QCD

To reduce the size of this section's PostScript file, we have divided it into two PostScript files. We present the following index:

PART 1

PART 2

9.6. QCD in heavy-quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of α_s at the heavy-quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. The total decay width of the Υ is predicted by perturbative QCD [72]

$$\begin{aligned} R_\mu(\Upsilon) &= \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)} \\ &= \frac{10(\pi^2-9)\alpha_s^3(M)}{9\pi\alpha_{\rm em}^2} \\ &\quad \times \left[1+\frac{\alpha_s}{\pi}\left(-19.4+\frac{3\beta_0}{2}\left(1.162+\ln\left(\frac{2M}{M_\Upsilon}\right)\right)\right)\right] . \end{aligned} \tag{9.16}$$

Data are available for the Υ, Υ', Υ'', and J/ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 32.5 \pm 0.9$) [73] that the theoretical errors will dominate. There are theoretical corrections to this simple formula due to the relativistic nature of the $Q\overline{Q}$ system; $v^2/c^2 \sim 0.1$ for the Υ. They are more severe for the J/ψ. There are also nonperturbative corrections of the form Λ^2/m_Υ^2; again these are more severe for the J/ψ. A fit to Υ, Υ', and Υ'' [74] gives $\alpha_s(M_Z) = 0.113 \pm 0.001$ (expt.). The results from each state separately and also from the J/ψ are consistent with each other. There is an uncertainty of order ± 0.005 from the choice of scale; the error from v^2/c^2 corrections is a little larger. The ratio of widths $\dfrac{\Upsilon \to \gamma gg}{\Upsilon \to ggg}$ has been measured by the CLEO collaboration who use it to determine $\alpha_s(9.45\ \text{GeV}) = 0.163 \pm 0.002 \pm 0.014$ [76] which corresponds to $\alpha_s(M_Z) = 0.110 \pm 0.001 \pm 0.007$. The error is dominated by theoretical uncertainties associated with the scale choice. The theoretical uncertainties due to the production of photons in fragmentation [75] are small [76].

9.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)}\left[1+\frac{\alpha_s}{\pi}+C_2\left(\frac{\alpha_s}{\pi}\right)^2+C_3\left(\frac{\alpha_s}{\pi}\right)^3+\cdots\right] , \tag{9.17}$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [77].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 35.2 of this *Review*. This result is only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [78].

The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

A comparison of the theoretical prediction of Eq. (9.17) (corrected for the b-quark mass), with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV, gives [79] $\alpha_s(35\ \text{GeV}) = 0.146 \pm 0.030$. The size of the order α_s^3 term is of order 40% of that of the order α_s^2 and 3% of the order α_s. If the order α_s^3 term is not included, a fit to the data yields α_s (34 GeV) $= 0.142 \pm 0.03$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the Z at LEP and SLC, Γ_h/Γ_μ probe, the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.783 \pm 0.029$ gives $\alpha_s(M_Z) = 0.124 \pm 0.0043$ [80]. There are theoretical errors arising from the values of top-quark and Higgs masses which enter due to electroweak corrections to the Z width and from the choice of scale.

While this method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the Z to quarks [81]. The presence of new physics which changes these couplings via electroweak radiative corrections would invalidate the value of $\alpha_s(M_Z)$. However, given the excellent agreement [82] of the many measurements at the Z, there is no reason not to use the value of $\alpha_s(M_Z) = 0.1214 \pm 0.0031$ from the global fits of the various precision measurements at LEP/SLC and the W and top masses in the world average (see the section on "Electroweak model and constraints on new physics," Sec. 10 of this *Review*)

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. A recent review should be consulted for more details [83] of the issues mentioned briefly here. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [84], energy-energy correlations [85], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to qqg$: [86]

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \ , \tag{9.18}$$

where

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{9.19}$$

are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order α_s^2 corrections to this process have been computed, as well as the 4-jet final states such as $e^+e^- \to qqgg$ [87].

There are many methods used by the e^+e^- experimental groups to determine α_s from the event topology. The jet-counting algorithm, originally introduced by the JADE collaboration [88], has been used by many other groups. Here, particles of momenta p_i and p_j are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is then iterated until no more pairs of particles or pseudo-particles remain. The remaining number is then defined to be the number of jets in the event, and can be compared to the QCD prediction. The Durham algorithm is slightly different: in computing the mass of a pair of partons, it uses $M^2 = 2\min(E_1^2, E_2^2)(1 - \cos\theta_{ij})$ for partons of energies E_i and E_j separated by angle θ_{ij} [89].

