ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS

To reduce the size of this section's PostScript file, we have divided it into three PostScript files. We present the following index:

PART 1

Page #	Section name	
1	10.1	Introduction
2	10.2	Renormalization and radiative corrections
6	10.3	Cross-section and asymmetry formulas

PART 2

Page #	Section name	
12	10.4	W and Z decays
13	10.5	Experimental results

PART 3

Page #	Section name	
23	10.6	Constraints on new physics
29		References

10.6. Constraints on new physics

The Z pole, W mass, and neutral-current data can be used to search for and set limits on deviations from the Standard Model. In particular, the combination of these indirect data with the direct CDF and DØ value for m_t allows stringent limits on new physics. We will mainly discuss the effects of exotic particles (with heavy masses $M_{\rm new} \gg M_Z$ in an expansion in $M_Z/M_{\rm new}$) on the gauge boson self-energies. (Brief remarks are made on new physics which is not of this type.) Most of the effects on precision measurements can be described by three gauge self-energy parameters S, T, and U. We will define these, as well as related parameters, such as ρ_0, ϵ_i, and $\widehat{\epsilon}_i$, to arise from new physics only. *I.e.*, they are equal to zero ($\rho_0 = 1$) exactly in the Standard Model, and do not include any contributions from m_t or M_H, which are treated seperately. Our treatment differs from most of the original papers. We also allow a $Zb\overline{b}$ vertex correction parameter γ_b.

Many extensions of the Standard Model are described by the ρ_0 parameter:

$$\rho_0 \equiv M_W^2/(M_Z^2\,\widehat{c}_Z^{\,2}\,\widehat{\rho}) \ , \tag{10.42}$$

which describes new sources of SU(2) breaking that cannot be accounted for by Higgs doublets or m_t effects. In the presence of $\rho_0 \neq 1$, Eq. (10.42) generalizes Eq. (10.9*b*), while Eq. (10.9*a*) remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.12)–(10.14), (10.28), and Γ_Z in Eq. (10.35). There is now enough data to determine ρ_0, $\sin^2\theta_W$, m_t, and α_s simultaneously. In particular, the direct CDF and DØ events and R_b yield m_t independent of ρ_0, the asymmetries yield $\widehat{s}_Z^{\,2}$, R_ℓ gives α_s, and M_Z and the widths constrain ρ_0. From the global fit,

$$\rho_0 = 0.9998 \pm 0.0008\ (+0.0014) \ , \tag{10.43}$$

$$\widehat{s}_Z^{\,2} = 0.23126 \pm 0.00019\ (+0.00010) \ , \tag{10.44}$$

$$\alpha_s = 0.1219 \pm 0.0034\ (-0.0009) \ , \tag{10.45}$$

$$m_t = 174 \pm 5\ \ {\rm GeV} \ , \tag{10.46}$$

where the central values are for $M_H = M_Z$ and in parentheses we show the effect of changing M_H to 300 GeV. (As in the case $\rho_0 = 1$, the best fit value for M_H is below its direct lower limit.) The allowed regions in the $\rho_0 - \widehat{s}_Z^{\,2}$ plane are shown in Fig. 10.2.

The result in Eq. (10.43) is in remarkable agreement with the Standard Model expectation, $\rho_0 = 1$. It can be used to constrain higher-dimensional Higgs representations to have vacuum expectation values of less than a few percent of those of the doublets. Indeed, the relation between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters which one may conveniently choose to be α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\widehat{s}_Z^{\,2}$ and ρ_0 can be considered dependent parameters.

