$K^{\pm}$ – THIS IS PART 1 OF 3

To reduce the size of this section's PostScript file, we have divided it into three PostScript files. We present the following index:

PART 1

PART 2

PART 3

$K^{\pm}$

$I(J^P) = \frac{1}{2}(0^-)$

THE CHARGED KAON MASS

Revised 1994 by T.G. Trippe, (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

$$m_{K^{\pm}} = 493.677 \pm 0.013 \text{ MeV (S = 2.4)} , \qquad (1)$$

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

$$m_{K^{\pm}} = 493.677 \pm 0.005 \text{ MeV} ,$$
$$\chi^2 = 22.9 \text{ for 5 D.F., Prob. } = 0.04\% , \qquad (2)$$

where the high χ^2 and correspondingly low χ^2 probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

$$m_{K^{\pm}} = 493.696 \pm 0.007 \text{ MeV} \qquad \text{DENISOV 91}$$
$$m_{K^{\pm}} = 493.636 \pm 0.011 \text{ MeV (S = 1.5)} \qquad \text{GALL 88}$$

$$\text{Average} = 493.679 \pm 0.006 \text{ MeV}$$
$$\chi^2 = 21.2 \text{ for 1 D.F., Prob. } = 0.0004\% , \qquad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high χ^2.

The GALL 88 measurement was made using four different kaonic atom transitions, K^- Pb $(9 \rightarrow 8)$, K^- Pb $(11 \rightarrow 10)$, K^- W $(9 \rightarrow 8)$, and K^- W $(11 \rightarrow 10)$. The $m_{K^\pm}$ values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1Their K^- Pb $(9 \rightarrow 8)$ $m_{K^\pm}$is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

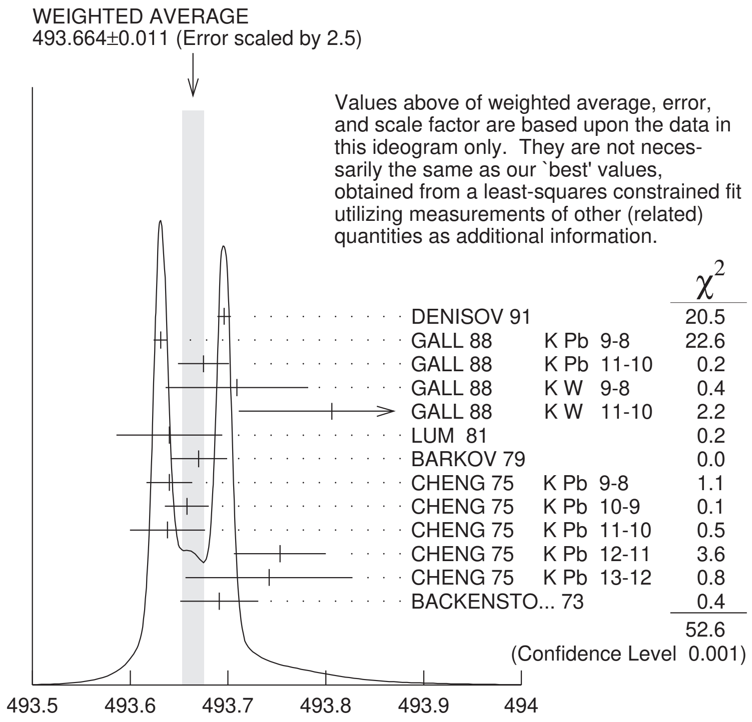
$$m_{K^\pm} = 493.636 \pm 0.007 \ ,$$

$$\chi^2 = 7.0 \ \text{for 3 D.F., Prob.} \ = 7.2\% \ . \tag{4}$$

This is a low but acceptable χ^2 probability so, to be conservative, GALL 88 scaled up the error on their average by S=1.5 to obtain their published error ± 0.011 shown in Eq. (3) above and used in the Particle Listings average.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88 K^- Pb $(9 \rightarrow 8)$ measurement yield two well-separated peaks. One might suspect the GALL 88 K^- Pb $(9 \rightarrow 8)$ measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the K^- Pb $(9 \rightarrow 8)$ transition, we have separated the CHENG 75 data, which also used K^- Pb, into its separate transitions. Fig. 1shows that the CHENG 75 and GALL 88 K^- Pb $(9 \rightarrow 8)$ values are consistent, suggesting the possibility of a common effect such as contaminant nuclear γ rays near the K^- Pb $(9 \rightarrow 8)$ transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a χ^2 of 52.6 as shown in Fig. 1and the first line of Table 1, yielding an unacceptable χ^2 probability of 0.00005%. The second line of Table 1