There are theoretical ambiguities in the way this process is carried out. Quarks and gluons are massless, whereas the observed hadrons are not, so that the massive jets that result from this scheme cannot be compared directly to the massless jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless. These schemes result in the same data giving a slightly different values [90,91] of α_s. These differences can be used to determine a systematic error. In addition, since what is observed are hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons, so that detector corrections can be applied. The QCD matrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [92–95] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP/SLD experiments.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center-of-mass energy. While there is no justification for doing so, if the value is allowed to float in the fit to the data, the data tend to prefer values of order $\sqrt{s}/10$ GeV for some variables, whereas others have only a preferred range of $M > 3$ GeV [91,96]; the exact value depends on the variable that is fitted.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contain terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events), the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [97]. If the jet recombination methods are used higher-order terms involve $\alpha_s^n \ln^m(y_0)$, these too can be resummed [98]. The resummed results give better agreement with the data at large values of T. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading-log corrections. Different schemes for combining the order α_s^2 and the resummations are available [99]. These different schemes result in shifts in α_s of order ± 0.002. An average of the recent results at the Z resonance from SLD [91], OPAL [100], L3 [101], ALEPH [102], and DELPHI [103], using the combined α_s^2 and resummation fitting to a large set of shape variables, gives $\alpha_s(M_Z) = 0.122 \pm 0.007$. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale, and the effects of hadronization Monte Carlos on the different quantities fitted.

Similar studies on event shapes have been undertaken at TRISTAN, at PEP/PETRA, and at CLEO. A combined result from various shape parameters by the TOPAZ collaboration gives $\alpha_s(58\ \text{GeV}) = 0.125 \pm 0.009$, using the fixed order QCD result, and $\alpha_s(58\ \text{GeV}) = 0.132 \pm 0.008$ (corresponding to $\alpha_s(M_Z) = 0.123 \pm 0.007$), using the same method as in the SLD and LEP average [104]. The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. The results are consistent with those from Z decay, but have larger errors. We use $\alpha_s(34\ \text{GeV}) = 0.14 \pm 0.02$ [105]. A recent analysis by the TPC group [106] gives $\alpha_s(29\ \text{GeV}) = 0.160 \pm 0.012$, using the same method as TOPAZ. This value corresponds to $\alpha_s(M_Z) = 0.131 \pm 0.010$

The CLEO collaboration fits to the order α_s^2 results for the two jet fraction at $\sqrt{s} = 10.53$ GeV, and obtains $\alpha_s(10.93) = 0.164 \pm 0.004$ (expt.) $\pm\, 0.014$ (theory) [107]. The dominant systematic error arises from the choice of scale (μ), and is determined from the range of α_s that results from fit with $\mu = 10.53$ GeV, and a fit where μ is allowed to vary to get the lowest χ^2. The latter results in $\mu = 1.2$ GeV. Since the quoted result corresponds to $\alpha_s(1.2) = 0.35$, it is by no means clear that the perturbative QCD expression is reliable and the resulting error should, therefore, be treated with caution. A fit to many different variables as is done in the LEP/SLC analyses would give added confidence to the quoted error.

Recently studies have been carried out at $\sim$130 GeV [108]. These can be combined to give $\alpha_s(130\ \text{GeV}) = 0.114 \pm 0.008$. Preliminary data from $\sim$ 165 GeV [109] are consistent with the decrease in α_s expected at the higher energy.

Since the errors in the event shape measurements are dominantly systematic, and are common to the experiments, the results from PEP/PETRA, TRISTAN, LEP, SLC, and CLEO are combined to give $\alpha_s(M_Z) = 0.121 \pm 0.007$. All of the experiments are consistent with this average and, taken together, provide verification of the running of the coupling constant with energy.

The total cross section $e^+e^- \to b\bar{b} + X$ near threshold can be used to determine α_s [110]. The result quoted is $\alpha_s(M_Z) = 0.109 \pm 0.001$. The relevant process is only calculated to leading order and the BLM scheme [9] is used. This results in $\alpha_s(0.632\ m_b)$. If $\alpha_s(m_b)$ is used, the resulting $\alpha_s(M_Z)$ shifts to ~ 0.117. This result is not used in the average.

