25. ACCELERATOR PHYSICS OF COLLIDERS

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25.1. Luminosity

This article provides background for the High-Energy Collider Parameter Tables that follow. Of prime importance in a collider run is the *integrated luminosity*; the ratio of yield to cross section. Integrated luminosity is the integral over time of the instantaneous *luminosity* denoted here by $\mathscr{L}$.

Today's colliders all employ bunched beams. If two bunches containing n_1 and n_2 particles collide head-on with frequency f, the luminosity is

$$\mathscr{L} = f \frac{n_1 n_2}{4\pi\sigma_x\sigma_y} \tag{25.1}$$

where σ_x and σ_y characterize the transverse beam profiles in the horizontal (bend) and vertical directions. In this form it is assumed that the bunches are identical in transverse profile, that the profiles are independent of position along the bunch, and the particle distributions are not altered during bunch passage.

Whatever the distribution at the source, by the time the beam reaches high energy, the normal form is a useful approximation as suggested by the σ-notation. In the case of an electron storage ring, synchrotron radiation leads to a Gaussian distribution in equilibrium, but even in the absence of radiation the central limit theorem of probability and the diminished importance of space charge effects produces a similar result.

The n's and σ's in Eq. (25.1) may change with time during a "store", and control of that time variation is a major factor in integrated luminosity. The integral achieved over a period such as a week is a measure of overall systems performance, as it will include such influences as turn-around time for refill. The formula needs a variety of modifications depending on the type of collider; for example, the angular distribution of particle velocities in a bunch may cause a significant variation in transverse beam size through the collision overlap region. This effect and others specific to collider type will be discussed in later sections.

In the Tables, luminosity is stated in units of cm^{-2}s^{-1}. Integrated luminosity, on the other hand, is usually quoted as the inverse of the standard measures of cross section such as femtobarns and, recently, attobarns.

Subsequent sections in this report enlarge briefly on the dynamics behind collider design, comment on the realization of collider performance in a selection of today's facilities, and end with some remarks on future possibilities.

25.2. Single Particle Dynamics

Today's operating HEP colliders are all alternating-gradient synchrotrons [1,2], and the material of this section reflects that circumstance. The single particle transverse motion in this focusing structure is not a simple sinusoid; rather it may be expressed in the form

$$x(s) = A\sqrt{\beta(s)}\,\cos[\psi(s) + \delta], \tag{25.2}$$

where s is path length in the beam direction, A and δ are constants of integration and the envelope of the motion is modulated by the *amplitude function*, β. The phase advances according to $d\psi/ds = 1/\beta$; that is, β also plays the role of a local $\lambda/2\pi$, and the *tune*, ν, is the number of such oscillations per turn about the closed path. In the neighborhood of an interaction point, the beam optics of the ring is configured so as to produce a near focus; the value of the amplitude function at this point is designated β^*.

The motion as it develops with s describes an ellipse in $\{x, x' \ (\equiv dx/ds)\}$ phase space the area of which is πA^2, where A is the constant above. If the interior of that ellipse is populated by an ensemble of particles, that area, given the name *emittance* and denoted by ε, would change only with beam energy in the absence of other processes. For a beam with a Gaussian distribution in x, x', the area containing one standard deviation σ_x is the definition of emittance in the Tables:

$$\varepsilon_x \equiv \pi \frac{\sigma_x^2}{\beta_x} \,, \tag{25.3}$$

with a corresponding expression in the other transverse direction, y. This definition includes 39% of the beam.

To complete the coordinates used to describe the motion, we add to the transverse phase space $\{x, x', y, y'\}$ the longitudinal variables $\{z, \delta p/p\}$, where z is the distance by which the particle leads the "ideal" particle along the design trajectory. Radiofrequency electric fields in the s direction provide the means for longitudinal oscillations, and the frequency determines the bunch length. The frequency of this system appears in the Tables as does $\delta p/p$ characterized as "energy spread".

For HEP bunch length is a significant quantity for a variety of reasons, but in the present context if the bunch length becomes larger than β^* the luminosity is adversely affected. This is because β grows parabolically as one proceeds away from the IP and so the beam size increases thus lowering the contribution to the luminosity from such locations. This is often called the "hour glass" effect as is the factor by which the luminosity is reduced.

The other major external electromagnetic field interaction in the single particle context is the production of synchrotron radiation due to centripetal acceleration, given by the Larmor formula multiplied by a relativistic magnification factor of γ^4 [3,4]. In the case of electron rings this process determines the equilibrium emittance through a balance between radiation damping and excitation of oscillations, and further serves as the barrier to future higher energy versions in this variety of collider.

25.3. Impediments to High Luminosity

Eq. (25.1) can be recast in terms of emittances and amplitude functions as

$$\mathscr{L} = f \frac{n_1 n_2}{4\sqrt{\varepsilon_x\, \beta_x^*\, \varepsilon_y\, \beta_y^*}} \ . \tag{25.4}$$

So to achieve high luminosity, all one has to do is make high population bunches of low emittance to collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible.

