound; a slightly different formula for r is given by Isidori and Paradisi, Phys. Lett. **B639**, 499 (2006) [[hep-ph/0605012](#)] that depends on a model dependent value of a parameter - ϵ_0 .
48. W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993); V. Cirigliano and I. Rosell, JHEP **0710**, 005 (2007).
49. Flavianet Working Group on Precise SM Tests in K Decays, <http://www.lnf.infn.it/wg/vus>. See M. Antonelli *et al.*, Nucl. Phys. Proc. Suppl. **181-182**, 83 (2008). For a recent detailed review, see M. Antonelli *et al.*, [[arXiv:0907.5386](#)].
50. P. A. Boyle *et al.*, Phys. Rev. Lett. **100**, 141601 (2008).
51. H. Leutwyler and M. Roos, Z. Phys. **C25**, 91 (1984).
52. N. Cabibbo, E. C. Swallow and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004).
53. S. Durr *et al.*, [[arXiv:1001.4692](#)] (2010).