$m_{K^\pm}$ (MeV)

Figure 1: Ideogram of $m_{K^\pm}$ mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

excludes both the GALL 88 and CHENG 75 measurements of the K^- Pb ($9 \rightarrow 8$) transition and yields a χ^2 probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88 K^- Pb ($9 \rightarrow 8$) [DENISOV 91] measurement and yields a χ^2 probability of 20% [8.6%]. Table 1 shows that removing both measurements of the K^- Pb ($9 \rightarrow 8$) transition produces the most consistent set of data, but that excluding only the GALL 88 K^- Pb $(9 \to 8)$ transition or DENISOV 91 also produces acceptable probabilities.

Table 1: $m_{K^\pm}$ averages for some combinations of Fig. 1data.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.664 ± 0.004	52.6	12	0.00005	all 13 measurements
493.690 ± 0.006	10.1	10	43	no K^- Pb(9→8)
493.687 ± 0.006	14.6	11	20	no GALL 88 K^- Pb(9→8)
493.642 ± 0.006	17.8	11	8.6	no DENISOV 91

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved ^{192}Ir and ^{198}Au calibration γ-ray energies. He estimates that CHENG 75 and BACKENSTOSS 73 $m_{K^\pm}$ values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88 K^- Pb $(9 \to 8)$ is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88 K^- Pb $(9 \to 8)$ transition, but we note that a future reanalysis of the CHENG 75 data could be useful because it might provide supporting evidence for such a rejection.

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear γ rays. Studies of γ rays following stopped π^- and Σ^- absorption in nucleii (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91

Table 2: $m_{K^\pm}$ averages for some combinations of Fig. 1data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

$m_{K^\pm}$ (MeV)	χ^2	D.F.	Prob. (%)	Measurements used
493.666 ± 0.004	53.9	12	0.00003	all 13 measurements
493.693 ± 0.006	9.0	10	53	no K^- Pb(9→8)
493.690 ± 0.006	11.5	11	40	no GALL 88 K^- Pb(9→8)
493.645 ± 0.006	23.0	11	1.8	no DENISOV 91

measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in K^- ^{12}C. The high resolution and the light nucleus reduce the probability for overlap by contaminant γ rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in π^- ^{12}C, which is good agreement with the calculated energy.

While we suspect that the GALL 88 K^- Pb $(9 \to 8)$ measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.

