Table of Contents

I. OVERVIEW

This review is an updating through November 1985 of the Review of Particle Properties [Particle Data Group (1984)], a compilation of experimental results on the properties of particles studied in elementary particle physics. These properties include masses, widths or lifetimes, branching ratios, and other experimentally determined properties. Where feasible, we provide a suggested "best" value of each parameter based on our own judgment, using the best available data. A discussion of some of the procedures that we apply, and a brief review of the historical performance of averages of measurements, may be found below (Section IV Part D).

The results of this compilation are presented in two sections, the "Summary Tables of Particle Properties" and the "Full Listings." The Summary Tables give our estimates of the properties of those states whose existence we consider well established. Our opinion of whether or not a particle is well established can change as new data become available. We attempt to be conservative, so particles awaiting confirmation are not included, even if they may be theoretically well understood.

All data used for the numerical estimates in the Summary Tables are included in the Full Listings, with references and our comments, if any. Those measurements considered recent enough or important enough to mention, but which for some reason were not used in the averaging, appear in parentheses. The Full Listings also contain information on unconfirmed particles and unsuccessful particle searches, as well as short "mini-reviews" about subjects of particular interest or data that have particular problems.

In the past, we have attempted to use the Full Listings as an archive of all reported data on particles of interest. This is no longer possible because the growth of information would require a 5 to 10% per year expansion in this Review. Therefore we refer interested readers to previous editions for references to data considered obsolete.

This edition we are implementing our new particle naming conventions [Barnett (1985) and Wohl (1984)], which primarily affect meson names. A few baryon states are renamed as well. In the Summary Tables of Particle Properties and the Full Listings each particle is listed by its new name, with the old name, if different, given below it. It is our hope that these new conventions, described in Section III below, if adopted by the community will bring order to the chaos of particle names and facilitate discussion and understanding. Since there will doubtless be a transition period during which the literature may contain a mixture of both old and new names, we will continue to list the old names with the new for several editions.

We categorize the particles into types, intended to correspond roughly to the different types of data and problems encountered:

STABLE PARTICLES — All particles stable under the strong interaction. These include the truly stable particles as well as those which decay weakly or electromagnetically, including the η, D, D_s (formerly called the F), Λ_c, W, Z^0, and so on.

MESONS — All meson resonances that decay strongly, including the ψ, χ, and Υ families.

BARYONS — All baryon resonances that decay strongly, including the resonant N and Δ families, dibaryon candidates, and so on.

This classification scheme is used to organize the Summary Tables and the Full Listings.

We include a section of "Miscellaneous Tables, Figures, and Formulae." These provide a quick reference for the practicing elementary particle physicist. They normally presuppose some understanding of the subject matter, and do not attempt to serve as a textbook. We welcome all suggestions and comments regarding topics for inclusion or deletion, any errors or confusing passages, etc.

A pocket-sized Particle Properties Data Booklet is available. This contains the complete Summary Tables of Particle Properties and the most frequently used parts of the Miscellaneous Section, but not the Full Listings. For North and South America, Australia, and the Far East, write to Technical Information Department, Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA. For all other areas, write to CERN Scientific Information Service, CH-1211 Geneva 23, Switzerland.

In 1984 we began a multiyear effort aimed at modernization and reorganization. In this edition, we have added Greek letters and larger fonts for headings in the Full Listings, as well as numerous more minor improvements in format. We are also modernizing our internal procedures, and some of these improvements are already in place.

II. AUTHORS AND CONSULTANTS

The primary responsibilities of the authors are as follows:

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(2) *Meson resonances*: M. Aguilar-Benitez, J.J. Hernandez, L. Montanet, F.C. Porter, M. Roos, K.R. Schubert, and N.A. Törnqvist.

(3) *Baryon resonances*: R.L. Crawford, G.P. Gopal, G. Höhler, D.M. Manley, L.D. Roper, and C.G. Wohl.

Consultants

Of increasing importance to the production of this Review is a world-wide network of consultants, experts in particular topics. We wish to mention the following people with thanks:

