

Fitting with Mass Differences

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A. The Problem

We want to do fits to particle masses for the case in which at least one experiment has quoted measured values for two masses and also the mass difference. We call these quantities A , B , and $D = A - B$. The experiment quotes $A \pm a$, $B \pm b$, and $D \pm d$. Because of systematic errors, the error d is smaller than it would be if the measurements of A and B were independent. Quantitatively, this is stated by $d^2 = a^2 + b^2 - 2abc$, where c is the correlation.

B. Time-Honored Solution

We have often used such data in fits by ignoring the measurement with the largest error and putting the other two into the fit. This is sometimes a good estimate. When the correlation is near zero, using A and B and ignoring D is good. When the correlation is near one, using D and the most accurate of A and B and ignoring the other one is good. But when the correlation is near to one-half, this method is poor.

C. Another Approach

We could use all three measurements in our fits if we could find modified errors for each quantity that give the quoted errors when we do a fit that assumes that these three quantities are not correlated. In this approach, we put into our fit $A \pm x$, $B \pm y$, and $D \pm z$. The equations that one must solve to obtain x , y , and z are:

$$\frac{1}{a^2} = \frac{1}{x^2} + \frac{1}{y^2 + z^2}, \quad \frac{1}{b^2} = \frac{1}{y^2} + \frac{1}{x^2 + z^2}, \quad \frac{1}{d^2} = \frac{1}{z^2} + \frac{1}{x^2 + y^2}$$

I have not found analytic solutions to these equations.

D. Solutions for Some Specific Cases

In an effort to understand the nature of the solutions, I investigated the following cases:

<u>Case</u>	<u>x/a or y/b</u>	<u>z/d</u>
1. $c=0$	1	∞
2. $c=1/2, a = b$	$\sqrt{3/2}$	$\sqrt{3/2}$
3. $c \approx 1$	$\sqrt{2}$	1

By inspection, I was able to find that

$$x = a\sqrt{1+c}, \quad y = b\sqrt{1+c}, \quad z = d\sqrt{\frac{1+c}{2c}}$$

is a solution for these three specific cases. It is pretty good for the set of cases with $a = b$, but is sometimes poor.

E. An Iterative Solution

I wrote a program, called COMER, that solves the equations by starting with the above approximate solution and iterating to find an exact solution. Sometimes this procedure converges and sometimes it does not. I believe that the program works and that the cases for which it does not converge are cases that really have no solutions.

F. Criterion for the Existence of a Solution

We know that d^2 cannot exceed $a^2 + b^2$, and I did not try to find a solution for such a case. However, the equations that we try to solve are symmetrical in the three measured errors. Therefore, we do not expect solutions to be found when $a^2 > b^2 + d^2$ or $b^2 > a^2 + d^2$. So, I believe that this method will work as long as none of the three errors a , b , and d is greater than the square root of the sum of the squares of the other two.