9.8. Scaling violations in fragmentation functions

Measurements of the fragmentation function $d_i(z, E)$, being the probability that a hadron of type i be produced with energy zE in e^+e^- collisions at $\sqrt{s} = 2E$, can be used to determine α_s. As in the case of scaling violations in structure functions, QCD predicts only the E dependence. Hence, measurements at different energies are needed to extract a value of α_s. Because the QCD evolution mixes the fragmentation functions for each quark flavor with the gluon fragmentation function, it is necessary to determine each of these before α_s can be extracted. The ALEPH collaboration has used data from energies ranging from $\sqrt{s} = 22$ GeV to $\sqrt{s} = 91$ GeV. A flavor tag is used to discriminate between different quark species, and the longitudinal and transverse cross sections are used to extract the gluon fragmentation function [111]. The result obtained is $\alpha_s(M_Z) = 0.126 \pm 0.007$ (expt.) $\pm\ 0.006$ (theory) [112]. The theory error is due mainly to the choice of scale. The OPAL collaboration [113] has also extracted the separate fragmentation functions. DELPHI [114] has also performed a similar analysis using data from other experiments at lower energy with the result $\alpha_s(M_Z) = 0.124 \pm 0.007 \pm 0.009$ (theory).The larger theoretical error is due to the larger range of scales that were used in the fit. These results can be combined to give $\alpha_s(M_Z) = 0.125 \pm 0.005 \pm 0.008$ (theory).

e^+e^- can also be used to study photon-photon interaction, which can be used to measure the structure function of a photon [115]. This process was included in earlier versions of this *Review* [115] which can be consulted for details on older measurements [116–119]. More recent data has become available from LEP [120,121] and from TRISTAN [122,123] which show Q^2 dependence of the structure function that is consistent with QCD expectations.

9.9. Jet rates in *ep* collisions

At lowest order in α_s, the ep scattering process produces a final state of (1+1) jets, one from the proton fragment and the other from the quark knocked out by the process $e + quark \rightarrow e + quark$. At next order in α_s, a gluon can be radiated, and hence a (2+1) jet final state produced. By comparing the rates for these (1+1) and (2+1) jet processes, a value of α_s can be obtained. A NLO QCD calculation is available [124]. The basic methodology is similar to that used in the jet counting experiments in e^+e^- annihilation discussed above. Unlike those measurements, the ones in ep scattering are not at a fixed value of Q^2. In addition to the systematic errors associated with the jet definitions, there are additional ones since the structure functions enter into the rate calculations. Results from H1 [125] and ZEUS [126] can be combined to give $\alpha_s(M_Z) = 0.118 \pm 0.001$ (expt.) $\pm\ 0.008$ (syst.). The contributions to the systematic errors from experimental effects (mainly the hadronic energy scale) in the case of ZEUS (H1) are comparable to (smaller than) the theoretical ones arising from scale choice, structure functions, and jet definitions. The theoretical errors are common to the two measurements; therefore, we have not reduced the systematic error after forming the average.

9.10. Lattice QCD

Lattice gauge theory calculations can be used to calculate, using non-perturbative methods, a physical quantity that can be measured experimentally. The value of this quantity can then be used to determine the QCD coupling that enters in the calculation. For a recent review of the methodology see Ref. 127. For example, the energy levels of a $Q\overline{Q}$ system can be determined and then used to extract α_s. The masses of the $Q\overline{Q}$ states depend only on the quark mass and on α_s. A limitation is that calculations cannot be performed for three light quark flavors. Results are available for zero ($n_f = 0$, quenched approximation) and two light flavors, which allow extrapolation to three. The coupling constant so extracted is in a lattice renormalization scheme, and must be converted to the $\overline{\mathrm{MS}}$ scheme for comparison with other results. Using the mass differences of Υ and Υ' and Υ'' and χ_b, Davies *et al.* [128] extract a value of $\alpha_s(M_Z) = 0.1174 \pm 0.0024$. A similar result with larger errors is reported by [129], where results are consistent with $\alpha_s(M_Z) = 0.111 \pm 0.006$. A combination of the results from quenched [130] and ($n_f = 2$) [131] gives $\alpha_s(M_Z) = 0.116 \pm 0.003$ [132]. Calculations [133] using the strength of the force between two heavy quarks computed in the quenched approximation obtains a value of α_s(5 GeV) that is consistent with these results. There have also been investigations of the running of α_s [134]. These show remarkable agreement with the two loop perturbative result of Eq. (9.4).