- R.A. Arndt (Virginia Polytechnic Inst. and State Univ.)
- W.B. Atwood (SLAC)
- V.I. Balbekov (Serpukhov)
- A. Baldini (University of Pisa)
- M.J. Berger (U.S. National Bureau of Standards)
- A. Bramon (Barcelona University)
- E. Browne (LBL)
- R.N. Cahn (LBL)
- W. Carithers (LBL)
- J. Carr (University of Colorado)
- COMPAS Group (IHEP, Serpukhov)
- S. Cooper (SLAC)
- F. Dydak (CERN)
- V.V. Ezhela (Serpukhov)
- G. Feldman (SLAC)
- V. Flaminio (University of Pisa)
- J.-M. Gaillard (CERN)
- M.K. Gaillard (LBL)
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- G. Hall (Imperial College, London)
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- J.H. Hubbell (U.S. National Bureau of Standards)
- K. Kleinknecht (Universität Dortmund)
- P. Langacker (University of Pennsylvania)
- G.M. Lewis (University of Glasgow)
- M.J. Losty (National Research Council, Canada)
- B. Lynn (SLAC)
- W.G. Moorhead (CERN)
- D.R.O. Morrison (CERN)
- K. Mursula (Nordita, Copenhagen)
- K. Olive (University of Minnesota)
- O.E. Overseth (University of Michigan)
- S.I. Parker (University of Hawaii)
- R. Partridge (Santa Cruz Institute for Particle Physics)
- N. Rivoire (CERN)
- S. Rudaz (University of Minnesota)
- F. Scheck (Universität Mainz)
- M. Shaevitz (Nevis Laboratory)
- M. Suzuki (LBL)
- B.N. Taylor (U.S. National Bureau of Standards)
- J.A. Thompson (University of Pittsburgh)
- W. Toki (SLAC)
- G.H. Trilling (LBL)
- R.D. Tripp (LBL)
- R. Waldi (Universität Heidelberg)
- K. Winter (CERN)
- L. Wolfenstein (Carnegie-Mellon University)

In addition, the Berkeley Particle Data Group has benefited from the advice of the PDG Advisory Committee, which meets annually to discuss matters of importance to the group, including the structure and content of this Review. The members of the 1985 committee are L. Wolfenstein (Carnegie-Mellon University) (chair), A. Kernan (University of California, Riverside), C.M. Lederer (University of California, Berkeley), C. Quigg (Fermilab), and R. Thun (University of Michigan).

The usefulness of this compilation depends in large part on interaction between the users and the authors and consultants. We appreciate comments, criticisms, and suggestions for improvements of all stages of data retrieval, evaluation, and presentation.

III. A NEW NAMING SCHEME FOR HADRONS

"Young man, if I could remember the names of these particles, I would have been a botanist."
Enrico Fermi

A. The need for a new scheme; guiding principles

We introduce in this edition a new naming scheme for the hadrons. Anyone who doubts the desirability of a better naming scheme is invited to give, without looking at the Meson Summary Table, the quantum numbers I, J, P, C, and G of the following established nonstrange mesons:

$S(975)$, $\delta(980)$, $H(1190)$, $B(1235)$, $D(1285)$, $\epsilon(1300)$, $\pi(1300)$, $E(1420)$, $\iota(1440)$, $\rho(1600)$, $\omega(1670)$, $A(1680)$, $\phi(1680)$, $g(1690)$, $\theta(1690)$, $h(2030)$.

There is no rhyme or reason to this alphabet soup of symbols – they convey nothing about the properties of the particles they name. Nor is the use of five different symbols, K, K^*, Q, L, and κ, to name just nine strange mesons informative or economical. The symbols for mesons containing heavy quarks and for ordinary baryons are fairly sensible, but it seems wise, while in the grip of reformist zeal, to make some rules regarding names of particles yet to be discovered, such as the whole spectrum of baryons containing one or more heavy quarks.

There are several obvious virtues any rational naming scheme ought to embody. The symbols ought to be as few and as simple as possible, with those already in common use retained where possible; the symbols ought to convey unambiguously the important quantum numbers of the particles they name; and the quark model ought to guide the whole scheme. There are, however, constraints: it is not practical, for example, to now rename the ϕ meson the ω', or to call the K meson containing an s quark (as opposed to an $\bar{s}$) a K instead of a $\bar{K}$. Some compromise between simplicity and long-established usage is unavoidable.

The new scheme adopted here has evolved over the last two years in response to much discussion both within the Particle Data Group and with the larger community. Preliminary versions of the scheme were presented at the 1984 Santa Fe Meeting of the Division of Particles and Fields [Wohl (1984)] and at the 1985 International Conference on Hadron Spectroscopy [Barnett (1985)]. Several thousand copies of the proposal, with an invitation to comment, were distributed in the spring of 1985. A *Physics Today* news report discussed the proposal [Schwarzschild (1985)], and it has been discussed in the CERN *Courier* (November 1985).

As indicated above, many of the mesons have been renamed. The Meson Summary Table in this edition gives both the new and old names, and a table of equivalent names will appear in foreseeable editions. Only two particles in the Stable Particle and Baryon Summary Tables are renamed (the F and the A^+ become the D_s and the Ξ_c^+).

B. "Neutral-flavor" mesons ($S = C = B = T = 0$)

Table I shows the naming scheme for mesons having the strangeness and all heavy-flavor quantum numbers equal to zero. The mesons are assumed to be quark-antiquark states. The rows of the table give the possible $q\bar{q}$ content. The columns give the possible parity/charge-conjugation states, PC = $-+$, $+-$, $--$, and $++$; these combinations correspond one-to-one with the angular-momentum state $^{2S+1}L_J$ of the $q\bar{q}$ system being $^1(L \text{ even})_J$, $^1(L \text{ odd})_J$, $^3(L \text{ even})_J$, or $^3(L \text{ odd})_J$.* In addition, the spin J is added to the main symbol as a subscript except for pseudoscalar and vector mesons ($L=0$ states in the quark model), and the mass is given for any meson that decays strongly.