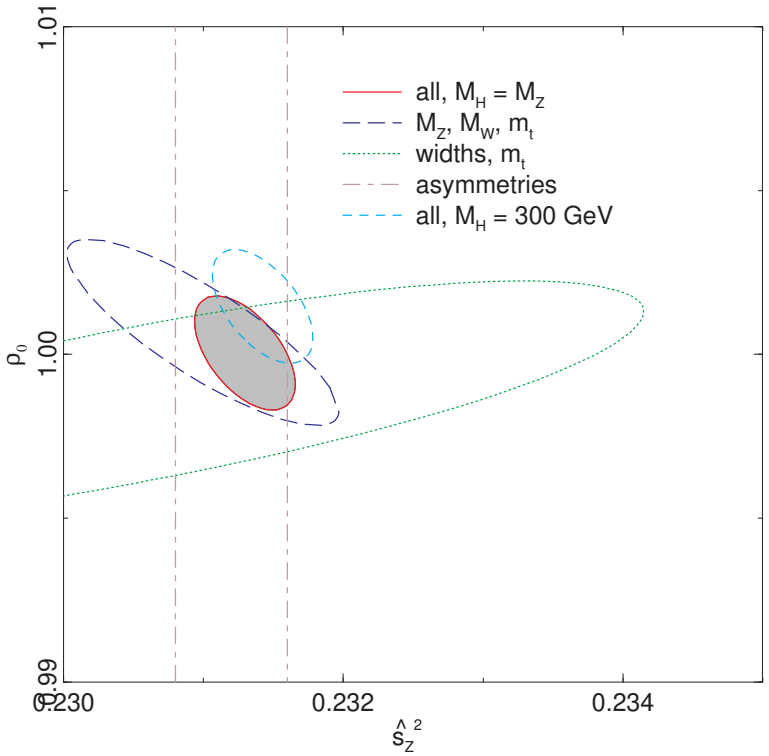

Figure 10.2: The allowed regions in $\sin^2\widehat{\theta}_W - \rho_0$ at 90% CL. m_t is a free parameter and $M_H = M_Z$ is assumed except for the dashed contour for all data which is for $M_H = 300$ GeV. The horizontal (width) band uses the experimental value of M_Z in Eq. (10.35).

Eq. (10.43) can also be used to constrain other types of new physics. For example, nondegenerate multiplets of heavy fermions or scalars break the vector part of weak SU(2) and lead to a decrease in the value of M_Z/M_W. A nondegenerate SU(2) doublet $\binom{f_1}{f_2}$ yields a positive contribution to ρ_t of [83]

$$\frac{CG_F}{8\sqrt{2}\pi^2}\Delta m^2 \ , \tag{10.47}$$

where

$$\Delta m^2 \equiv m_1^2 + m_2^2 - \frac{4m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1}{m_2} \geq (m_1 - m_2)^2 \ , \tag{10.48}$$

and $C = 1$ (3) for color singlets (triplets). Thus, in the presence of such multiplets, one has

$$\frac{3G_F}{8\sqrt{2}\pi^2}\sum_i \frac{C_i}{3}\Delta m_i^2 = \rho_0 - 1 \ , \tag{10.49}$$

where the sum includes fourth-family quark or lepton doublets, $\binom{t'}{b'}$ or $\binom{E^0}{E^-}$, and scalar doublets such as $\binom{\tilde{t}}{\tilde{b}}$ in supersymmetry (in the absence of $L-R$ mixing). This implies

$$\sum_i \frac{C_i}{3}\,\Delta m_i^2 < (49\ \text{GeV})^2 \ \text{ and } \ (83\ \text{GeV})^2 \tag{10.50}$$

for $M_H = M_Z$ and 300 GeV, respectively, at 90% CL.

Nondegenerate multiplets usually imply $\rho_0 > 1$. Similarly, heavy Z' bosons decrease the prediction for M_Z due to mixing and generally lead to $\rho_0 > 1$ [84]. On the other hand, additional Higgs doublets which participate in spontaneous symmetry breaking [85], heavy lepton doublets involving Majorana neutrinos [86], and the vacuum expectation values of Higgs triplets or higher-dimensional representations can contribute to ρ_0 with either sign. Allowing for the presence of heavy degenerate chiral multiplets (the S parameter, to be discussed below) affects the determination of ρ_0 from the data, at present leading to a smaller value.

A number of authors [87–92] have considered the general effects on neutral current and Z and W pole observables of various types of heavy (*i.e.*, $M_{\rm new} \gg M_Z$) physics which contribute to the W and Z self-energies but which do not have any direct coupling to the ordinary fermions. In addition to nondegenerate multiplets, which break the vector part of weak SU(2), these include heavy degenerate multiplets of chiral fermions which break the axial generators. The effects of one degenerate chiral doublet are small, but in technicolor theories there may be many chiral doublets and therefore significant effects [87].