There are several sources of error in these estimates of $\alpha_s(M_Z)$. The experimental error associated with the measurements of the particle masses is negligible. The conversion from the lattice coupling constant to the $\overline{\mathrm{MS}}$ constant is obtained using a perturbative expansion where one coupling expanded as a power series in the other. This series is only known to second order. A third order calculation exists only from the $n_f = 0$ case [135]. Its inclusion leads to a shift in the extracted value of $\alpha_s(M_Z)$ of $+0.002$. Other theoretical errors arising from the limited statistics of the Monte-Carlo calculation, extrapolation in n_f, and corrections for light quark masses are smaller than this.

The result with a more conservative error $\alpha_s(M_Z) = 0.117 \pm 0.003$ will be used in the average.

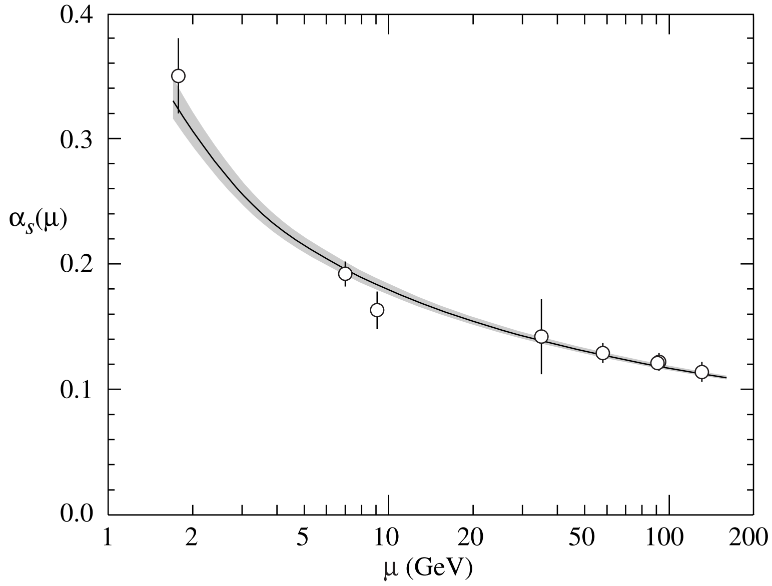

Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ.

9.11. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory, the interface of soft and hard QCD as manifest, for example, by hard diffractive processes, and QCD effects in hadron spectroscopy.

We have focused on those high-energy processes which currently offer the most quantitative tests of perturbative QCD. Figure 9.1 shows the values of $\alpha_s(M_Z)$ deduced from the various experiments. Figure 9.2 shows the values and the values of Q where they are measured. This figure clearly shows the experimental evidence for the variation of $\alpha_s(Q)$ with Q.

An average of the values in Fig. 9.1 gives $\alpha_s(M_z) = 0.1189$, with a total χ^2 of 3.3 for eleven fitted points, showing good consistency among the data. The error on the average, assuming that all of the errors in the contributing results are uncorrelated, is ± 0.0015, and is an underestimate. Almost all of the values used in the average are dominated by systematic, usually theoretical errors. Only some of these, notably from the choice of scale, are correlated. Two of the results with the smallest errors are the ones from τ decay and lattice gauge theory. If these errors are increased to ± 0.006, the average is unchanged and the error increases to 0.0020. We quote our average value as $\alpha_s(M_Z) = 0.119 \pm 0.002$, which corresponds to $\Lambda^{(5)} = 219^{+25}_{-23}$ MeV using Eq. (9.5a), only the two-loop result (*i.e.* dropping the last term in Eq. (9.5a)) gives $\Lambda^{(5)} = 237^{+26}_{-24}$ MeV. Future experiments can be expected to improve the measurements of α_s somewhat. Precision at the 1% level may be achievable if the systematic and theoretical errors can be reduced [136].

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