Experimental determination of the mass, quark content, and quantum numbers I, J, P, and C (or G) of a meson thus fixes its symbol. Conversely, these properties may be inferred unambiguously from the symbol.

If the main symbol cannot be assigned, because the quantum numbers (other than J) are unknown, the symbol X is used temporarily. Sometimes it is not known whether a meson is mainly the isospin-0 mix of $u\bar{u}$ and $d\bar{d}$ or is mainly $s\bar{s}$; the prime (or symbol ϕ) may be used to distinguish two such mixing states.

Names have been assigned for the anticipated $t\bar{t}$ mesons. No suggestion is made here for names for mesons (should any be found) with the "exotic" quantum numbers that a $q\bar{q}$ system cannot have, namely $J^{PC} = 0^{--}$, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, $\cdots$. Glueballs or other mesons that are not $q\bar{q}$ states would (if the quantum numbers are *not* exotic) be named just as if they were $q\bar{q}$ states, since they will probably be difficult to distinguish from such states and will likely mix with them.

The results of all this are as follows. None of the lowest

Table I. Symbols for mesons with the strangeness and all heavy-flavor quantum numbers equal to zero.

	$J^{PC} =$	0^{-+}	1^{+-}	1^{--}	0^{++}
		2^{-+}	3^{+-}	2^{--}	1^{++}
		⋮	⋮	⋮	⋮
$q\bar{q}$ content	$^{2S+1}L_J =$	$^1(L \text{ even})_J$	$^1(L \text{ odd})_J$	$^3(L \text{ even})_J$	$^3(L \text{ odd})_J$
$u\bar{d}$, $d\bar{d}-u\bar{u}$, $d\bar{u}$ ($I=1$)		π	b	ρ	a
$d\bar{d}+u\bar{u}$ and/or $s\bar{s}$ ($I=0$)		η,η'	h,h'	ω,ϕ	f,f'
$c\bar{c}$		η_c	h_c	ψ †	χ
$b\bar{b}$		η_b	h_b	Υ	χ_b
$t\bar{t}$		η_t	h_t	θ	χ_t

†The J/ψ remains the J/ψ.

* The relations between the quantum numbers are
$P=(-1)^{L+1}$ $C=(-1)^{L+S}$ $G=(-1)^{L+S+I}$,
where of course the C quantum number (charge conjugation) is only relevant to charge-zero mesons.

mass pseudoscalar or vector mesons (π, η, and η'; ρ, ω, and ϕ) change names, nor do any of the $c\bar{c}$ or $b\bar{b}$ mesons. Established mesons whose names change slightly are:

Old name	New name	Old name	New name
$H(1190)$	$h_1(1190)$	$A_2(1320)$	$a_2(1320)$
$B(1235)$	$b_1(1235)$	$f'(1525)$	$f_2'(1525)$
$f(1270)$	$f_2(1270)$	$\omega(1670)$	$\omega_3(1670)$
$A_1(1270)$	$a_1(1270)$	$\phi(1850)$	$\phi_J(1850)$

Established mesons whose names change completely are:

Old name	New name	Old name	New name
$S(975)$	$f_0(975)$	$\iota(1440)$	$\eta(1440)$
$\delta(980)$	$a_0(980)$	$A_3(1680)$	$\pi_2(1680)$
$D(1285)$	$f_1(1285)$	$g(1690)$	$\rho_3(1690)$
$\epsilon(1300)$	$f_0(1300)$	$\theta(1690)$	$f_2(1720)$
$E(1420)$	$f_1(1420)$	$h(2030)$	$f_4(2030)$

The $S(975)$, $D(1285)$, $\epsilon(1300)$, $E(1420)$, $\theta(1690)$, and $h(2030)$ all become f mesons; the new scheme reveals that all have PC=++ and are $^3(L \text{ odd})_J$ states.

C. Mesons with nonzero S, C, B, and/or T

Since the strangeness or a heavy flavor is nonzero, none of the mesons here are eigenstates of charge conjugation, and in each of them one of the quarks must be heavier than the other. The rules are:

(1) The main symbol is an upper-case Roman letter indicating the heavier quark as follows:‡
$s \to K \quad c \to D \quad b \to B \quad t \to T$.
(2) If the lighter quark is not a u or a d quark, its identity is given by a subscript.
(3) If the spin-parity is in the "normal" series $J^P = 0^+, 1^-, 2^+, \cdots$, a superscript "*" is added.
(4) The spin is added as a subscript unless the meson is a pseudoscalar or a vector ($L=0$ states in the quark model).