Such effects can be described by just three parameters, S, T, and U at the (electroweak) one loop level. (Three additional parameters are needed if the new physics scale is comparable to M_Z [93].) T is proportional to the difference between the W and Z self-energies at $Q^2 = 0$ (*i.e.*, vector SU(2)-breaking), while S ($S+U$) is associated with the difference between the Z (W) self-energy at $Q^2 = M_{Z,W}^2$ and $Q^2 = 0$ (axial SU(2)-breaking). In the $\overline{\rm MS}$ scheme [20]

$$\begin{aligned}
\alpha(M_Z)T &\equiv \frac{\Pi^{\rm new}_{WW}(0)}{M_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{M_Z^2}\ , \\
\frac{\alpha(M_Z)}{4\widehat{s}^2_Z\widehat{c}^2_Z}S &\equiv \frac{\Pi^{\rm new}_{ZZ}(M_Z^2) - \Pi^{\rm new}_{ZZ}(0)}{M_Z^2}\ , \\
\frac{\alpha(M_Z)}{4\widehat{s}^2_Z}(S+U) &\equiv \frac{\Pi^{\rm new}_{WW}(M_W^2) - \Pi^{\rm new}_{WW}(0)}{M_W^2}\ ,
\end{aligned} \tag{10.51}$$

where $\Pi^{\rm new}_{WW}$ and $\Pi^{\rm new}_{ZZ}$ are, respectively, the contributions of the new physics to the W and Z self-energies. S, T, and U are defined with a factor of α removed, so that they are expected to be of order unity in the presence of new physics. They are related to other

parameters ($\widehat{\epsilon}_i$, h_i, S_i) defined in [20,88,89] by

$$\begin{aligned} T &= h_V = \widehat{\epsilon}_1/\alpha \ , \\ S &= h_{AZ} = S_Z = 4\widehat{s}_Z^2\widehat{\epsilon}_3/\alpha \ , \\ U &= h_{AW} - h_{AZ} = S_W - S_Z = -4\widehat{s}_Z^2\widehat{\epsilon}_2/\alpha \ . \end{aligned} \tag{10.52}$$

A heavy nondegenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 = \frac{1}{1-\alpha T} \simeq 1 + \alpha T \ , \tag{10.53}$$

where ρ_0 is given in Eq. (10.49). The effects of nonstandard Higgs representations cannot be separated from heavy nondegenerate multiplets unless the new physics has other consequences, such as vertex corrections. Most of the original papers defined T to include the effects of loops only. However, we will redefine T to include all new sources of SU(2) breaking, including nonstandard Higgs, so that T and ρ_0 are equivalent by Eq. (10.53).

A multiplet of heavy degenerate chiral fermions yields

$$S = C\sum_i \Big(t_{3L}(i) - t_{3R}(i)\Big)^2/3\pi \ , \tag{10.54}$$

where $t_{3L,R}(i)$ is the third component of weak isospin of the left- (right-) handed component of fermion i and C is the number of colors. For example, a heavy degenerate ordinary or mirror family would contribute $2/3\pi$ to S. In technicolor models with QCD-like dynamics, one expects [87] $S \sim 0.45$ for an isodoublet of technifermions, assuming $N_{TC} = 4$ technicolors, while $S \sim 1.62$ for a full technigeneration with $N_{TC} = 4$; T is harder to estimate because it is model dependent. In these examples one has $S \geq 0$. However, the QCD-like models are excluded on other grounds (flavor-changing neutral currents, and too-light quarks and pseudo-Goldstone bosons [94]). In particular, these estimates do not apply to models of walking technicolor [94], for which S can be smaller or even negative [95]. Other situations in which $S < 0$, such as loops involving scalars or Majorana particles, are also possible [96]. Supersymmetric extensions of the Standard Model generally give very small effects [97]. Most simple types of new physics yield $U = 0$, although there are counter-examples, such as the effects of anomalous triple-gauge vertices [89].

The Standard Model expressions for observables are replaced by

$$\begin{aligned} M_Z^2 &= M_{Z0}^2 \frac{1-\alpha T}{1 - G_F M_{Z0}^2 S/2\sqrt{2}\pi} \ , \\ M_W^2 &= M_{W0}^2 \frac{1}{1 - G_F M_{W0}^2 (S+U)/2\sqrt{2}\pi} \ , \end{aligned} \tag{10.55}$$

where M_{Z0} and M_{W0} are the Standard Model expressions (as functions of m_t and M_H) in the $\overline{\rm MS}$ scheme. Furthermore,

$$\begin{aligned} \Gamma_Z &= \frac{1}{1-\alpha T} M_Z^3 \beta_Z \ , \\ \Gamma_W &= M_W^3 \beta_W \ , \\ A_i &= \frac{1}{1-\alpha T} A_{i0} \ , \end{aligned} \tag{10.56}$$

where β_Z and β_W are the Standard Model expressions for the reduced widths Γ_{Z0}/M_{Z0}^3 and Γ_{W0}/M_{W0}^3, M_Z and M_W are the physical masses, and A_i (A_{i0}) is a neutral current amplitude (in the Standard Model).