While there are no fundamental limits to this process, there are certainly challenges. Here we have space to mention only a few of these. The beam-beam tune shift appears in the Tables. A bunch in one beam presents a (nonlinear) lens to a particle in the other beam resulting in changes to the particle's transverse oscillation tune with a range characterized by the parameter [5]

$$\xi_{y,2} = \frac{r_2 n_1 \beta^*_{y,2}}{2\pi\gamma_2 \sigma_{y,1}(\sigma_{x,1} + \sigma_{y,1})} \tag{25.5}$$

where $r_2 = e^2/(4\pi\epsilon_0 m_2 c^2)$ is the classical radius of the affected particle. The transverse oscillations are susceptible to resonant perturbations from a variety of sources such as imperfections in the magnetic guide field, so certain values of the tune must be avoided. Accordingly, the tune spread arising from ξ is limited, but limited to a value difficult to predict. But a glance at the Tables shows that electrons are more forgiving than protons thanks to the damping effects of synchrotron radiation; the ξ-values for the former are about an order of magnitude larger than those for protons.

A subject of present intense interest is the *electron-cloud effect* [6,7]; actually a variety of related processes come under this heading. They typically involve a buildup of electron density in the vacuum chamber due to emission from the chamber walls stimulated by electrons or photons originating from the beam itself. For instance, there is a process closely resembling the multipacting effects familiar from radiofrequency system commissioning. Low energy electrons are ejected from the walls by photons from positron or proton beam-produced synchrotron radiation. These electrons are accelerated toward a beam bunch, but by the time they reach the center of the vacuum chamber the bunch has gone and so the now-energetic electrons strike the opposite wall to produce more secondaries. These secondaries are now accelerated by a subsequent bunch, and so on. Among the disturbances that this electron accumulation can produce is enhancement of the tune spread within the bunch; the near-cancellation of bunch induced electric and magnetic fields is no longer in effect.

The benefits of low emittance are clear in Eq. (25.4), so a few words are in order on that subject. For electron synchrotrons, radiation damping provides an automatic route. For hadrons, particularly antiprotons, two inventions have played a prominent role. Stochastic cooling [8] was employed first in the Sp$\bar{\text{p}}$S and subsequently in the Tevatron. Electron cooling [9] is currently also in use in the Tevatron complex. Further innovations are underway due to the needs of potential future projects; these are noted in the final section.

25.4. Comments on Present Facilities

Collider accelerator physics of course goes far beyond the elements of the preceding sections. In this section elaboration is made on various issues associated with some of the recently operating colliders, particularly issues which impact integrated luminosity. The various colliders utilizing hadrons have important unique differences and hence are broken out separately. As space is limited, general references are provided where much further information can be obtained.

25.4.1. ***LHC*** : [10] Once commissioning is complete, the superconducting Large Hadron Collider will emerge as the world's highest energy collider. To meet its luminosity goals the LHC will have to contend with a high beam current of 0.5 A, leading to stored energies of several hundred MJ per beam. Component protection, beam collimation, and controlled energy deposition will be given very high priorities. Additionally, at energies of 5-7 TeV per particle, synchrotron radiation will move from being a curiosity to a challenge in a hardron accelerator for the first time. At design beam current the system must remove roughly 7 kW at 1.8 K due to synchrotron radiation. As the photons are emitted their interactions with the vacuum chamber wall can generate free electrons, with consequent "electron cloud" development. Much care was taken to design a special liner for the chamber to mitigate this issue.

The two proton beams are contained in separate pipes throughout most of the circumference, but naturally must be brought together into a single pipe at the interaction points (IP's). The large number of bunches, and subsequent short bunch spacing, would lead to approximately 30 head-on collisions through 120 m of common beam pipe at each IP. Thus, a small crossing angle is employed, which reduces the luminosity by about 15%. Still, the bunches moving in one direction will have long-range encounters with the counter-rotating bunches and the resulting perturbations of the particle motion constitute a continued course of study.

As with all hadron colliders, emittance preservation and optimization throughout the injector chain and through to collision conditions is paramount to obtaining the highest luminosity possible.

25.4.2. ***Tevatron*** : [11] The route to high integrated luminosity in the Tevatron has been governed by the antiproton production rate, the turn-around time to produce another store, and the resulting optimization of store time. The overall reliability of the accelerator complex plays a crucial role, as it can take many hours to produce an adequate number of antiprotons for collisions. The first superconducting synchrotron in history, the Tevatron has operated as the highest energy collider for approximately 25 years.