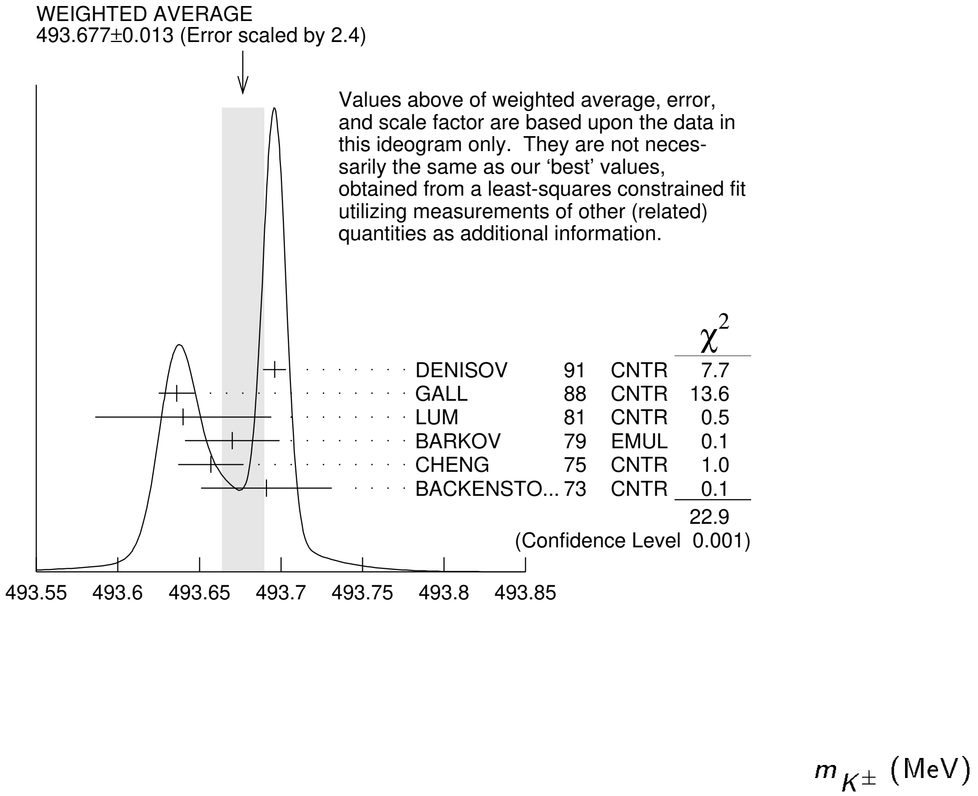
$K^{\pm}$ MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
493.677±0.016 OUR FIT	Error includes scale factor of 2.8.				
493.677±0.013 OUR AVERAGE	Error includes scale factor of 2.4. See the ideogram below.				
493.696±0.007	[1] DENISOV	91	CNTR	−	Kaonic atoms
493.636±0.011	[2] GALL	88	CNTR	−	Kaonic atoms
493.640±0.054	LUM	81	CNTR	−	Kaonic atoms
493.670±0.029	BARKOV	79	EMUL	±	$e^+ e^- \rightarrow K^+ K^-$
493.657±0.020	[2] CHENG	75	CNTR	−	Kaonic atoms
493.691±0.040	BACKENSTO...	73	CNTR	−	Kaonic atoms
• • • We do not use the following data for averages, fits, limits, etc. • • •					
493.631±0.007	GALL	88	CNTR	−	K^- Pb (9→ 8)
493.675±0.026	GALL	88	CNTR	−	K^- Pb (11→ 10)
493.709±0.073	GALL	88	CNTR	−	K^- W (9→ 8)
493.806±0.095	GALL	88	CNTR	−	K^- W (11→ 10)
493.640±0.022±0.008	[3] CHENG	75	CNTR	−	K^- Pb (9→ 8)
493.658±0.019±0.012	[3] CHENG	75	CNTR	−	K^- Pb (10→ 9)
493.638±0.035±0.016	[3] CHENG	75	CNTR	−	K^- Pb (11→ 10)
493.753±0.042±0.021	[3] CHENG	75	CNTR	−	K^- Pb (12→ 11)
493.742±0.081±0.027	[3] CHENG	75	CNTR	−	K^- Pb (13→ 12)
493.662±0.19	KUNSELMAN	74	CNTR	−	Kaonic atoms
493.78 ±0.17	GREINER	65	EMUL	+	
493.7 ±0.3	BARKAS	63	EMUL	−	
493.9 ±0.2	COHEN	57	RVUE	+	

[1] Error increased from 0.0059 based on the error analysis in IVANOV 92.

[2] This value is the authors' combination of all of the separate transitions listed for this paper.

[3] The CHENG 75 values for separate transitions were calculated from their Table 7 transition energies. The first error includes a 20% systematic error in the noncircular contaminant shift. The second error is due to a ±5 eV uncertainty in the theoretical transition energies.



$m_{K^+} - m_{K^-}$

Test of *CPT*.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
$-$0.032±0.090	1.5M	[4] FORD	72	ASPK	±

[4] FORD 72 uses $m_{\pi^+} - m_{\pi^-} = +28 \pm 70$ keV.

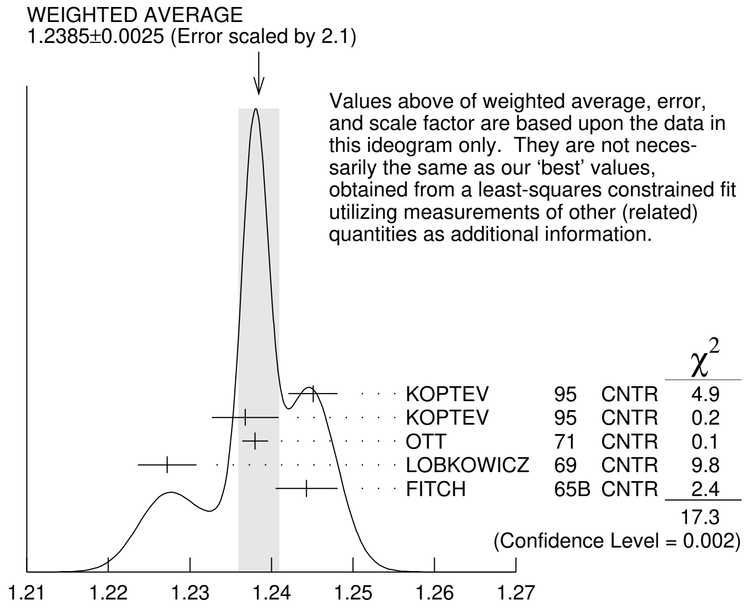
$K^\pm$ MEAN LIFE

VALUE (10^{-8} s)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1.2386±0.0024 OUR FIT	Error includes scale factor of 2.0.					
1.2385±0.0025 OUR AVERAGE	Error includes scale factor of 2.1. See the ideogram below.					
1.2451±0.0030	250k	KOPTEV	95	CNTR		K at rest, U target
1.2368±0.0041	150k	KOPTEV	95	CNTR		K at rest, Cu target
1.2380±0.0016	3M	OTT	71	CNTR	+	K at rest
1.2272±0.0036		LOBKOWICZ	69	CNTR	+	K in flight
1.2443±0.0038		FITCH	65B	CNTR	+	K at rest

