• *Bayes' theorem*:

$$f_4(x|y) = \frac{f_3(y|x)\, f_1(x)}{f_2(y)} = \frac{f_3(y|x)\, f_1(x)}{\int f_3(y|x)\, f_1(x)\, dx} \ . \tag{16.7}$$

• *Correlation coefficient and covariance*:

$$\mu_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\ dx\ dy \ , \tag{16.8}$$

$$\rho_{xy} = E\left[(x-\mu_x)(y-\mu_y)\right]/\sigma_x\,\sigma_y \equiv \mathrm{Cov}(x,y)/\sigma_x\,\sigma_y \ , \tag{16.9}$$

$$\sigma_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)^2\ f(x,y)\ dx\ dy \ . \ \ \text{Note } \rho_{xy}^2 \le 1.$$

• *Independence*: x,y are independent if and only if $f(x,y) = g(x)\ h(y)$; then $\rho_{xy} = 0$, $E[u(x)\ v(y)] = E[u(x)]\ E[v(y)]$ and $\mathrm{Var}(x+y) = \mathrm{Var}(x) + \mathrm{Var}(y)$.

• *Change of variables*: From $\boldsymbol{x} = (x_1,\ldots,x_n)$ to $\boldsymbol{y} = (y_1,\ldots,y_n)$: $g(\boldsymbol{y}) = f\left[w_1(\boldsymbol{y}),\ldots,w_n(\boldsymbol{y})\right]|J|$, where $w_i(\boldsymbol{y})$ expresses x_i as a function of $\boldsymbol{y}$ and $|J|$ is the absolute value of the determinant of the Jacobian $J_{ij} = \partial x_i/\partial y_j$. For discrete variables, use $|J| = 1$.

16.2. Characteristic functions

$$\phi(u) = E(e^{iux}) = \int e^{iux} f(x)dx \ . \tag{16.12}$$

$$i^{-n}\frac{d^n\phi}{du^n}\bigg|_{u=0} = \int x^n f(x)dx = \alpha_n \ . \tag{16.13}$$

If $f_1(x)$ and $f_2(y)$ have characteristic functions $\phi_1(u)$ and $\phi_2(u)$, then the characteristic function of the weighted sum $ax+by$ is $\phi_1(au)\phi_2(bu)$. The addition rules for common distributions (*e.g.*, that the sum of two numbers from Gaussian distributions also has a Gaussian distribution) easily follow from this observation. See the full *Review* for further properties of c.f.

16.3. Some probability distributions

See Table 16.1.

16.3.3. *Normal or Gaussian distribution*:

Its cumulative distribution, for mean 0 and variance 1, is usually tabulated as the *error function*

$$F(x;0,1) = \tfrac{1}{2}\left[1 + \mathrm{erf}(x/\sqrt{2})\right] \ . \tag{16.19}$$

For mean μ and variance σ^2, replace x by $(x-\mu)/\sigma$. This distribution is readily available on spreadsheets such as Excel.