Though the bunches in the Tevatron are collided without a crossing angle they are of long enough extent that the luminous region takes on an "hour glass" shape along the direction of motion as the beam is focused toward the interaction point. This leads to its own reduction in luminosity, roughly 40% in this case. Unlike the LHC, the beams in the Tevatron circulate in a single vacuum pipe and thus are placed on separated orbits which wrap around each other in a helical pattern outside of the interaction regions. Hence, long-range encounters play an important role here as well, though the effects can be different than in the LHC where the encounters are more or less “in phase” with each other through a single interaction region. In the Tevatron, the 70 long-range encounters are distributed about the synchrotron and their mitigation is limited by the available aperture.

In recent years the antiproton bunch intensities have approached those of the proton bunches, and their emittances have been greatly reduced using improved beam cooling, so much so that detrimental effects on the proton beam have become apparent. The antiproton beam emittance is now adjusted prior to collision conditions to optimize the proton bunch lifetime during the store. Eq. (25.1) can be re-written as

$$\mathscr{L} = \frac{2 f_0 \gamma}{\beta^* r_0}\, \xi\, \frac{\mathcal{H}}{1 + \bar{\varepsilon}/\varepsilon}\, \overline{N}_{tot} \;\approx 10^{30}\mathrm{cm}^{-2}\mathrm{s}^{-1}\, \left(\overline{N}_{tot}/10^{10}\right)$$

where f_0 is the revolution frequency, $\gamma = E/mc^2$, β^* is the amplitude function at the IP, r_0 is the classical radius of the proton, ξ is the beam-beam tune shift parameter, $\mathcal{H}$ is the hour glass factor, and $\bar{\varepsilon}/\varepsilon$ is the ratio of the emittances of the antiproton to proton beams. After many years the first three factors have become saturated operationally, so that the luminosity depends almost entirely upon the production of antiprotons. In these units, the Tevatron has achieved luminosities of over 350, where its original design luminosity was 1 [12].

25.4.3. ***e^+e^- Rings*** : As should be expected, synchrotron radiation plays a major role in the design and optimization of the e^+e^- colliders. While vacuum stability and electron clouds can be of concern in the positron rings, synchrotron radiation along with the restoration of longitudinal momentum by the RF system have the positive effect of generating very small transverse beam sizes and small momentum spread. Further reduction of beam size at the interaction points using standard beam optics techniques and successfully contending with high beam currents has led to record luminosities in these rings, far exceeding those of hadron colliders. To maximize integrated luminosity the beam can be “topped off” by injecting new particles without removing existing ones – a feature difficult to imitate in hadron colliders.

Asymmetric energies of the two beams have allowed for the enhancement of B-physics research and for interesting interaction region designs. As the bunch spacing can be quite short, the lepton beams sometimes pass through each other at an angle and hence have reduced luminosity. Recently, however, the invention of high frequency “crab crossing” schemes have been successfully tested wherein bunches are rotated at the IP to produce full restoration of the luminous region. KEK-B has attained over 1 fb^{-1} of integrated luminosity in a single day, and its upgrade plans is aiming for initial luminosities of 8×10^{35} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ [13].

25.4.4. ***HERA*** : [14] Now decommissioned, HERA was the first facility to employ both applications of superconductivity: magnets and accelerating structures. Its next-generation cold-iron superconducting magnets for the proton beam were the culmination of lessons learned from the Tevatron experience and extensive development of the technology since then. The HERA team felt comfortable with a larger dynamic range of the magnet system, enabling the use of the existing DESY complex for injection. Though the HERA magnets could reach fields consistent with energies above 1 TeV, other accelerator systems precluded operation above 920 GeV.

The lepton beam (positrons or electrons) were provided by the existing complex, and were accelerated to 27.5 GeV using conventional magnets. The interaction region where the beams had common vacuum chambers had the interesting feature that the lepton beam could be manipulated without detrimental effects on the proton beam due to the large difference in magnetic rigidity. A 4-times higher frequency RF system was used at collision to generate shorter bunches, thus helping alleviate the hour glass effect at the collision points. As in any high energy lepton storage ring, the lepton beam naturally would become transversely polarized (within about 40 minutes, for HERA). “Spin rotators” were implemented on either side of an IP to produce longitudinal polarization at the experiment.

25.4.5. ***RHIC*** : [15] The Relativistic Heavy Ion Collider employs superconducting magnets, and collides combinations of fully-stripped ions such as H-H (p-p), Au-Au, Cu-Cu, and d-Au.

The high charge per particle (+79 for gold, for instance) makes intra-beam scattering of particles within the bunch of special concern, even for seemingly modest bunch intensities. Another special feature of accelerating heavy ions in RHIC is that the beams experience a “transition energy” during acceleration – a point where the derivative with respect to momentum of the revolution period is zero. This is more typical of low-energy accelerators, where the necessary phase jump required of the RF system is implemented rapidly and little time is spent near this condition. In the case of RHIC with heavy ions, the superconducting magnets do not ramp very quickly and the period of time spent crossing transition is long and must be dealt with carefully. For p-p operation the beams are always above their transition energy and so this condition is completely avoided.