• • • We do not use the following data for averages, fits, limits, etc. • • •

1.2415 ± 0.0024	400k	[5] KOPTEV	95	CNTR		K at rest
1.221 ± 0.011		FORD	67	CNTR	±	
1.231 ± 0.011		BOYARSKI	62	CNTR	+	
1.25 $^{+0.22}_{-0.17}$		BARKAS	61	EMUL		
1.27 $^{+0.36}_{-0.23}$	51	BHOWMIK	61	EMUL		
1.31 ± 0.08	293	NORDIN	61	HBC	−	
1.24 ± 0.07		NORDIN	61	RVUE	−	
1.38 ± 0.24	33	FREDEN	60B	EMUL		
1.21 ± 0.06		BURROWES	59	CNTR		
1.60 ± 0.3	52	EISENBERG	58	EMUL		
0.95 $^{+0.36}_{-0.25}$		ILOFF	56	EMUL		

[5] KOPTEV 95 report this weighted average of their U-target and Cu-target results, where they have weighted by $1/\sigma$ rather than $1/\sigma^2$.



$K^{\pm}$ mean life (10^{-8} s)

$(\tau_{K^+} - \tau_{K^-}) / \tau_{\text{average}}$

This quantity is a measure of *CPT* invariance in weak interactions.

VALUE (%)	*DOCUMENT ID*		*TECN*
0.11 ±0.09 OUR AVERAGE	Error includes scale factor of 1.2.		
0.090 ± 0.078	LOBKOWICZ	69	CNTR
0.47 ± 0.30	FORD	67	CNTR

RARE KAON DECAYS

Revised November 1997 by L. Littenberg (BNL) and G. Valencia (Iowa State University)

A. Introduction: There are several useful reviews on rare kaon decays and related topics [1–10]. The current activity in rare kaon decays can be divided roughly into four categories:

1. Searches for explicit violations of the Standard Model
2. Measurements of Standard Model parameters
3. Searches for CP violation
4. Studies of strong interactions at low energy.

The paradigm of Category 1 is the lepton flavor violating decay $K_L \to \mu e$. Category 2 includes processes such as $K^+ \to \pi^+ \nu\overline{\nu}$, which is sensitive to $|V_{td}|$. Much of the interest in Category 3 is focussed on the decays $K_L \to \pi^0 \ell\overline{\ell}$, where $\ell \equiv e, \mu, \nu$. Category 4 includes reactions like $K^+ \to \pi^+\ell^+\ell^-$ which constitute a testing ground for the ideas of chiral perturbation theory. Other reactions of this type are $K_L \to \pi^0\gamma\gamma$, which also scales a CP-conserving background to CP violation in $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \gamma\ell^+\ell^-$, which could possibly shed light on long distance contributions to $K_L \to \mu^+\mu^-$.

B. Explicit violations of the Standard Model: Most of the activity here is in searches for lepton flavor violation (LFV). This is motivated by the fact that many extensions of the minimal Standard Model violate lepton flavor and by the potential to access very high energy scales. For example, the tree-level exchange of a LFV vector boson of mass M_X that couples to left-handed fermions with electroweak strength and without mixing angles yields $\mathrm{B}(K_L \to \mu e) = 3.3 \times 10^{-11} (91\ \mathrm{TeV}/M_X)^4$ [5]. This simple dimensional analysis may be used to read from Table 1 that the reaction $K_L \to \mu e$ is already probing scales of nearly 100 TeV. Table 1 summarizes the present experimental situation vis a vis LFV, along with the expected near-future progress. The decays $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^\mp \mu^\pm$ (or $K_L \to \pi^0 e^\mp \mu^\pm$) provide complementary information on potential family number violating interactions since the former is sensitive to axial-vector (or pseudoscalar) couplings and the latter is sensitive to vector (or scalar) couplings.