Thus the pseudoscalar and vector K, K^*, D, D^*, and B mesons do not change names. Established mesons whose names do change are:

Old name	New name	Old name	New name
$Q_1(1280)$	$K_1(1280)$	$L(1770)$	$K_2(1770)$
$\kappa(1350)$	$K_0^*(1350)$	$K^*(1780)$	$K_3^*(1780)$
$Q_2(1400)$	$K_1(1400)$	$K^*(2060)$	$K_4^*(2060)$
$K^*(1430)$	$K_2^*(1430)$	F	D_s

‡Two different conventions exist in the literature for the sign of the flavor of b quarks. We have adopted the convention that *the sign of the flavor of a quark is the same sign as its charge*, which is true for all flavors. Thus the strangeness of the s quark is negative, the charm of the c quark is positive, and the bottom of the b quark is negative. In addition, I_3 of the u and d quarks is positive and negative, respectively. The effect of this convention is as follows: Any *flavor* carried by a *charged* meson has the *same* sign as its *charge*. Thus the K^+, D^+, and B^+, have positive strangeness, charm, and bottom, respectively, and all have positive I_3. The D_s^+ (formerly the F^+) has positive charm and strangeness. Furthermore, the $\Delta(\text{flavor}) = \Delta Q$ rule, which is best known for the kaons, applies to every flavor.

The most notable change is that of the F (the $c\bar{s}$ state) to a D_s. However, with the prospect of B_s, B_c, T_s, and similar mesons, there is no consistent and economical alternative. The rules can lead to cumbersome symbols, such as a D_{s2}^*, but such particles are unlikely to be often seen.

D. Baryons

The symbols N, Δ, Λ, Σ, Ξ, and Ω have been used for 20 years for the baryons made of light quarks (u, d, and s quarks), and no change is made to these symbols here. They tell the isospin and quark content, and the same information ought to be conveyed by the symbols used for the baryons containing one or more heavy quarks (c, b, and t quarks). The following system was invented earlier and independently by Hendry and Lichtenberg (1978) and by Samios (1980). The rules are (see also Fig. 1):

(1) Baryons with *three* u and/or d quarks are N's (isospin 1/2) or Δ's (isospin 3/2).
(2) Baryons with *two* u and/or d quarks are Λ's (isospin 0) or Σ's (isospin 1). If the third quark is a heavy quark (not an s quark) its identity is given by a subscript. This nomenclature is already used for the $\Lambda_c(2281)$, $\Sigma_c(2450)$, and $\Lambda_b(5500)$.
(3) Baryons with *one* u or d quark are Ξ's (isospin 1/2). One or two subscripts are used if one or both of the remaining quarks are heavy: thus Ξ_c, Ξ_{cc}, Ξ_b, etc. The possible but not established $A(2460)$ is renamed the $\Xi_c(2460)$.
(4) Baryons with *no* u or d quarks are Ω's (isospin 0), and subscripts indicate any heavy-quark content. The possible but not established $T(2740)$ is renamed the $\Omega_c(2740)$.

In short, the total number of u and d quarks together with the isospin determine the main symbol, and subscripts indicate any content of heavy quarks. A Σ always has isospin 1, an Ω always has isospin 0, etc. The only baryons whose names change are the A and the T.

Note in Fig. 1 that the SU(4) 20-plet that contains the basic SU(3) octet has an Ω_c and an Ω_{cc} although it has no Ω. It has *two* Ξ_c's, which would be distinguished by mass (they might also be distinguished by a prime on the heavier of the two).

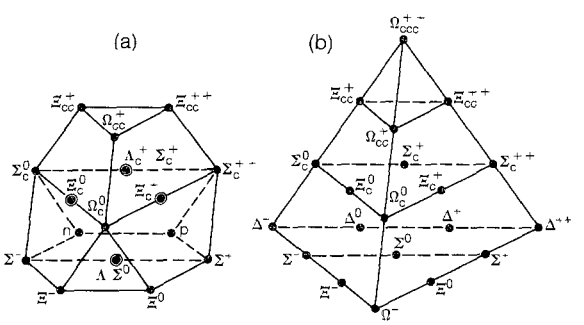

Fig. 1. SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

IV. PROCEDURES

A. Selection and treatment of data

The Full Listings contain a complete record of all *relevant* data known to us. As a general rule, we do not include results from preprints or conference reports. It is our experience that preprinted results often change before publication. In some cases, such results may be cited but not used in computing the estimates given in the Summary Tables. There are a few exceptions to this exclusion, which we decide on a case-by-case basis after consultation with the experimenters.

As mentioned earlier, we no longer attempt to maintain an archival record of data of historical importance only. We do, however, quote the references of discoveries, even when the data are no longer useful.

If data are included in the Full Listings but not used in the final average given in the Summary Tables, they are enclosed in parentheses. We give explanatory comments in many such cases. If no comment is given, the reason the data were excluded is one or more of the following:

- The data are superseded or included in later results.
- No error was given.
- The data were contained in a preprint or conference report.
- The result involves some assumptions we do not wish to incorporate.
- The measurement has poor signal-to-noise ratio, low statistical significance, or is otherwise of much poorer quality than other data available.
- The measurement is clearly inconsistent with other results which appear to be highly reliable (see discussion in Section IV Part D below).
- The measurement is not independent of other results, e.g., it is from one of several partial-wave analyses, all of which use the same data, rendering averaging meaningless.