RHIC is also distinctive in its ability to accelerate and collide polarized proton beams. As proton beam polarization must be maintained from its low-energy source, successful acceleration through the myriad of depolarizing resonance conditions in high energy circular accelerators has taken years to accomplish. A record energy of 250 GeV per proton with $\sim$35% final polarization per beam has recently been realized.

25.5. Future Prospects

Present design activity emphasizes a lepton collider as the next major HEP project following initial results from the LHC. Synchrotron radiation precludes a higher energy successor to LEP. Four alternatives are noted in this section: two approaches to an electron-positron linear collider, a muon ring collider, and potential use of a plasma as the acceleration medium.

25.5.1. *Electron-Positron Linear Colliders* :

The only linear collider ever operated is the ground-breaking Stanford Linear Collider (SLC), which ran from 1989 until 1998. A major problem confronting a future high energy, high luminosity single pass collider design is the power requirement, so measures must be taken to keep the demand within bounds as illustrated in a transformed Eq. (25.1) as developed in the *TESLA Design Report* [16]:

$$\mathscr{L} = \frac{1}{4\pi r_e^{3/2}} \frac{P_b}{E_{cm}} \left(\frac{\pi \delta_E}{\gamma \varepsilon_y} \right)^{1/2} H_D . \tag{25.6}$$

Here, r_e is the classical electron radius, P_b is the total power of both beams and E_{cm} their cms energy. Management of P_b leads to an upward push on the product of collision frequency and bunch population with an attendant rise in the energy radiated due to the electromagnetic field on one bunch acting on the particles of the other. The fractional spread in the collision energy that results from this radiation is represented by δ_E and keeping a significant fraction of the luminosity within a percent or so of the nominal energy represents a design goal. A consequence is the use of flat beams, where δ_E is managed by the beam width, and luminosity adjusted by the beam height, thus the explicit appearance of the vertical emittance ε_y. The final factor in Eq. (25.6), H_D, represents the enhancement of luminosity due to the pinch effect during bunch passage.

The approach designated the International Linear Collider (ILC) is presented in the Tables, and the contrast with the collision-point parameters of the circular colliders is striking, though reminiscent in direction of those of the SLAC Linear Collider that are no longer shown. The ILC *Reference Design Report* [17] has a baseline of a cms energy of 500 GeV with upgrade provision for 1 TeV, and luminosity comparable to the LHC. The ILC is based on superconducting accelerating structures of the 1.3 GHz TESLA variety.

At CERN, a design effort is underway on the Compact Linear Collider (CLIC), each linac of which is itself a two-beam accelerator, in that a high energy, low current beam is fed by a low energy, high current driver [18]. The CLIC design employs normal conducting 12 GHz accelerating structures at a gradient of 100 MeV/m, some three times the current capability of the superconducting ILC cavities. The design cms energy is 3 TeV.

25.5.2. *Muon Collider* :

The muon to electron mass ratio of 210 implies less concern about synchrotron radiation by a factor of about 2×10^9 and its 1.6 μs lifetime means that it will last for some $150B$ turns in a ring about half of which is occupied by bend magnets with average field B (tesla). Design effort became serious in the mid 1990s and a collider outline emerged quickly [19].

Removal of the synchrotron radiation barrier reduces collider facility scale to a level compatible with on-site placement at some locations. If a Higgs particle is detected the $(m_\mu/m_e)^2$ cross section advantage in s-channel production would be valuable. And a neutrino factory could potentially be realized in the course of construction [20].

The challenges to luminosity achievement were clear and very attractive for R&D: targetting, collection, and emittance reduction are three that come immediately to mind. The proton source will deliver a beam power of several MW; muon collection would be aided by ultra-high magnetic fields, with solenoids to produce them currently under development. The emittance requirements have inspired fascinating investigations into phase space manipulation that are finding application in other facilities. A summary of the status may be found in a recent presentation to the HEPAP P5 Subpanel [21].

25.5.3. *Plasma Acceleration* :

At the 1956 CERN Symposium, a paper by Veksler in which he suggested acceleration of protons to the TeV scale using a bunch of electrons anticipated current interest in plasma acceleration [22]. A half-century later this is more than a suggestion, with the demonstration, as a striking example, of energy enhancement of 28.5 GeV at SLAC [23].

How plasma acceleration will find application in an HEP facility is not yet clear, given the likely necessity of sequential impulses. Active R&D is underway; for recent discussion of parameters for a laser-plasma based electron positron collider, see, for example, relevant papers in the recent Advanced Accelerator Concepts Workshop [24]. In the relatively near-term, there is the likelihood of application outside of HEP in compact multi-GeV accelerators [25].

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