Table 1: Searches for lepton flavor violation in K decay

Mode	90% CL upper limit	Exp't	Yr./Ref.	(Near-) future aim
$K^+ \to \pi^+ e\mu$	$2.1 \cdot 10^{-10}$	BNL-777	90/11	$3 \cdot 10^{-12}$ (BNL-865)
$K_L \to \mu e$	$3.3 \cdot 10^{-11}$	BNL-791	93/12	$3 \cdot 10^{-12}$ (BNL-871)
$K_L \to \pi^0 e\mu$	$3.2 \cdot 10^{-9}$	FNAL-799	94/13	$5 \cdot 10^{-11}$ (KTeV)

Another forbidden decay currently being pursued is $K^+ \to \pi^+ X^0$, where X^0 is a very light, noninteracting particle (*e.g.* hyperphoton, axion, familon, etc.). Recently the upper limit on this process has been improved to 3×10^{-10} [15]. Data already collected by BNL-787 are expected to yield a further factor in sensitivity to this process.

C. Measurements of Standard Model parameters: Until recently, searches for $K^+ \to \pi^+ \nu\overline{\nu}$ have been motivated by the possibility of observing non-SM physics because the sensitivity attained was far short of the SM prediction for this decay [16] and long-distance contributions were known to be negligible [2]. However, BNL-787 has attained the sensitivity at which the observation of an event can no longer be unambiguously attributed to non-SM physics. The previous 90% CL upper limit [14] is 2.4×10^{-9}, but running with an upgraded beam and detector

BNL-787 recently observed one candidate event, corresponding to a branching ratio of $(4.2^{+9.7}_{-3.5}) \times 10^{-10}$ [15]. Further data already collected are expected to increase the sensitivity by more than a factor 2, and there are plans to collect data representing a further large increase in sensitivity. This reaction is now interesting from the point of view of constraining SM parameters. The branching ratio can be written in terms of the very well-measured rate of K_{e3} as [2]:

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) = \frac{\alpha^2 \mathrm{B}(K^+ \to \pi^o e^+ \nu)}{V_{us}^2 2\pi^2 \sin^4\theta_W} \times \sum_{l=e,\mu,\tau} |V_{cs}^* V_{cd} X_{NL}^{\ell} + V_{ts}^* V_{td} X(m_t)|^2 \quad (1)$$

to eliminate the *a priori* unknown hadronic matrix element. Isospin breaking corrections to the ratio of matrix elements reduce this rate by 10% [17]. In Eq. (1) the Inami-Lim function $X(m_t)$ is of order 1 [18], and X_{NL}^{ℓ} is several hundred times smaller. This form exhibits the strong dependence of this branching ratio on $|V_{td}|$. QCD corrections, which are contained in X_{NL}^{ℓ}, are relatively small and now known [10] to $\leq 10\%$. Evaluating the constants in Eq. (1) with $m_t = 175$ GeV, one can cast this result in terms of the CKM parameters A, ρ and η (see our Section on "The Cabibbo-Kobayashi-Maskawa mixing matrix") [10]

$$\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu}) \approx 1.0 \times 10^{-10} A^4 [\eta^2 + (\rho_o - \rho)^2] \quad (2)$$

where $\rho_o \equiv 1 + (\frac{2}{3} X_{NL}^{e} + \frac{1}{3} X_{NL}^{\tau})/(A^2 V_{us}^4 X(m_t)) \approx 1.4$. Thus, $\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})$ determines a circle in the ρ, η plane with center $(\rho_o, 0)$ and radius $\approx \frac{1}{A^2}\sqrt{\frac{\mathrm{B}(K^+ \to \pi^+ \nu\overline{\nu})}{1.0 \times 10^{-10}}}$.

The decay $K_L \to \mu^+\mu^-$ also has a short distance contribution sensitive to the CKM parameter ρ. For $m_t = 175$ GeV it is given by [10]:

$$\mathrm{B}_{\mathrm{SD}}(K_L \to \mu^+\mu^-) \approx 1.7 \times 10^{-9} A^4(\rho_o' - \rho)^2 \tag{3}$$