The $Z \to b\overline{b}$ vertex is sensitive to certain types of new physics which primarily couple to heavy families. It is useful to introduce an additional parameter γ_b by [98]

$$\Gamma(Z \to b\overline{b}) = \Gamma^0(Z \to b\overline{b})(1 + \gamma_b) \ , \tag{10.57}$$

where Γ^0 is the Standard Model expression (or the expression modified by S, T, and U). Experimentally, R_b is 1.3 σ above the Standard Model expectations, favoring a positive γ_b. Extended technicolor interactions generally yield negative values of γ_b of a few percent [99], although it is possible to obtain a positive γ_b in models for which the extended technicolor group does not commute with the electroweak gauge group [100] or for which diagonal interactions related to the extended technicolor dominate [101]. Topcolor and topcolor-assisted technicolor models do not generally give a significant contribution to γ_b because the extended technicolor contribution to m_t is small [102]. Supersymmetry can yield (typically small) contributions of either sign [103,104].

The data allow a simultaneous determination of $\widehat{s}^2_Z$ (*e.g.*, from the Z pole asymmetries), S (from M_Z), U (from M_W), T (*e.g.*, from the Z decay widths), α_s (from R_ℓ), m_t (from CDF and DØ), and γ_b (from R_b) with little correlation among the Standard Model parameters:

$$\begin{aligned} S &= -0.16 \pm 0.14\,(-0.10) \ , \\ T &= -0.21 \pm 0.16\,(+0.10) \ , \\ U &= \ \ 0.25 \pm 0.24\,(+0.01) \ , \\ \gamma_b &= \ \ 0.007 \pm 0.005 \ , \end{aligned} \tag{10.58}$$

and $\widehat{s}^2_Z = 0.23118 \pm 0.00023$, $\alpha_s = 0.1191 \pm 0.0051$, $m_t = 175 \pm 5$ GeV, where the uncertainties are from the inputs. The central values assume $M_H = M_Z$, and in parentheses we show the change for $M_H = 300$ GeV. The parameters in Eq. (10.58) which by definition are due to new physics only, are all consistent with the Standard Model values of zero near the 1σ level, although at present there is a slight tendency for negative S and T, and positive U and γ_b. With the latest value of R_b, the extracted $\alpha_s = 0.1191 \pm 0.0051$ is now in perfect agreement with other determinations, even in the presence of the large class of new physics allowed in this fit. Its error is slightly higher than in Eq. (10.38) for the Standard Model, but the central value is independent of M_H. Using Eq. (10.53) the value of ρ_0 corresponding to T is 0.9984 ± 0.0012 (+0.0008). The values of the $\widehat{\epsilon}$ parameters defined in Eq. (10.52) are

$$\begin{aligned} \widehat{\epsilon}_3 &= -0.0013 \pm 0.0012\,(-0.0009) \ , \\ \widehat{\epsilon}_1 &= -0.0016 \pm 0.0012\,(+0.0008) \ , \\ \widehat{\epsilon}_2 &= -0.0022 \pm 0.0021\,(-0.0001) \ . \end{aligned} \tag{10.59}$$

There is a strong correlation between γ_b and the predicted α_s (the correlation coefficient is -0.69), just as in the model with $S = T = U = 0$ [17]. For $\gamma_b = 0$ one obtains

$\alpha_s = 0.1239 \pm 0.0037$, with little change in the other parameters. The largest correlation coefficient ($+0.73$) is between S and T. The allowed region in $S-T$ is shown in Fig. 10.3. From Eq. (10.58) one obtains $S < 0.03\,(0.08)$ and $T < 0.09\,(0.15)$ at $90\,(95)\%$ CL for $M_H = M_Z$ (S) and 300 GeV (T). If one fixes $M_H = 600$ GeV and requires the constraint $S \geq 0$ (as is appropriate in QCD-like technicolor models) then $S < 0.12\,(0.15)$. Allowing arbitrary S, an extra generation of ordinary fermions is now excluded at the 99.2% CL. This is in agreement with a fit to the number of light neutrinos, $N_\nu = 2.993 \pm 0.011$. The favored value of S is problematic for simple technicolor models with many techni-doublets and QCD-like dynamics, as is the value of γ_b. Although S is consistent with zero, the electroweak asymmetries, especially the SLD left-right asymmetry, favor $S < 0$. The simplest origin of $S < 0$ would probably be an additional heavy Z' boson [84], which could mimic $S < 0$. Similarly, there is a slight indication of negative T, while, as discussed above, nondegenerate scalar or fermion multiplets generally predict $T > 0$.