In some cases, *none* of the measurements is entirely reliable and no statistically meaningful average is quoted. For example, the masses of many of the baryon resonances, obtained from partial-wave analyses, are quoted as a range thought to probably include the true value rather than as an average with error. This is discussed in more detail in some of the mini-reviews in the Baryon Full Listings.

For upper limits, we normally quote in the Summary Tables the strongest limit available from a single experiment. We do not average or combine upper limits except in a very few cases where they may be re-expressed as measured numbers with Gaussian errors.

For quantum number assignments, we indicate in the Summary Tables those which are either well established or probable. In the Meson Summary Table, we underline those we consider well established; the others are inferred from whatever experimental evidence is available. In the Stable Particle Summary Table, nearly all quantum numbers are well established and we do not underline; those which are not well established are indicated by a footnote.

As is customary, we assume that antiparticles are the result of operating with *CPT* on particles, so both share the same spins, masses, and mean lives. There is an entry in the Miscellaneous Section, Tests of Conservation Laws, listing tests of *CPT* and other conservation laws.

B. Criteria for new states

An experimentalist who sees indications of a new state will of course want to know what has been seen in that region in the past. Hence, we include in the Full Listings all reported states which have not been, in our opinion, disproved by better (e.g., more reliable) data.

For the Summary Tables we are much more conservative. We include only those reported states which we feel have a large chance of survival. One's betting odds for survival are of course subjective; therefore no precise criteria can be defined. For more detailed discussions, see the mini-reviews in the Full Listings. In what follows we shall attempt to specify some guidelines.

(a) When energy-independent partial-wave analyses are available (mostly for πN resonances), approximate Breit-Wigner behavior of the amplitude appears to us to be the most satisfactory test for a resonance. We can check that the Argand plot follows roughly a left-hand circle, and that the "speed" of the amplitude also shows a maximum near the resonance energy; further, there should be data well above the resonance, showing that the speed again decreases. Indeed, proper behavior of the partial-wave amplitude often establishes a resonance even if its elasticity is too small to make a noticeable peak in the cross section.

(b) When there are insufficient data to perform energy-independent analyses, one often resorts to energy-dependent partial-wave analyses. In this case Breit-Wigner behavior is an input. We usually require that resonance solutions be found by several different analyses, preferably in different channels ($\bar{K}N \to \bar{K}N$, $\pi\Sigma$, etc.), before putting the claim in the Summary Tables.

(c) Stable particles, most meson resonances, Ξ resonances, and high-mass N, Δ, Λ, and Σ resonances fall into a category for which no partial-wave analyses exist. In general, we accept such states if they are experimentally reliable, of high statistical significance (4.5σ or better), or observed in several different production processes.

(d) Partial-wave analyses of three-body final states ($\pi N \to \pi\pi N$) are also available. While these analyses are based on the isobar model ($\pi N \to \rho N$, $\pi\Delta$, etc.) and are subject to theoretical objections of varying importance, they provide increasingly reliable information on inelastic decay modes of otherwise-established resonances.

C. Statistical Procedures

We divide this discussion on obtaining averages and errors into two sections:

1. The unconstrained case, or "simple averaging;" and
2. The constrained case.

In what follows, the term "error" means one standard deviation (1σ); that is, for central value $\overline{x}$ and error $\delta\overline{x}$, the range $\overline{x} \pm \delta\overline{x}$ constitutes a 68.3% confidence interval.

1. Unconstrained averaging

We use a standard Gaussian procedure with a "scale factor" applied to the errors as our method of averaging the data. The Student's t-distribution, the basis of an earlier experiment of ours in data averaging, would give more conservative (and perhaps more realistic) errors at the two-standard-deviation (2σ) and higher level, but we do not choose to quote such errors. It is worth bearing in mind,

however, that a 2σ error might more realistically be somewhat larger than twice a 1σ error, owing to the non-Gaussian character of some sets of real measurements. This is a persistent problem in data averaging arising from the existence of mildly discrepant measurements.

We begin by assuming that measurements of a given quantity are uncorrelated, and thus we calculate a weighted average and error

$$\bar{x} \pm \delta\bar{x} = \left(\sum_i w_i x_i / \sum_i w_i\right) \pm \left(\sum_i w_i\right)^{-1/2},$$

$$w_i = \left[1/(\delta x_i)^2\right], \tag{1}$$

where x_i and δx_i are the value and error, respectively, reported by the ith experiment, and the sums run over N experiments. We also calculate χ^2 and compare it with its expectation value; assuming that the measurements obey a Gaussian distribution, this is $N - 1$.

If $\chi^2/(N-1)$ is less than or equal to 1, and there are no known problems with the data, we accept the above results.

If $\chi^2/(N-1)$ is very large, or if there is prior knowledge of extremely large inconsistencies among experiments, we may choose not to average the data at all. Alternatively, we may quote the calculated average, but then give an educated guess as to the error; such a guess is generally a quite conservative estimate designed to take into account known problems with the data.

