RARE KAON DECAYS

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A. Introduction: There are several recent reviews on rare kaon decays and related topics [1–13]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focussed on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+\ell^+\ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0\gamma\gamma$, which also scales a CP-conserving background to CP violation in $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \gamma\ell^+\ell^-$, which could possibly shed light on long distance contributions to $K_L \to \mu^+\mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 3.3\times10^{-11}(91\ \mathrm{TeV}/M_X)^4$ [7]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of nearly 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to axial-vector (or pseudoscalar) couplings and the latter is sensitive to vector (or scalar) couplings.

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e\mu$	2.1E-10	BNL-777	90/14	3E-12 (BNL-865)
$K_L \to \mu e$	3.3E-11	BNL-791	93/15	2E-12 (BNL-871)
$K_L \to \pi^0 \mu e$	3.5E-9	FNAL-799	94/16	E-11 (FNAL799II)

Another forbidden decay currently being pursued is $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, etc.). The published upper limit on this process [17] is 1.7×10^{-9}, but recently this has been improved to 5.2×10^{-10} [18]. Data already collected by BNL-787 are expected to yield another substantial factor in sensitivity to this process.

C. Measurements of Standard Model parameters: Until recently searches for $K^+ \to \pi^+ \nu\overline{\nu}$ have been motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [19] and long-distance contributions were known to be negligible [3,20]. However, BNL-787 is approaching the sensitivity at which the observation of an event could no longer be unambiguously attributed to non-SM physics. The published 90% c.l. upper limit [17] is 5.2×10^{-9}, but this has been recently improved to 2.4×10^{-9} [18], and extensive recent running with an upgraded beam and detector is expected to further improve this significantly. This reaction is now becoming interesting from the point of view of constraining SM parameters where the branching ratio is expected to be of order 10^{-10}, and can be written as [3]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \qquad (1)$$

where $X(m_t)$ is of order 1, and X_{NL}^{ℓ} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. It also makes manifest the fact that the *a priori* unknown hadronic matrix element drops out in the comparison to the very well-measured rate of K_{e3} decay. QCD corrections, which are contained in X^{ℓ}_{NL}, are relatively small and now known [21] to $\leq 10\%$. Evaluating the constants in Eq. (1) with $m_t = 175$ GeV, one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [21].

$$\begin{aligned} &\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \\ &\qquad \approx 1.2 \times 10^{-10} A^4 [\eta^2 + \frac{2}{3}(\rho^e_o - \rho)^2 + \frac{1}{3}(\rho^{\tau}_o - \rho)^2] \end{aligned} \tag{2}$$

where $\rho^{\ell}_o \equiv 1 + \frac{X^{\ell}_{NL}}{A^2 \lambda^4 X(m_t)}$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$; $\rho_o \equiv \frac{2}{3}\rho^e_o + \frac{1}{3}\rho^{\tau}_o \approx 1.4$, and radius $\approx \frac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})}{1.2\times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ. For $m_t = 175$ GeV it is given by [21]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 1.9 \times 10^{-9} A^4 (\rho'_o - \rho)^2 \tag{3}$$

where ρ'_o depends on the charm quark mass and is around 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (6.8 \pm 0.3) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9}$ listed in the current edition. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present, it is not possible to compute this long-distance component reliably and, therefore, it is not possible to constrain ρ from this mode. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$, and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is parameterized in terms of α_K^*, discussed in the Form Factors section of the K_L^0 Particle Properties Listings).

D. Searches for CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [3,22]. The Standard Model predicts a branching ratio of order 10^{-10}; for $m_t = 175$ GeV it is given approximately by [21]:

$$\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \approx 5 \times 10^{-10} A^4\eta^2 \ . \tag{4}$$

The current upper bound is $\mathrm{B}(K_L \to \pi^0\nu\overline{\nu}) \leq 5.8 \times 10^{-5}$ [23] and FNAL799II (KTeV) is expected to place a bound of order 10^{-8} [24].

The decay $K_L \to \pi^0 e^+e^-$ also has sensitivity to the product $A^4\eta^2$. It has a direct CP-violating component that depends on the value of the top-quark mass, and that for $m_t = 175$ GeV is given by [25]:

$$\mathrm{B_{dir}}(K_L \to \pi^0 e^+e^-) \approx 7 \times 10^{-11} A^4\eta^2 \ . \tag{5}$$

However, like $K_L \to \mu^+\mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects. It has an indirect CP-violating component given by:

$$\mathrm{B_{ind}}(K_L \to \pi^0 e^+e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+e^-) \ , \tag{6}$$

that has been estimated to be less than 10^{-12} [26], but that will not be known precisely until a measurement of $K_S \to \pi^0 e^+e^-$ is available [6,27]. There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0\gamma\gamma$.

An analysis of $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory has been carried out in terms of a parameter a_V [28,29] that determines both the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$. A fit to the distribution has given $-0.32 < a_V <$

0.19 [30]; a value that suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ is significantly smaller than the direct CP-violating component [30]. However, there remains some uncertainty in the interpretation of $K_L \to \pi^0 \gamma\gamma$ in terms of a_V. Analyses that go beyond chiral perturbation theory have found larger values of a_V, indicating a sizable CP-conserving component for $K_L \to \pi^0 e^+ e^-$. The real part of the CP-conserving contribution to $K_L \to \pi^0 e^+ e^-$ is also unknown.

Finally, BNL-845 observed a potential background to $K_L \to \pi^0 e^+ e^-$ from the decay $K_L \to \gamma\gamma e^+ e^-$ [31]. This was later confirmed with an order of magnitude larger sample by FNAL-799 [32], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 10^{-11} [33], comparable to the signal level. Because of this, the observation of $K_L \to \pi^0 e^+ e^-$ will depend on background subtraction with good statistics.

