STANDARD MODEL OF ELECTROWEAK INTERACTIONS

To reduce the size of this section's PostScript file, we have divided it into three PostScript files. We present the following index:

PART 1

PART 2

PART 3

10.3. *W* and *Z* decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226 \pm 1 \text{ MeV} \tag{10.36a}$$

$$\Gamma(W^+ \to u_i\overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi}\, |V_{ij}|^2 \approx (705 \pm 4)\, |V_{ij}|^2 \text{ MeV} \tag{10.36b}$$

$$\Gamma(Z \to \psi_i\overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi} \left[g_V^{i2} + g_A^{i2} \right] \tag{10.36c}$$

$$\approx \begin{cases} 167.2 \pm 0.1 \text{ MeV } (\nu\overline{\nu}), \;\; 84.0 \pm 0.1 \text{ MeV } (e^+e^-), \\ 300.6 \pm 0.3 \text{ MeV } (u\overline{u}), \;\; 383.3 \pm 0.3 \text{ MeV } (d\overline{d}), \\ 375.9 \mp 0.2 \text{ MeV } (b\overline{b}). \end{cases}$$

For leptons $C = 1$, while for quarks $C = 3\Big(1 + \alpha_s(M_V)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses represents the universal QCD corrections for massless quarks [50]. The $Z \to f\overline{f}$ widths contain a number of additional corrections [51]: universal (non-singlet) top-mass contributions [52]; fermion mass effects and further QCD corrections proportional to m_q^2 [53] (m_q is the running quark mass evaluated at the Z scale) which are different for vector and axial-vector partial widths; and singlet contributions starting from two loop order which are large, strongly top-mass dependent, family universal and flavor non-universal [54]. All QCD effects are known and included up to three loop order with the exception of order $\alpha_s^3 m_b^2$ corrections which are very small. The QED factor $1 + 3\alpha q_f^2/4\pi$ and order $\alpha\alpha_s$ corrections [55] have to be included, as well. Expressing the widths in terms of $G_F M_{W,Z}^3$ incorporates the bulk of the low-energy radiative corrections [16,56]. The electroweak corrections are incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, the widths are proportional to $\rho_i \sim 1 + \rho_t$. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [57] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b.

For 3 fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.497 \pm 0.002 \text{ GeV} \tag{10.37}$$

$$\Gamma_W \approx 2.09 \pm 0.01 \text{ GeV} \quad . \tag{10.38}$$

The numerical values for the widths assume $M_Z = 91.1884 \pm 0.0022$ GeV, $M_W = 80.26 \pm 0.16$ GeV, $\alpha_s = 0.123$, and $m_t = 180 \pm 7$ GeV, where the α_s and m_t values are predicted by the global fits for $M_H = 300$ GeV. The uncertainties for Γ_W and Γ_Z are dominated by ΔM_W and Δm_t, respectively. The uncertainty in α_s, ± 0.004, introduces an additional uncertainty of 0.13% in the hadronic widths, corresponding to ± 2 MeV in Γ_Z.

These predictions are to be compared with the experimental results $\Gamma_Z = 2.4963 \pm 0.0032$ GeV and $\Gamma_W = 2.08 \pm 0.07$ GeV.

