17.5.4. *Poisson processes with background* [18] :

Let n_0 be the observed number of events, μ_B be the known Poisson parameter for the sum of all backgrounds, and μ_S be the unknown Poisson parameter for the signal. Reasonable (conservative) upper limits for μ_S at a given ε can be obtained from

$$1 - \varepsilon = 1 - \frac{e^{-(\mu_B+N)} \displaystyle\sum_{n=0}^{n_0} \frac{(\mu_B + N)^n}{n!}}{e^{-\mu_B} \displaystyle\sum_{n=0}^{n_0} \frac{\mu_B^n}{n!}} \ . \tag{17.35}$$

where N is adjusted until this relation obtains. N is the desired upper limit for μ_S, and has true probability $\geq 1 - \varepsilon$ of being larger than μ_S, provided μ_B is known with negligible error.

If $\mu_B + \mu_S \gg 1$, it may be more convenient to approximate all Poisson p.d.f.'s as Gaussians with mean = variance and, estimating $\widehat{\mu}_S = n_0 - \mu_B$, use the Gaussian techniques of Sec. 17.5 or 17.5.1.

17.6. Propagation of errors

If we have N measured random variables y_i, with covariance matrix V, we can transform to a new set of variables $f_n \equiv f_n(y)$, $j = 1, \ldots, M$ $(M \leq N)$ and obtain best estimates for the f_n from

$$\widehat{f}_n \approx f_n(\,\widehat{y}\,) + \frac{1}{2} \sum_{k,n}^{N} V_{kn}(\,\widehat{y}\,) \left[\frac{\partial^2 f_n}{\partial y_k \, \partial y_n}\right]_{\widehat{y}} \tag{17.36}$$

The $\widehat{y}$ are our best estimates for the y's. These will have approximate covariance matrix

$$V_{ij}(\,\widehat{f}\,) \approx \sum_{n,m} \left.\frac{\partial f_i}{\partial y_n}\right|_{\widehat{y}} \left.\frac{\partial f_j}{\partial y_m}\right|_{\widehat{y}} V_{nm}(\widehat{y}\,) \ . \tag{17.37}$$

For $M = N = 1$, these relations become

$$\begin{aligned} \widehat{f} &\approx f(\,\widehat{y}\,) + \frac{1}{2}\sigma^2 f''(\,\widehat{y}\,) \\ V(\,\widehat{f}\,) &\approx \sigma^2 [f'(\,\widehat{y}\,)]^2 \ , \end{aligned} \tag{17.38}$$

where the primes denote differentiation with respect to y, evaluated at $\widehat{y}$. If the transformation is approximately linear, it is customary to neglect the second-derivative terms in $\widehat{f}$ entirely.

See the full *Review* for further discussion and all references.