Finally, if $\chi^2/(N-1)$ is greater than 1, but not greatly so, we still average the data, but then also do the following:

(a) We plot an ideogram to display the pattern of the data. Sometimes only one or two data points lie apart from the main body; other times the data split into two or more roughly equal-sized groups. The reader may use this information in deciding upon an alternative average, but caution is urged, as "outlying" data points are sometimes the "correct" ones. An example of such an ideogram is given in Fig. 2 below. Each experiment appearing in the plot is

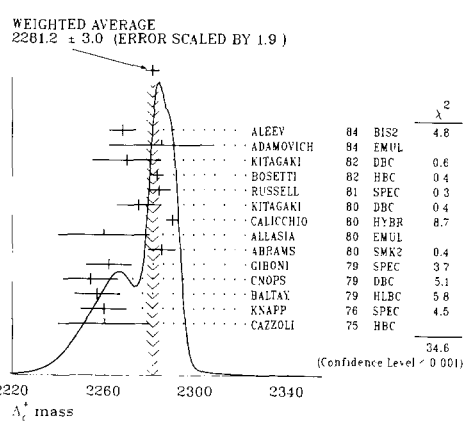

Fig. 2. Ideogram of measurements of the Λ_c^+ mass. The "data point" at the top shows the position of the weighted average, while the width of the error bar (and the shaded pattern beneath it) shows the error in the average after scaling by the SCALE factor. Only those experiments indicated by + error flags were precise enough to be accepted in the calculation of the SCALE factor; the column on the far right gives the χ^2 contribution of each of these experiments. The less precise experiments were included in the calculation of the weighted average, but not SCALE; they have $\perp$ error flags.

represented by a Gaussian with central value x_i, error δx_i, and area proportional to $1/\delta x_i$. The choice of area is somewhat arbitrary; it assumes that an experimenter will work to reduce the systematic errors until they are slightly smaller (but seldom much smaller) than the statistical errors. Thus, as a physicist collects more events, he or she will use them both to reduce the statistical errors and to study the biases. Our confidence that a significant systematic error has not been made in a given experiment, as compared with other contradictory experiments, then tends to go up as $1/\delta x_i$.

But why not assign a weight $1/(\delta x_i)^2$, as is done when computing a weighted average? We feel that this assignment is equivalent to assuming that large systematic errors are as infrequent as large statistical fluctuations, and that this assumption is unrealistic.

We emphasize the difference between least-squares averaging (where the weighting factor is the inverse square of the error) and the ideograms prepared for visual display. The former arithmetic is of course best if one has unbiased data whose errors are well understood. In particular, the error analysis assumes that the true error on each datum is sampled from a Gaussian whose width is correctly reported. Then we obtain a narrow Gaussian distribution centered at the weighted mean for the answer. The ideogram (often multipeaked and certainly not Gaussian) is based on the opposite hypothesis that some of the input is systematically in error. The idea behind least-squares averaging is that experiments 1, 2, 3, etc., are *all* valid (so we should multiply their probabilities). Our *ideograms* are based on the assumption that 1 *or* 2 *or* 3, etc., is valid, "hedged" with $1/\delta x_i$ betting odds; we then add their probabilities. Both approaches cannot simultaneously be right; we allow the reader to choose. However, we quote the least-squares result in the Summary Tables. This is the most precise value if the data satisfy the appropriate assumptions. A glance at the ideogram will show that the difference between the two approaches is usually not severe.

(b) The second way in which we try to take account of $\chi^2/(N-1)$ being greater than 1 is to scale up our quoted error $\delta\bar{x}$ in Eq. (1) by a factor

$$\text{SCALE} = \left[\chi^2/(N-1)\right]^{1/2}. \tag{2}$$

Our reasoning is as follows. Since we do not know which of the experiments are more than one standard deviation away from the correct value, we assume that all experimentalists underestimated their errors by the same scale factor (2). If we scale up all input errors by this factor, χ^2 becomes $N - 1$, and of course the output error scales up by the same factor.

If we are to combine experiments with widely varying errors, we modify this procedure slightly. This is because it is the more precise experiments that most influence not only the average value $\bar{x}$, but also the error $\delta\bar{x}$. Now, on the average, the low-precision experiments each contribute about unity to both the numerator and the denominator of SCALE, hence the χ^2 contribution of the sensitive experiments is diluted, i.e., reduced. Therefore, we evaluate SCALE by using *only* experiments for which the errors are not much greater than those of the more precise experiments, i.e., only those experiments with errors less than δ_0, where the ceiling δ_0 is (arbitrarily) chosen to be

$$\delta_0 = 3N^{1/2}\delta\bar{x}.$$

Here $\delta\bar{x}$ is the unscaled error of the mean of all the experiments. Note that if each experiment had the same error δx_i, then $\delta\bar{x}$ would be $\delta x_i/N^{1/2}$, so each individual experiment would be well under the ceiling on SCALE.

This scaling approach has the property that if there are two values with comparable errors separated by much more than their stated errors (with or without a number of other experiments of lower accuracy), the error on the mean value $\delta\bar{x}$ is increased so that it is approximately half the interval between the two discrepant values.