10.4. Experimental results

The values of the principal Z-pole observables are listed in Table 10.3, along with the Standard Model predictions for $M_Z = 91.1884 \pm 0.0022$, $m_t = 180 \pm 7$ GeV (for $M_H = 300$ GeV), 60 GeV $< M_H <$ 1 TeV, and $\alpha_s = 0.123 \pm 0.004$. Note that, the values of the Z-pole observables (as well as M_W) differ from those in the Particle Listings because they include recent preliminary results [47,49,59]. The values and predictions of M_W [59], the Q_W for cesium [36,41], and recent results from deep inelastic and $\nu_\mu e$ scattering are also listed. The agreement is generally excellent. Major exceptions are $R_b = \Gamma(b\bar{b})/\Gamma(\text{had})$ which is 3.7σ above the Standard Model prediction, and $R_c = \Gamma(c\bar{c})/\Gamma(\text{had})$ which is 2.4σ below. These are strongly correlated: if R_c is fixed at the Standard Model value of 0.172, then one obtains [47] $R_b = 0.2205 \pm 0.0016$, which is still 3.0σ too high. Within the Standard Model framework, these values must be considered large statistical fluctuations or systematic errors. However, R_b tends to favor small values of m_t, and when combined with other observables, small values for M_H. Many types of new physics could contribute to R_b (see also Sec. 14 on "Constraints on New Physics from Electroweak Analyses" in this *Review*). The implications of this possiblity for the value of $\alpha_s(M_Z)$ extracted from the fits are discussed below. The left-right asymmetry $A^0_{LR} = 0.1551 \pm 0.0040$ [49] based on all data from 1992–1995 has moved closer to the Standard Model expectation of 0.144 ± 0.003 than the previous value 0.1637 ± 0.0075, from 1992–1993. However, because of the smaller error A^0_{LR} is still 2.3σ above the Standard Model prediction. There is also an experimental difference of $\sim 1.5\sigma$ between the SLD value of $A^0_e = A^0_{LR}$ and the LEP value $A^0_{\ell\text{LEP}} \sim 0.147 \pm 0.004$ obtained from $A^{(0,\ell)}_{FB}$, $A^0_e(P_\tau)$, $A^0_\tau(P_\tau)$ assuming lepton family universality. Finally, the forward-backward asymmetry into τ's, $A^{0\tau}_{FB} = 0.0206 \pm 0.0023$ [47], is 2.2σ above the Standard Model prediction and 1.6σ above the average 0.0162 ± 0.0014 of A^{0e}_{FB} and $A^{0\mu}_{FB}$. This is small enough to be a fluctuation, so lepton-family universality will be assumed. The observables in Table 10.3 (including correlations on the LEP observables), as well as all low-energy neutral-current data [14,15], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from the Z-pole observables and M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [14], shown in Table 10.4, are in impressive agreement with each other, indicating the quantitative success of the Standard Model. The one discrepancy is the value $\widehat{s}^2_Z = 0.2302 \pm 0.0005$ from A^0_{LR} which is 2.1σ below the value (0.2315 ± 0.0004) from the global fit to all data and 2.6σ below the value 0.2318 ± 0.0004 obtained from all data other than A^0_{LR}.

The data allow a simultaneous determination of $\sin^2\theta_W$, m_t, and the strong coupling $\alpha_s(M_Z)$. The latter is determined mainly from Γ_Z and R, and is only weaky correlated

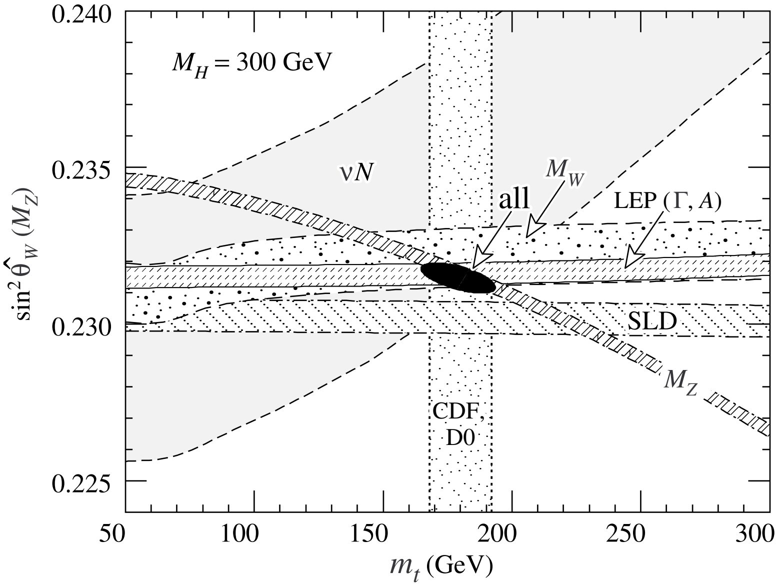

Figure 10.1: One-standard-deviation uncertainties in $\sin^2\hat{\theta}_W$ as a function of m_t, the direct CDF and DØ range 180 ± 12 GeV, and the 90% CL region in $\sin^2\hat{\theta}_W - m_t$ allowed by all data, assuming $M_H = 300$ GeV.