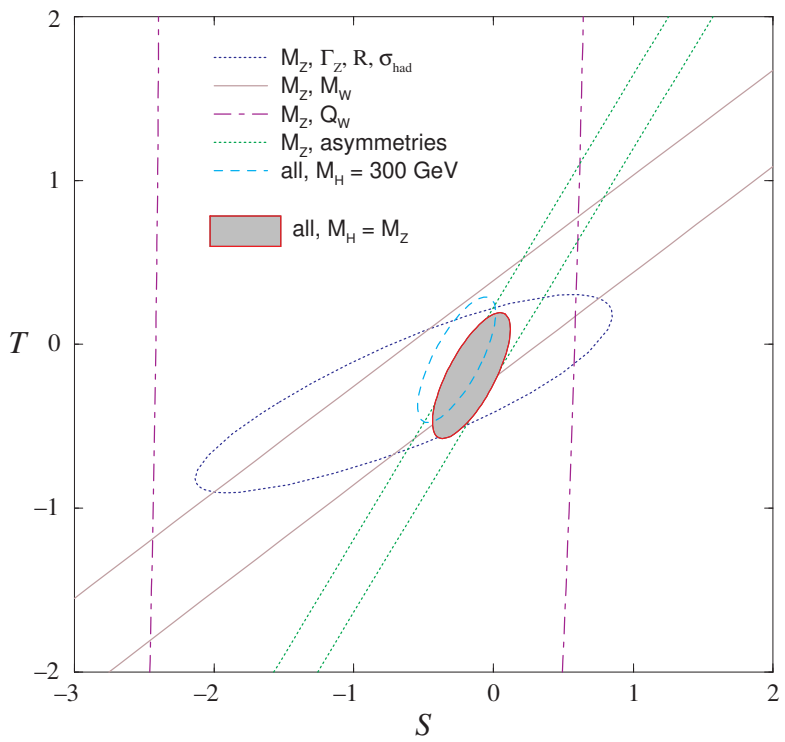

Figure 10.3: 90% CL limits on S and T from various inputs. S and T represent the contributions of new physics only. (Uncertainties from m_t are included in the errors.) The contours assume $M_H = M_Z$ except for the dashed contour for all data which is for $M_H = 300$ GeV. The fit to M_W and M_Z assumes $U = 0$, while U is arbitrary in the other fits.

There is no simple parametrization that is powerful enough to describe the effects of every type of new physics on every possible observable. The S, T, and U formalism

describes many types of heavy physics which affect only the gauge self-energies, and it can be applied to all precision observables. However, new physics which couples directly to ordinary fermions, such as heavy Z' bosons [84] or mixing with exotic fermions [105] cannot be fully parametrized in the S, T, and U framework. It is convenient to treat these types of new physics by parametrizations that are specialized to that particular class of theories (*e.g.*, extra Z' bosons), or to consider specific models (which might contain, *e.g.*, Z' bosons and exotic fermions with correlated parameters). Constraints on various types of new physics are reviewed in [17,106,107]. Fits to models with technicolor, extended technicolor, and supersymmetry are described, respectively, in [100], [108], and [109]. An alternate formalism [110] defines parameters, ϵ_1, ϵ_2, ϵ_3, ϵ_b in terms of the specific observables M_W/M_Z, $\Gamma_{\ell\ell}$, $A_{FB}^{(0,\ell)}$, and R_b. The definitions coincide with those for $\widehat{\epsilon}_i$ in Eqs. (10.51) and (10.52) for physics which affects gauge self-energies only, but the ϵ's now parametrize arbitrary types of new physics. However, the ϵ's are not related to other observables unless additional model-dependent assumptions are made. Another approach [111–113] parametrizes new physics in terms of gauge-invariant sets of operators. It is especially powerful in studying the effects of new physics on nonabelian gauge vertices. The most general approach introduces deviation vectors [106]. Each type of new physics defines a deviation vector, the components of which are the deviations of each observable from its Standard Model prediction, normalized to the experimental uncertainty. The length (direction) of the vector represents the strength (type) of new physics.

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