We wish to emphasize the fact that our scaling procedures for *errors* in no way affect central values. In addition, if one wishes to recover the unscaled error $\delta\bar{x}$, one need only divide the given error by the SCALE factor for that error.

2. Systematic errors and correlated measurements

Many experimental groups have now adopted the convention of presenting results with statistical and systematic errors explicitly indicated. Because of a lack of space in the Full Listings, we usually do not quote the two errors separately and at present we combine them.[§] In general we add the statistical and the systematic errors in quadrature. A comment is printed whenever this is done. When averages are calculated as described above in Eq. (1), the weight w_i is based upon the combined error.

It may happen that two measurements have correlated errors. For example, a group may improve the statistical or systematic errors by further data-taking or analysis. In this case we use only the improved result for averaging. The earlier result may still appear in the Listings (in parentheses), but in general we omit such obsolete entries.

A second case of correlated measurements is the occurrence of a common systematic error in experiments which are statistically independent. If two results $A_1 \pm \sigma_1 \pm S$ and $A_2 \pm \sigma_2 \pm S$ have completely correlated systematic errors S, one must first average $A_1 \pm \sigma_1$ and $A_2 \pm \sigma_2$ and then combine the resulting statistical error with S. One obtains, however, the same result by a second procedure, averaging $A_1 \pm X\cdot\sigma_1$ and $A_2 \pm X\cdot\sigma_2$ where

$$X = \left[1 + S^2/\sigma_1^2 + S^2/\sigma_2^2\right]^{1/2}. \tag{3}$$

The second procedure has the advantage that the modified entries $A_i \pm X\cdot\sigma_i$ may be averaged with further independent data as in Eq. (1). We therefore adopt this second procedure when appropriate.

3. Constrained fits

Except for trivial cases, all branching ratios and rate measurements are analyzed by making a simultaneous least-squares fit to all the data and extracting the partial decay fractions P_i, the width Γ, the partial widths Γ_i, and the associated error matrix.

Assume, for a simple example, that a state has only three partial decay fractions, P_1, P_2, and P_3 ($\sum P_i = 1$), which have been measured in four different ratios, $R_1, \cdots, R_4$, where, e.g., $R_1 = P_1/P_2$, $R_2 = P_1/P_3$, etc.[**] Further assume that *each* ratio r has been measured by N_r experiments (we designate each experiment with a subscript x, e.g., R_{1x}). We then find the best values of P_1, P_2, and P_3 by minimizing χ^2:

$$\chi^2 = \sum_{r=1}^{4}\left[\sum_{x=1}^{N_r}\left(\frac{R_{rx} - R_r(P_1,P_2,P_3)}{\delta R_{rx}}\right)^2\right]. \tag{3}$$

In addition to the fitted values $\bar{P}_i$, we calculate an error matrix $\langle\delta\bar{P}_i\delta\bar{P}_j\rangle$. We tabulate the diagonal elements of $\delta\bar{P}_i = \langle\delta\bar{P}_i\delta\bar{P}_i\rangle^{1/2}$ (except that some errors are scaled as discussed below). In the Full Listings we give the complete error matrix; we also calculate the fitted value of each ratio, for comparison with the input data, and list it below the relevant input, along with a simple unconstrained average of the same input.

Two further comments on the example above:

(1) There was no connection between measurements of the width and the branching ratios. But often we also have information on partial widths Γ_i as well as total width Γ. In this case we must introduce Γ as a parameter into the fit, along with the relations $\Gamma_i = \Gamma P_i$, $\sum\Gamma_i = \Gamma$. When appropriate, we tabulate the Γ_i along with the P_i, and give error matrices in the Full Listings.

(2) We do *not* allow for correlations between input data. We *do* try to pick those ratios and widths which are as independent and as close to the original data as possible.

For *asymmetric* errors, we use a continuous function of $\delta(P)^+$ and $\delta(P)^-$ in the fitting. When no errors are reported, we merely list the data for inspection.

Inconsistent constrained data. According to Eq. (3), the double sum for χ^2 is first summed over experiments $x = 1$ to N_r, leaving a single sum over ratios

$$\chi^2 = \sum_r \chi_r^2.$$

We test for SCALE factors after the fit. Knowing the fitted χ_r^2 and its expectation value $\langle\chi_r^2\rangle$, we form SCALE factors (just as before), i.e.,

$$(\mathrm{SCALE})_r^2 = \chi_r^2/\langle\chi_r^2\rangle,$$

and if any $(\mathrm{SCALE})_r$ is greater than 1, all N_r of the measurements of that particular ratio are equally penalized by having their errors increased by $(\mathrm{SCALE})_r$. We then recycle the full fit, yielding new values $\delta\bar{P}_i'$ for the errors in the partial decay modes, as well as new central values $\bar{P}_i'$.

Because of the constraint ($\sum P_i = 1$), some of the new SCALE factors may still be greater than 1. If this is so, the whole procedure (i.e., increasing errors by the new SCALE factors and recycling through the fit) is repeated until the process converges.