with the other variables. The global fit to all data, including the CDF/DØ value $m_t = 180 \pm 12$ GeV, yields

$$\hat{s}^2_Z = 0.2315 \pm 0.0002 \pm 0.0003$$

$$m_t = 180 \pm 7^{+12}_{-13} \text{ GeV}$$

$$\alpha_s(M_Z) = 0.123 \pm 0.004 \pm 0.002 , \tag{10.39}$$

where the central values are for a Higgs mass of 300 GeV, and the second error bars are for $M_H \to 1000(+)$ or $60(-)$ GeV. In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The $\hat{s}^2_Z$ error is dominated by m_t, and $\hat{s}^2_Z$ and m_t have a strong negative correlation of ~ -0.62. In the on-shell scheme one has $s^2_W = 0.2236 \pm 0.0008$, the larger error due to the stronger sensitivity to m_t. The extracted value of α_s is based on a formula which has almost no theoretical uncertainty (if one assumes the exact validity of the Standard Model), and is in excellent agreement with the values 0.122 ± 0.007 from jet-event shapes in e^+e^- annihilation, and the average 0.118 ± 0.003 from all data (including the Z-lineshape data), as described in our Section 9 on "Quantum Chromodynamics" in this *Review*. However, it is higher than some of the individual values extracted from low-energy data, such as deep-inelastic scattering (0.112 ± 0.002 (exp) ± 0.004 (scale)) or lattice calculations of the $b\bar{b}$ and $c\bar{c}$ spectra (0.115 ± 0.003). It has been suggested [60] that there is a real discrepancy. However, caution is requred since most of the determinations are dominated by theory errors.

The value of R_b is more than 3σ above the Standard Model expectation. If this is not just a fluctuation but is due to a new physics contribution to the $Z \rightarrow b\bar{b}$ vertex (many types would couple preferentially to the third family), the value of $\alpha_s(M_Z)$ extracted from the hadronic Z width would be reduced [15]. Allowing for this possibility one obtains $\alpha_s(M_Z) = 0.101 \pm 0.008$. (See also Sec. 14 on "Constraints on New Physics from Electroweak Analyses." in this *Review*)

In principle the low value of R_c could also be due to new physics. However, allowing for new physics contributions to R_c alone, one obtains $\alpha_s(M_Z) = 0.19 \pm 0.03$, which is clearly inconsistent with low-energy determinations. Allowing new contributions to both R_b and R_c yields the slightly lower but still high value of $\alpha_s(M_Z) = 0.16 \pm 0.04$. We will, therefore, take the view that the R_c value is a fluctuation. We keep the experimental values $R_b = 0.2219(17)$ and $R_c = 0.1540(74)$ and their correlation (-0.35) in all fits, but do not allow any special vertex corrections for $Z \rightarrow c\bar{c}$. This is effectively equivalent to using the lower value 0.2205(16) that the LEP experimenters obtain for R_b when they constrain R_c to the Standard Model value of 0.172.

One can also carry out a fit to the indirect data alone, *i.e.*, without including the value $m_t = 180 \pm 12$ GeV observed directly by CDF and DØ. (The indirect prediction is for the pole mass, which should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 179 \pm 8^{+17}_{-20}$ GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The results of fits to various combinations of the data are shown in Table 10.5 and the relation between $\widehat{s}^{\,2}_Z$ and m_t for various observables in Fig. 10.1.

The data indicate a preference for a small Higgs mass. This is because there is a strong correlation between the quadratic ρ_t terms and logarithmic M_H effects in all of the indirect data except the $Z \rightarrow b\bar{b}$ vertex. The latter favor a smaller m_t and therefore a smaller M_H. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000\ \text{GeV}) - \chi^2(M_H = 60\ \text{GeV}) = 7.9$. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the allowed range; including the direct constraint $M_H \geq 60$ GeV, the best fit is for $M_H = 60$ GeV, with the limit $M_H < 320(430)$ GeV at 90(95)% CL. However, one should be cautious because the M_H constraint is driven almost entirely by R_b and A_{LR}, both of which deviate from the Standard Model prediction. Using $\alpha(M_Z)$ and $\widehat{s}^{\,2}_Z$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [61] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is consistent with the experimental $\alpha_s(M_Z) = 0.121(4)(1)$ from the Z-lineshape (using the lower M_H range appropriate for supersymmetry) and with the average 0.118 ± 0.003 (see our Section 9 on "Quantum Chromodynamics" in this *Review*), but is high compared to some low-energy determinations of α_s [60]. Nonsupersymmetric unified theories predict the low value $\alpha_s(M_Z) = 0.073 \pm 0.001 \pm 0.001$.