where ρ_o' depends on the charm quark mass and is around 1.2. This decay, however, is dominated by a long-distance contribution from a two-photon intermediate state. The absorptive (imaginary) part of the long-distance component is calculated in terms of the measured rate for $K_L \to \gamma\gamma$ to be $\mathrm{B}_{\mathrm{abs}}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$; and it almost completely saturates the observed rate $\mathrm{B}(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9}$ listed in the current edition. The difference between the observed rate and the absorptive component can be attributed to the (coherent) sum of the short-distance amplitude and the real part of the long-distance amplitude. In order to use this mode to constrain ρ it is, therefore, necessary to know the real part of the long-distance contribution. Unlike the absorptive part, the real part of the long-distance contribution cannot be derived from the measured rate for $K_L \to \gamma\gamma$. At present, it is not possible to compute this long-distance component reliably and, therefore, it is not possible to constrain ρ from this mode. It is expected that studies of the reactions $K_L \to \ell^+\ell^-\gamma$, and $K_L \to \ell^+\ell^-\ell'^+\ell'^-$ for $\ell, \ell' = e$ or μ will improve our understanding of the long distance effects in $K_L \to \mu^+\mu^-$ (the current data is parameterized in terms of α_K^*, discussed on page 24 of the K_L^0 Particle Properties Listing in our 1997 WWW update).

D. Searches for CP violation: The mode $K_L \to \pi^0\nu\overline{\nu}$ is dominantly CP-violating and free of hadronic uncertainties [2,19]. The Standard Model predicts a branching ratio

$\sim 10^{-11} - 10^{-10}$; for $m_t = 175$ GeV it is given approximately by [10]:

$$\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \approx 4.1 \times 10^{-10} A^4 \eta^2 \ . \qquad (4)$$

The current published upper bound is $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu}) \leq 5.8 \times 10^{-5}$ [20] and KTeV (FNAL799II) is expected to place a bound of order 10^{-8} [21]. The KTeV group has recently quoted a preliminary result of 1.8×10^{-6} [22]. If lepton flavor is conserved, the 90% CL bound on $K^+ \to \pi^+ \nu\bar{\nu}$ provides the model independent bound $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.1 \times 10^{-8}$ [23]. A recent proposal, BNL-926 [24], aims to make a $\sim 15\%$ measurement of $\mathrm{B}(K_L \to \pi^0 \nu\overline{\nu})$. There is also a Fermilab EOI [25] with comparable goals.

The decay $K_L \to \pi^0 e^+ e^-$ also has sensitivity to the product $A^4 \eta^2$. It has a direct CP-violating component that depends on the value of the top-quark mass, and that for $m_t = 175$ GeV is given by [10]:

$$\mathrm{B}_{\mathrm{dir}}(K_L \to \pi^0 e^+ e^-) \approx 6.7 \times 10^{-11} A^4 \eta^2 \ . \qquad (5)$$

However, like $K_L \to \mu^+ \mu^-$ this mode suffers from large theoretical uncertainties due to long distance strong interaction effects. It has an indirect CP-violating component given by:

$$\mathrm{B}_{\mathrm{ind}}(K_L \to \pi^0 e^+ e^-) = |\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} \mathrm{B}(K_S \to \pi^0 e^+ e^-) \ , \qquad (6)$$

that has been estimated to be less than 10^{-12} [26], but that will not be known precisely until a measurement of $K_S \to \pi^0 e^+ e^-$ is available [4,27]. There is also a CP-conserving component dominated by a two-photon intermediate state that cannot be computed reliably at present. This component has an absorptive part that can be, in principle, determined from a detailed analysis of $K_L \to \pi^0 \gamma\gamma$.

An analysis of $K_L \to \pi^0\gamma\gamma$ within chiral perturbation theory has been carried out in terms of a parameter a_V [28] that determines both the rate and the shape of the distribution $d\Gamma/dm_{\gamma\gamma}$. A fit to the distribution has given $-0.32 < a_V < 0.19$ [29]; a value that suggests that the absorptive part of the CP-conserving contribution to $K_L \to \pi^0 e^+e^-$ is significantly smaller than the direct CP-violating component [29]. However, there remains some uncertainty in the interpretation of $K_L \to \pi^0\gamma\gamma$ in terms of a_V. Analyses that go beyond chiral perturbation theory have found larger values of a_V, helping with understanding the rate in that process [30]. This would indicate a sizeable CP-conserving component to $K_L \to \pi^0e^+e^-$. The real part of the CP-conserving contribution to $K_L \to \pi^0e^+e^-$ is also unknown. The related process, $K_L \to \pi^0\gamma e^+e^-$, is an additional background in some region of phase space [31].

Finally, BNL-845 observed a potential background to $K_L \to \pi^0e^+e^-$ from the decay $K_L \to \gamma\gamma e^+e^-$ [32]. This was later confirmed with an order of magnitude larger sample by FNAL-799 [33], which measured additional kinematic quantities. It has been estimated that this background will enter at the level of 10^{-11} [34], comparable to the signal level. Because of this, the observation of $K_L \to \pi^0e^+e^-$ will depend on background subtraction with good statistics.

