PARTICLE DATA GROUP NOTES **PDG–93–04**

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Effect of a Form Factor on dE/dx from Close Collisions

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Point 1:

Close collisions and distant collisions contribute approximately equally to dE/dx. A form factor is possibly relevant only for the close collision half. We consider these close collisions in the "free electron" approximation.

Point 2:

We parameterize the pion-electron interaction with a form factor showing a ρ-meson propagator:

$$F\left(Q^2\right) = \frac{m_\rho^2}{m_\rho^2 + Q^2} \ , \quad \text{where } m_\rho = 0.77 \text{ GeV} \ .$$

Calculation:

$$s = m_\pi^2 + m_e^2 + 2m_e E_\pi \,(\text{lab})$$

$$Q^2_{\text{max}} = s - 2\left(m_\pi^2 + m_e^2\right) + \frac{\left(m_\pi^2 - m_e^2\right)^2}{s}$$

$$T_{\text{max}} = Q^2_{\text{max}}/2m_e \quad \text{(maximum kinetic energy transfer)}$$

The form factor multiplies the cross section given by Rossi [1], p. 16, Eq. 2.3.6, which in the notation we are using then reads

$$\frac{d\sigma}{dT} = \frac{A}{T^2}\left(1 - \beta^2 \frac{T}{T_{\text{max}}}\right)\left|F\left(2m_e T\right)\right|^2$$

Upon multiplication by T and integration over T, we obtain the equivalent of Rossi's Eq. 2.5.4:

$$\frac{dE}{dx}\,(\text{close}) = A\int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT}{T}\left(1 - \beta^2 \frac{T}{T_{\text{max}}}\right)\left|F\left(2m_e T\right)\right|^2 \equiv A\left(\mathcal{I}_1 - \mathcal{I}_2\right)$$

where T_{min} is the dividing energy between close and distant collisions (called η by Rossi) Following Rossi, I put $T_{\text{min}} = 10^5\,\text{eV} = 10^{-4}\,\text{GeV}$. Now

$$\mathcal{I}_1 = \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT}{T}\,\frac{m_\rho^4}{\left(m_\rho^2 + 2m_e T\right)^2}$$

My CRC Tables yield

$$\mathcal{I}_1 = \ln\left(\frac{T_{\text{max}}}{T_{\text{min}}}\right) - \ln\left(\frac{m_\rho^2 + 2m_e T_{\text{max}}}{m_\rho^2 + 2m_e T_{\text{min}}}\right) + \frac{m_\rho^2}{m_\rho^2 + 2m_e T_{\text{max}}} - \frac{m_\rho^2}{m_\rho^2 + 2m_e T_{\text{min}}} \ .$$

For $m_\rho^2 \to \infty$,$\mathcal{I}_1 \to \ln\left(\frac{T_{\rm max}}{T_{\rm min}}\right)$ [the usual result].

Similarly,

$$\begin{aligned}
\mathcal{I}_2 &= \frac{\beta^2 m_\rho^4}{T_{\rm max}} \int_{T_{\rm min}}^{T_{\rm max}} \frac{dT}{\left(m_\rho^2 + 2m_e T\right)^2} \\
&= \frac{\beta^2}{T_{\rm max}} \frac{m_\rho^4}{2m_e} \left[\frac{1}{m_\rho^2 + 2m_e T_{\rm min}} - \frac{1}{m_\rho^2 + 2m_e T_{\rm max}}\right] \\
&= \beta^2 \left(\frac{m_\rho^2}{2m_e T_{\rm max}}\right) \left[\frac{1}{(1 + 2m_e T_{\rm min}/m_\rho^2)} - \frac{1}{(1 + 2m_e T_{\rm max}/m_\rho^2)}\right]
\end{aligned}$$

In the limit $m_\rho^2 \to \infty$, $\mathcal{I}_2 \to \beta^2(1 - T_{\rm min}/T_{\rm max})$,

$$\beta^2 = 1 - 1/\gamma^2 = 1 - m_\pi^2/E_\pi^2 \ .$$

Results:

I wrote a short program on my Macintosh (using THINK Pascal) to compute the various quantities:

$$\begin{gathered}
Q^2_{\rm max}, \quad T_{\rm max}, \\
\ln\left(T_{\rm max}/T_{\rm min}\right) - \beta^2 \left(1 - T_{\rm min}/T_{\rm max}\right) \ = \text{usual close term}, \\
\text{and} \quad R = \frac{(\mathcal{I}_1 - \mathcal{I}_2)}{[\text{usual close term}]}
\end{gathered}$$

Everything is in GeV or GeV^2 or dimensionless. Inputs: $m_\rho = 0.77$, $m_e = 5.11 \times 10^{-4}$, $T_{\rm min}$ cutoff $= 10^{-4}/$ GeV. Attached is a page of output,* showing the effect for pions. The quantity called "dE/dx ratio" is R, the ratio of dE/dx(close) with the form factor to dE/dx(close) with no form factor.

We see that for pions of 850 GeV there is a 6% reduction with the form factor, corresponding to roughly 3% reduction in dE/dx. The reduction is even smaller for protons.

Addendum:

It is useful to summarize the result as the usual close-collision term minus a correction. Using the approximations $2m_e T_{\rm min}/m_\rho^2 << 1$ and $T_{\rm min}/T_{\rm max} << 1$, and defining $x = 2m_e T_{\rm max}/m_\rho^2$, I find

$$\begin{gathered}
\frac{dE}{dx}\,(\text{close}) = A\left[\ln\left(T_{\rm max}/T_{\rm min}\right) - \beta^2 - f\left(x\right)\right] \\
\text{where} \quad f\left(x\right) = \ln\left(1 + x\right) + \frac{1}{\gamma^2}\frac{x}{1 + x}
\end{gathered}$$

Reference:

1. B. Rossi, *High Energy Particles*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1952.

* Not available in postscript for web listing. Request copy by mail from the PDG, LBNL.