One can also determine the radiative correction parameters Δr: including the CDF and DØ data, one obtains $\Delta r = 0.039 \pm 0.003$ and $\Delta\widehat{r}_W = 0.068 \pm 0.0013$, where the error includes m_t and M_H, in excellent agreement with the predictions 0.038 ± 0.005 and 0.0705 ± 0.0007.

Table 10.3: Principal LEP and other recent observables, compared with the Standard Model predictions for $M_Z = 91.1884 \pm 0.0022$ GeV, 60 GeV $< M_H <$ 1 TeV, the global best fit value $m_t = 180 \pm 7$ GeV (for $M_H = 300$ GeV), $\alpha_s = 0.123 \pm 0.004$, and $\alpha_s(M_Z)^{-1} = 128.90 \pm 0.09$. The LEP averages [58] of the ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [58]. $\bar{s}^2_\ell(A^{(0,q)}_{FB})$ is the effective angle extracted from the hadronic-charge asymmetry. A^0_{LR} includes all data from 1992–1995 [48,49]. The values of $\Gamma(\ell\bar{\ell})$, Γ(had), and Γ(inv) are not independent of Γ_Z, R, and $\sigma_{\rm had}$. The M_W value is from CDF, UA2, and DØ [59]. M_W and M_Z are correlated, but the effect is negligible due to the tiny M_Z error. The two values of s^2_W from deep-inelastic scattering are from CCFR [31] and the global average, respectively. The $g^{\nu e}_{V,A}$ are from CHARM II [34]. The second error in Q_W (for cesium) is theoretical [41]. Older low-energy results are not listed but are included in the fits. In the Standard Model predictions, the first uncertainty is from M_Z and Δr, while the second is from m_t and M_H. The $\Delta\alpha_s = 0.004$ uncertainty leads to additional errors of 0.002 (Γ_Z), 0.02 (R), 0.02 (σ), 2.0 (Γ(had)).

Quantity	Value	Standard Model
M_Z (GeV)	91.1884 ± 0.0022	input
Γ_Z (GeV)	2.4963 ± 0.0032	$2.497 \pm 0.001 \pm 0.002$
R	20.788 ± 0.032	$20.77 \pm 0.004 \pm 0.002$
$\sigma_{\rm had}(nb)$	41.488 ± 0.078	$41.45 \pm 0.002 \pm 0.004$
R_b	0.2219 ± 0.0017	$0.2156 \pm 0 \pm 0.0003$
R_c	0.1540 ± 0.0074	$0.172 \pm 0 \pm 0$
$A^{(0,\ell)}_{FB}$	0.0172 ± 0.0012	$0.0155 \pm 0.0004 \pm 0.0004$
$A^0_\tau(P_\tau)$	0.1418 ± 0.0075	$0.144 \pm 0.002 \pm 0.002$
$A^0_e(P_\tau)$	0.1390 ± 0.0089	$0.144 \pm 0.002 \pm 0.002$
$A^{(0,b)}_{FB}$	0.0997 ± 0.0031	$0.101 \pm 0.001 \pm 0.001$
$A^{(0,c)}_{FB}$	0.0729 ± 0.0058	$0.072 \pm 0.001 \pm 0.001$
A^0_{LR}	0.1551 ± 0.0040	$0.144 \pm 0.002 \pm 0.002$
$\overline{A}_b$	0.841 ± 0.053	$0.934 \pm 0 \pm 0$
$\overline{A}_c$	0.606 ± 0.090	$0.667 \pm 0.001 \pm 0.001$
$\bar{s}^2_\ell(A^{(0,q)}_{FB})$	0.2325 ± 0.0013	$0.2319 \pm 0.0002 \pm 0.0002$
$\Gamma(\ell\bar{\ell})$ (MeV)	83.93 ± 0.14	$83.97 \pm 0.01 \pm 0.06$
Γ(had) (MeV)	1744.8 ± 3.0	$1743.8 \pm 0.2 \pm 1.2$
Γ(inv) (MeV)	499.9 ± 2.5	$501.6 \pm 0 \pm 0.3$
M_W (GeV)	80.26 ± 0.16	$80.34 \pm 0.01 \pm 0.04$
Q_W	$-71.04 \pm 1.58 \pm 0.88$	$-72.88 \pm 0.05 \pm 0.03$
$s^2_W = 1 - \frac{M^2_W}{M^2_Z}$	0.2218 ± 0.0059 0.2260 ± 0.0048	$0.2237 \pm 0.0002 \pm 0.0008$
$g^{\nu e}_A$	-0.503 ± 0.017	$-0.507 \pm 0 \pm 0.0004$
$g^{\nu e}_V$	-0.035 ± 0.017	$-0.037 \pm 0.0005 \pm 0.0003$

