The marginal distribution of any x_i is Gaussian. V is symmetric and positive definite; therefore $(\boldsymbol{x}-\boldsymbol{\mu})^T\ V^{-1}\ (\boldsymbol{x}-\boldsymbol{\mu}) = \chi^2(n) \geq 0$ traces an n-dimensional ellipsoid in $\boldsymbol{x}$ space centered at $\boldsymbol{\mu}$ for $\chi^2(n)$ constant. The probability that $\boldsymbol{x}$ will lie *outside* this ellipsoid with certain usual CL's may be obtained from Fig. 16.3, provided $\overrightarrow{\mu}$ and V are known correctly. We can always find a nonsingular $n \times n$ matrix H such that $HH^T = V$. Then the change of variables to $\boldsymbol{z} = H^{-1}(\boldsymbol{x}-\boldsymbol{\mu})$ yields $\boldsymbol{z}$ such that each z_i is an independent Gaussian with mean 0, variance 1.

16.3.4. χ^2 *distribution (continuous)*: The probability of $\chi^2 > z$, as shown in Fig. 16.1, is given by

$$\mathrm{CL}(\chi^2) = \int_{\chi^2}^{\infty} f(z;\, n)\, dz \equiv 1 - F(z;\, n)\ . \tag{16.22}$$

Since the mean of the χ^2 distribution is equal to n, one expects in a "reasonable" experiment to obtain $\chi^2 \approx n$. While caution is necessary because of the width and skewness of the distribution, the "reduced χ^2" $\equiv \chi^2/n$ is a sometimes useful quantity. Figure 16.3 shows χ^2/n for useful CL's as a function of n.

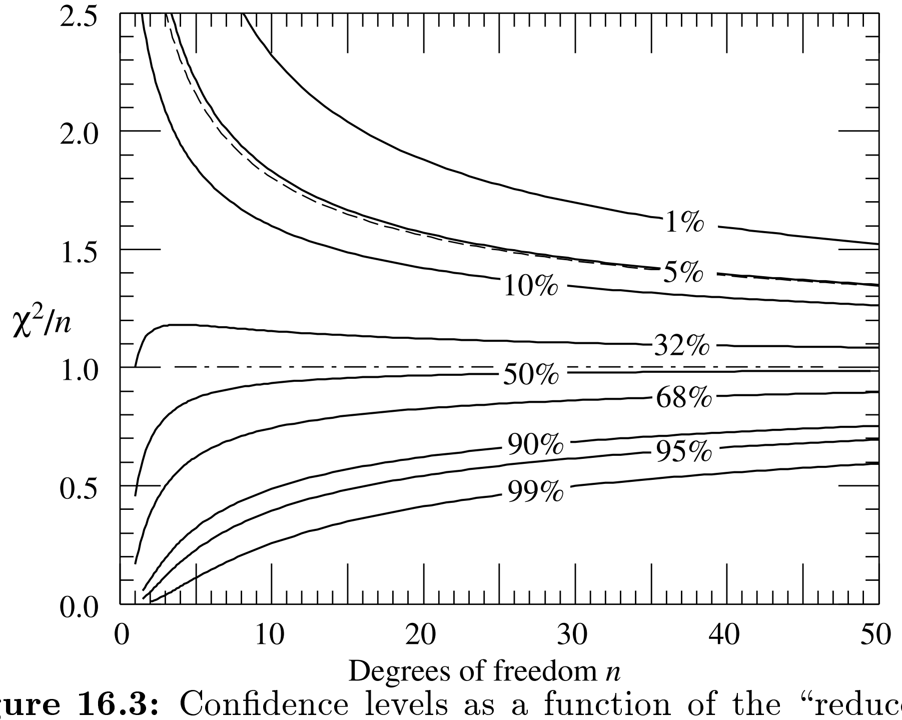

Figure 16.3: Confidence levels as a function of the "reduced χ^2" $\equiv \chi^2/n$ and the number of degrees of freedom n. Curves are labeled by the probability that a measurement will give a value of χ^2/n greater than that given on the y axis; *e.g.*, for $n = 10$, a value $\chi^2/n \gtrsim 1.8$ can be expected 5% of the time.

See the full *Review* for further discussion and all references.