17. STATISTICS

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The following is a condensed version of the same section from the full *Review.* Refer to that version for all discussions and references. (All equation and figure numbering are taken from the full *Review.*)

17.1. General [1–6]

A *statistic* is any function of random variables (for example, measurements with random errors) plus known quantities. It is therefore also a random variable and we may compute its mean and variance. An *estimator* is any statistic intended as an estimate of a parameter of the assumed underlying p.d.f. for the data; write $\widehat{\alpha}$ as an estimator for α.

17.2. Data with a common mean

Assume N independent measurements y_i of the same unknown quantity μ;

(1) Let each y_i have the same variance σ^2. Then

$$\widehat{\mu} = \frac{1}{N}\sum_{i=1}^{N} y_i = E(y) \qquad (17.2)$$

is an unbiased estimator for μ with variance σ^2/N. If σ^2 is also unknown, then

$$\widehat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \widehat{\mu})^2 = \frac{N}{N-1}\Big(E(y^2) - \widehat{\mu}^2\Big) \qquad (17.3)$$

is an unbiased estimator for σ^2. If μ is known and $\widehat{\sigma}^2$ is desired, use $\widehat{\sigma}^2 = \sum(y_i - \mu)^2/N$.

(2) If the y_i have different, known, variances σ_i^2, use

$$\widehat{\mu} = \frac{1}{w}\sum^{N} w_i\, y_i \ , \qquad (17.4)$$

where $w_i = 1/\sigma_i^2$ and $w = \sum_{i=1}^{N} w_i$. The variance of $\widehat{\mu}$ is $1/w$.

17.3. The method of maximum likelihood

17.3.1. *General*:

We suppose that a set of independently measured quantities x_i came from a p.d.f. $f(x;\boldsymbol{\alpha})$, where $\boldsymbol{\alpha}$ is an unknown set of parameters. The method of maximum likelihood consist of finding the set of values,