The current upper bound for the process $K_L \to \pi^0 e^+ e^-$ is 4.3×10^{-9} [34]. For the closely related muonic process, the upper bound is $\mathrm{B}(K_L \to \pi^0 \mu^+ \mu^-) \leq 5.1 \times 10^{-9}$ [35]. FNAL799II expects to reach a sensitivity $\lesssim 10^{-11}$ for both reactions [36].

E. Other long distance dominated modes: The decays $K^+ \to \pi^+ \ell^+ \ell^-$ ($\ell = e$ or μ) are described by chiral perturbation theory in terms of one parameter, ω^+ [37]. This parameter determines both the rate and distribution $d\Gamma/dm_{\ell\ell}$ for these processes. A careful study of these two reactions can provide a measurement of ω^+ and a test of the chiral perturbation theory description. A simultaneous fit to the rate and spectrum of $K^+ \to \pi^+ e^+ e^-$ gives: $\omega^+ = 0.89^{+0.24}_{-0.14}$; $\mathrm{B}(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}$ [38]. These two results satisfy the prediction of chiral perturbation theory within two standard deviations [6]. Improved statistics for this mode and a measurement of the mode $K^+ \to \pi^+ \mu^+ \mu^-$ are thus desired. BNL-787 has observed the process $K^+ \to \pi^+ \mu^+ \mu^-$ [39] at about the predicted level, but the result is not yet accurate enough to provide additional constraints.

References

1. W. Marciano, Rare Decay Symposium, Ed. D. Bryman *et al.*, World Scientific 1 (1988).
2. D. Bryman, Int. J. Mod. Phys. **A4**, 79 (1989).
3. J. Hagelin and L. Littenberg, Prog. in Part. Nucl. Phys. **23**, 1 (1989).
4. A. Buras and M. Harlander, *Review Volume on Heavy Flavors*, ed. A. Buras and M. Lindner, World Scientific, Singapore (1992).
5. R. Battiston *et al.*, Phys. Reports **214**, 293 (1992).
6. L.Littenberg and G. Valencia, Ann. Rev. Nucl. and Part. Sci. **43**, 729 (1993).
7. J. Ritchie and S. Wojcicki, Rev. Mod. Phys. **65**, 1149 (1993).
8. U. Meissner, Rept. on Prog. in Phys. **56**, 903 (1993).
9. B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65**, 1113 (1993).
10. N. Bilic and B. Guberina, Fortsch. Phys. **42**, 209 (1994).
11. G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, *Radiative Non-Leptonic Kaon Decays*, in The DAΦNE Physics Handbook (second edition), eds. L. Maiani, G. Pancheri and N. Paver (Frascati), Vol. I, 265 (1995).
12. E. de Rafael, *Chiral Lagrangians and Kaon CP Violation*, in *CP Violation and the Limits of the Standard Model*, Proc. TASI'94, ed. J.F. Donoghue (World Scientific, Singapore, 1995).
13. A. Pich, Rept. on Prog. in Phys. **58**, 563 (1995).
14. A. M Lee *et al.*, Phys. Rev. Lett. **64**, 165 (1990).
15. K. Arisaka *et al.*, Phys. Rev. Lett. **70**, 1049 (1993).
16. P. Gu, Albuquerque DPF Workshop, 1994.
17. M.S. Atiya *et al.*, Phys. Rev. Lett. **70**, 2521 (1993).
18. S. Adler *et al.*, Phys. Rev. Lett. **76**, 1421 (1996).
19. I. Bigi and F. Gabbiani, Nucl. Phys. **B367**, 3 (1991).
20. D. Rein and L.M. Sehgal, Phys. Rev. **D39**, 3325 (1989); M. Lu and M.B. Wise, Phys. Lett. **B324**, 461 (1994).
21. G. Buchalla and A. Buras, Nucl. Phys. **B412**, 106 (1994).
22. L. Littenberg, Phys. Rev. **D39**, 3322 (1989).
23. M. Weaver *et al.*, Phys. Rev. Lett. **72**, 3758 (1994).
24. M. Weaver, "The Albuquerque Meeting", Ed. S. Seidel, World Scientific, 1026 (1995).
25. A. Buras *et al.*, Nucl. Phys. **B423**, 349 (1994).

26. G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303**, 665 (1988).
27. J.F. Donoghue and F. Gabbiani, Phys. Rev. **D51**, 2187 (1995).
28. G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **189B**, 363 (1987).
29. G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **237B**, 481 (1990).
30. G.D. Barr *et al.*, Phys. Lett. **242B**, 523 (1990);
    G.D. Barr *et al.*, Phys. Lett. **284B**, 440 (1992).
31. W.M. Morse *et al.*, Phys. Rev. **D45**, 36 (1992).
32. T. Nakaya *et al.*, Phys. Rev. Lett. **73**, 2169 (1994).
33. H.B. Greenlee, Phys. Rev. **D42**, 3724 (1990).
34. D.A. Harris *et al.*, Phys. Rev. Lett. **71**, 3918 (1993).
35. D.A. Harris *et al.*, Phys. Rev. Lett. **71**, 3914 (1993).
36. "Heavy Flavor Physics and CP Violation", in *Particle Physics - Perspectives and Opportunities, Report of the DPF Committee on Long-Term Planning*, Ed. R. Peccei *et al.*, 91 (1995).
37. G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B291**, 692 (1987).
38. C. Alliegro *et al.*, Phys. Rev. Lett. **68**, 278 (1992).
39. J.S. Haggerty, *Proceedings of the XXVII International Conference on High Energy Physics*, Ed. P.J. Bussey and I.G. Knowles, 1341 (1995);
    M. Ardebili, Ph D. thesis, Princeton University (1995).