The current upper bound for the process $K_L \to \pi^0e^+e^-$ is 4.3×10^{-9} [35]. For the closely related muonic process, the upper bound is $\mathrm{B}(K_L \to \pi^0\mu^+\mu^-) \leq 5.1 \times 10^{-9}$ [36]. KTeV expects to reach a sensitivity of roughly 10^{-11} for both reactions [21].

E. Other long distance dominated modes: The decays $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e$ or μ) are described by chiral perturbation theory in terms of one parameter, ω^+ [37]. This parameter determines both the rate and distribution $d\Gamma/dm_{\ell\ell}$ for these processes. A careful study of these two reactions can provide a measurement of ω^+ and a test of the chiral perturbation theory description. A simultaneous fit to the rate and spectrum of $K^+ \to \pi^+ e^+ e^-$ gives: $\omega^+ = 0.89^{+0.24}_{-0.14}$; $\mathrm{B}(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}$ [38]. These two results satisfy the prediction of chiral perturbation theory within two standard deviations [4]. Improved statistics for this mode and a measurement of the mode $K^+ \to \pi^+ \mu^+ \mu^-$ are thus desired. BNL-787 has recently measured $B(K^+ \to \pi^+ \mu^+ \mu^-) = (5.0 \pm 1.0) \times 10^{-8}$ [39] which is at about the predicted level, but the result is not yet accurate enough to provide additional constraints.

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K^+ DECAY MODES

K^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\mu^+\nu_\mu$		(63.51 ± 0.18) %	S=1.3
Γ_2	$e^+\nu_e$		$(1.55\pm0.07)\times10^{-5}$	
Γ_3	$\pi^+\pi^0$		(21.16 ± 0.14) %	S=1.1
Γ_4	$\pi^+\pi^+\pi^-$		(5.59 ± 0.05) %	S=1.8
Γ_5	$\pi^+\pi^0\pi^0$		(1.73 ± 0.04) %	S=1.2
Γ_6	$\pi^0\mu^+\nu_\mu$		(3.18 ± 0.08) %	S=1.5
	Called $K^+_{\mu3}$.			
Γ_7	$\pi^0 e^+\nu_e$		(4.82 ± 0.06) %	S=1.3
	Called K^+_{e3}.			
Γ_8	$\pi^0\pi^0 e^+\nu_e$		$(2.1\pm0.4)\times10^{-5}$	
Γ_9	$\pi^+\pi^- e^+\nu_e$		$(3.91\pm0.17)\times10^{-5}$	
Γ_{10}	$\pi^+\pi^-\mu^+\nu_\mu$		$(1.4\pm0.9)\times10^{-5}$	
Γ_{11}	$\pi^0\pi^0\pi^0 e^+\nu_e$		$< 3.5\times10^{-6}$	CL=90%
Γ_{12}	$\pi^+\gamma\gamma$	[a]	$(1.10\pm0.32)\times10^{-6}$	
Γ_{13}	$\pi^+3\gamma$	[a]	$< 1.0\times10^{-4}$	CL=90%
Γ_{14}	$\mu^+\nu_\mu\nu\overline{\nu}$		$< 6.0\times10^{-6}$	CL=90%
Γ_{15}	$e^+\nu_e\nu\overline{\nu}$		$< 6\times10^{-5}$	CL=90%
Γ_{16}	$\mu^+\nu_\mu e^+e^-$		$(1.3\pm0.4)\times10^{-7}$	
Γ_{17}	$e^+\nu_e e^+e^-$		$(3.0\ ^{+3.0}_{-1.5})\times10^{-8}$	
Γ_{18}	$\mu^+\nu_\mu\mu^+\mu^-$		$< 4.1\times10^{-7}$	CL=90%

Γ_{19}	$\mu^+ \nu_\mu \gamma$		[a,b]	$(5.50 \pm 0.28) \times 10^{-3}$	
Γ_{20}	$\pi^+ \pi^0 \gamma$		[a,b]	$(2.75 \pm 0.15) \times 10^{-4}$	
Γ_{21}	$\pi^+ \pi^0 \gamma$(DE)		[a,c]	$(1.8 \pm 0.4) \times 10^{-5}$	
Γ_{22}	$\pi^+ \pi^+ \pi^- \gamma$		[a,b]	$(1.04 \pm 0.31) \times 10^{-4}$	
Γ_{23}	$\pi^+ \pi^0 \pi^0 \gamma$		[a,b]	$(7.5 {}^{+5.5}_{-3.0}) \times 10^{-6}$	
Γ_{24}	$\pi^0 \mu^+ \nu_\mu \gamma$		[a,b]	$< 6.1 \times 10^{-5}$	CL=90%
Γ_{25}	$\pi^0 e^+ \nu_e \gamma$		[a,b]	$(2.62 \pm 0.20) \times 10^{-4}$	
Γ_{26}	$\pi^0 e^+ \nu_e \gamma$(SD)		[d]	$< 5.3 \times 10^{-5}$	CL=90%
Γ_{27}	$\pi^0 \pi^0 e^+ \nu_e \gamma$			$< 5 \times 10^{-6}$	CL=90%

