16. PROBABILITY

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The following is a much shortened version of Sec. 16 of the full *Review*. (Equation, section, and figure numbers follow the *Review*.)

16.1. General [1–5]

- *Probability density function* (p.d.f.): x or x_i is a *random variable*.
  Continuous: $f(x;\theta)dx$ = probability x is between x to $x+dx$, given parameter(s) θ;
  Discrete: $f(x_i;\theta)$ = probability x equal to x_i, given θ.
- *Cumulative distribution function*:

Continuous: $$F(a) = \int_{-\infty}^{a} f(x;\theta)\,dx \; ; \tag{16.1}$$

Discrete: $$F(a) = \sum_{i=1}^{n} f(x_i;\theta) \; , \; x_1 \le x_2 \le \ldots \le x_n \le a \; .$$

- *Expectation values*: Given a function u:

Continuous: $$E\left[u(x)\right] = \int_{-\infty}^{\infty} u(x)\; f(x;\theta)\; dx \; ; \tag{16.2}$$

Discrete: $$E\left[u(x)\right] = \sum_{\text{all } i} u(x_i)\; f(x_i;\theta) \; .$$

Examples below will be for the continuous case only.

- *Moments*:

nth moment of a distribution: $$\alpha_n = E(x^n) \; , \tag{16.3a}$$

nth moment about the mean: $$m_n = E[(x-\alpha_1)^n] \; . \tag{16.3b}$$

Mean: $$\mu \equiv \alpha_1 \; . \tag{16.4a}$$

Variance: $$\sigma^2 \equiv \mathrm{Var}(x) \equiv m_2 = \alpha_2 - \mu^2 \; . \tag{16.4b}$$

Coefficient of skewness: $\gamma_1 \equiv m_3/\sigma^3$.

- *Marginal* p.d.f.: Let x,y be two random variables with joint p.d.f. $f(x,y)$; suppress the parameters θ.

$$f_1(x) = \int_{-\infty}^{\infty} f(x,y)\; dy \; ; \qquad f_2(y) = \int_{-\infty}^{\infty} f(x,y)\; dx \; . \tag{16.5}$$

- *Conditional* p.d.f.:

$$f(x|y) = f(x,y)/f_2(y) \; ; \quad f(y|x) = f(x,y)/f_1(x) \; . \tag{16.6}$$