At the end, we have final estimated errors $\delta\bar{P}_i'$ for the $\bar{P}_i'$. If SCALE factors have been used, they normally will have caused a shift in the central fitted values $\bar{P}_i'$, as well as having given larger errors $\delta\bar{P}_i'$. Often we find that the shift $|\bar{P}_i - \bar{P}_i'|$ due to the SCALE factors is the same size as (or

[§] We are considering a revision of the format of the Full Listings which would allow separation of these types of error, and also allow presentation of asymmetric errors.

[**] We can handle any ratio R of the form $\sum\alpha_i P_i/\sum\beta_i P_i$, where α_i and β_i are constants, usually 1 or 0. The forms $R = P_i\cdot P_j$ and $R = (P_i\cdot P_j)^{1/2}$ are also allowed.

greater than) the $\delta\overline{P}_i'$. We have decided to incorporate this shift into our errors as a reflection of the uncertainty due to the introduction of the SCALE factor; we tabulate an error

$$(\delta\overline{P}_i)_{\text{tab}} = [(\delta\overline{P}_i')^2 + (\overline{P}_i - \overline{P}_i')^2]^{1/2} ,$$

where $\overline{P}_i$ is the fitted value of the ith partial decay mode before scaling, $\overline{P}_i'$ is its value after all scaling, and $\delta\overline{P}_i'$ is the error in $\overline{P}_i'$. The SCALE factors we finally list in such cases are defined by

$$(\text{SCALE})_i = (\delta\overline{P}_i)_{\text{tab}}/\delta\overline{P}_i .$$

However, in line with our policy of not letting SCALE affect the central values, we quote the values of $\overline{P}_i$ obtained from the original (unscaled) fit [which are always less than or equal to one standard deviation from $\overline{P}_i'$, by construction of $(\delta\overline{P}_i)_{\text{tab}}$].

D. Discussion

The entire question of averaging data containing discrepant values is nicely discussed by Taylor (1982). He considers a number of algorithms which attempt to incorporate data which are not completely consistent into a meaningful average. Problems occur because it is very difficult to develop a procedure which handles simultaneously in a reasonable way two basic types of situations: (a) data which seem to lie apart from the main body of the data are incorrect (contain unreported errors); and (b) the opposite (the main body of the data is systematically wrong). Unfortunately, as Taylor shows, case (b) is not infrequent. His conclusion is that the choice of procedure is less significant than the initial choice of data to include or exclude.

We place a great emphasis on the choice of data to include or exclude. Unfortunately, the volume of data precludes spending as much time on the problem as we would like. We address this problem by soliciting the help of as many outside experts (consultants) as possible. In the final analysis, however, it is often impossible to determine which (if either) of two discrepant measurements is correct. Our SCALE factor technique is an attempt to address this ignorance by increasing the error above that suggested by least-squares analysis. In effect, we are saying that present experiments do not allow a precise determination of this constant because of unresolvable discrepancies, and one must await further measurements. The reader is warned of this situation by the size of the SCALE factor; he or she is then able to go back to the literature (via the Full Listings) and redo the average as desired.

Our situation with regard to discrepant data is easier to handle than most of the cases Taylor considers, such as estimates of the fundamental constants like $\hbar$, etc. Most of the errors in his case are dominated by systematic effects. In particle properties data, statistical effects are often at least as large as systematic effects, and statistical errors are usually easier to estimate. A notable exception occurs in partial-wave analyses, where different techniques applied to the same data yield different results. In this case, as stated earlier, we often do not attempt an average, but just quote a range of values.

A brief history of Particle Data Group averages is given in Rosenfeld (1975). Updated versions of some of Rosenfeld's figures are shown in Fig. 3. The least-squares error is shown by the thick portion of the error bars; the full error bar exhibits the SCALE factor extension.

Some cases of rather wild fluctuation are shown; this usually represents the introduction of significant new data or the discarding of some older data. Older data are sometimes discarded in favor of more modern data if it is felt that the newer data had fewer systematic errors, had more checks on their systematic errors, made some corrections unknown at the time of the older experiments, or some such reason. Near the time at which a large jump takes place, the SCALE factor sometimes becomes large, reflecting the uncertainty introduced by the new existence of partly inconsistent data.

By and large, a full scan of our history plots shows a rather dull progression toward greater precision at a central value completely consistent with the first data point shown. These plots are available on request from the Berkeley Particle Data Group.

We conclude that the reliability of the combination of experimental data and Particle Data Group averaging procedures is usually good, but it is important to realize that fluctuations outside of the quoted errors can and do occur, perhaps with more frequency than expected for truly Gaussian errors.

ACKNOWLEDGMENTS

We thank all those who have assisted in the many phases of preparing this Review. In particular, we acknowledge the usefulness of feedback from the physics community, especially those who have made suggestions or pointed out errors.

The European members of the Particle Data Group wish to acknowledge the generous support of CERN, in particular Division EP.

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