Table 10.4: Values obtained for s^2_W (on-shell) and $\widehat{s}^{\,2}_Z(\overline{\mathrm{MS}})$ from various reactions assuming the global best fit value $m_t = 180 \pm 07$ GeV (for $M_H = 300$ GeV), and $\alpha_s = 0.123 \pm 0.004$. The uncertainties include the effect of 60 GeV $< M_H <$ 1 TeV. The determination from Γ_Z, R, and σ_{had} uses the experimental value of M_Z, so that the values obtained are from the vertices and not the overall scale.

Reaction	s^2_W	$\widehat{s}^{\,2}_Z$
M_Z	0.2237 ± 0.0010	0.2316 ± 0.0005
M_W	0.2242 ± 0.0011	0.2321 ± 0.0009
$\Gamma_Z,\ R, \sigma_{\mathrm{had}}$	0.2239 ± 0.0013	0.2317 ± 0.0013
$A^{(0,\ell)}_{FB}$	0.2228 ± 0.0009	0.2307 ± 0.0007
LEP asymmetries	0.2237 ± 0.0007	0.2316 ± 0.0003
A^0_{LR}	0.2223 ± 0.0008	0.2302 ± 0.0005
$\overline{A}_b, \overline{A}_c$	0.250 ± 0.021	0.259 ± 0.022
Deep inelastic (isocalar)	0.226 ± 0.004	0.234 ± 0.005
$\nu_\mu(\overline{\nu}_\mu)p \to \nu_\mu(\overline{\nu}_\mu)p$	0.205 ± 0.030	0.212 ± 0.031
$\nu_\mu(\overline{\nu}_\mu)e \to \nu_\mu(\overline{\nu}_\mu)e$	0.221 ± 0.007	0.228 ± 0.008
atomic parity violation	0.216 ± 0.008	0.223 ± 0.008
SLAC eD	0.216 ± 0.017	0.223 ± 0.018
All data	0.2236 ± 0.0008	0.2315 ± 0.0004

Table 10.5: Values of $\widehat{s}^2_Z$ and s^2_W (in parentheses), α_s, and m_t for various combinations of observables. The central values are for $M_H = 300$ GeV, and the second set of errors is for $M_H \to 1000(+)$, $60(-)$.

Data	$\widehat{s}^2_Z$ (s^2_W)	α_s (M_Z)	m_t (GeV)
Indirect + CDF + DØ	0.2315(2)(3)	0.123(4)(2)	$180 \pm 7^{+12}_{-13}$
	(0.2236 ± 0.0008)		
All indirect	0.2315(2)(2)	0.123(4)(2)	$179 \pm 8^{+17}_{-20}$
	(0.2236 ± 0.0009)		
All LEP	0.2318(3)(2)	0.124(4)(2)	$171 \pm 10^{+18}_{-20}$
	(0.2246 ± 0.0011)		
SLD + M_Z	0.2302(5)(0)	—	220^{+14+19}_{-15-24}
	(0.2184 ± 0.0020)		
Z pole	0.2314(3)(1)	0.123(4)(2)	181^{+8+18}_{-9-20}
(LEP + SLD)	(0.2234 ± 0.0010)		