Lepton Family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

Γ_{28}	$\pi^+ \pi^+ e^- \overline{\nu}_e$	*SQ*		$< 1.2 \times 10^{-8}$	CL=90%
Γ_{29}	$\pi^+ \pi^+ \mu^- \overline{\nu}_\mu$	*SQ*		$< 3.0 \times 10^{-6}$	CL=95%
Γ_{30}	$\pi^+ e^+ e^-$	*S1*		$(2.74 \pm 0.23) \times 10^{-7}$	
Γ_{31}	$\pi^+ \mu^+ \mu^-$	*S1*		$(5.0 \pm 1.0) \times 10^{-8}$	
Γ_{32}	$\pi^+ \nu \overline{\nu}$	*S1*		$(4.2 {}^{+9.7}_{-3.5}) \times 10^{-10}$	
Γ_{33}	$\mu^- \nu e^+ e^+$	*LF*		$< 2.0 \times 10^{-8}$	CL=90%
Γ_{34}	$\mu^+ \nu_e$	*LF*	[e]	$< 4 \times 10^{-3}$	CL=90%
Γ_{35}	$\pi^+ \mu^+ e^-$	*LF*		$< 2.1 \times 10^{-10}$	CL=90%
Γ_{36}	$\pi^+ \mu^- e^+$	*LF*		$< 7 \times 10^{-9}$	CL=90%
Γ_{37}	$\pi^- \mu^+ e^+$	*L*		$< 7 \times 10^{-9}$	CL=90%
Γ_{38}	$\pi^- e^+ e^+$	*L*		$< 1.0 \times 10^{-8}$	CL=90%
Γ_{39}	$\pi^- \mu^+ \mu^+$	*L*	[e]	$< 1.5 \times 10^{-4}$	CL=90%
Γ_{40}	$\mu^+ \overline{\nu}_e$	*L*	[e]	$< 3.3 \times 10^{-3}$	CL=90%
Γ_{41}	$\pi^0 e^+ \overline{\nu}_e$	*L*		$< 3 \times 10^{-3}$	CL=90%
Γ_{42}	$\pi^+ \gamma$				

[a] See the Particle Listings below for the energy limits used in this measurement.

[b] Most of this radiative mode, the low-momentum γ part, is also included in the parent mode listed without γ's.

[c] Direct-emission branching fraction.

[d] Structure-dependent part.

[e] Derived from an analysis of neutrino-oscillation experiments.

CONSTRAINED FIT INFORMATION

An overall fit to the mean life, 2 decay rate, and 20 branching ratios uses 60 measurements and one constraint to determine 8 parameters. The overall fit has a $\chi^2 = 78.1$ for 53 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\left\langle \delta p_i \delta p_j \right\rangle / (\delta p_i \cdot \delta p_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i \equiv \Gamma_i/\Gamma_{\text{total}}$. The fit constrains the x_i whose labels appear in this array to sum to one.

x_3	−58						
x_4	−41	−12					
x_5	−27	−4	21				
x_6	−48	−17	14	2			
x_7	−50	−16	34	6	39		
x_8	−3	−1	2	0	2	6	
Γ	7	2	−18	−4	−2	−6	0
	x_1	x_3	x_4	x_5	x_6	x_7	x_8

	Mode	Rate (10^8 s^{-1})	Scale factor
Γ_1	$\mu^+ \nu_\mu$	0.5128 ±0.0018	1.5
Γ_3	$\pi^+ \pi^0$	0.1708 ±0.0012	1.1
Γ_4	$\pi^+ \pi^+ \pi^-$	0.0452 ±0.0004	1.8
Γ_5	$\pi^+ \pi^0 \pi^0$	0.01399±0.00032	1.2
Γ_6	$\pi^0 \mu^+ \nu_\mu$ Called $K^+_{\mu 3}$.	0.0257 ±0.0006	1.5
Γ_7	$\pi^0 e^+ \nu_e$ Called K^+_{e3}.	0.0389 ±0.0005	1.3
Γ_8	$\pi^0 \pi^0 e^+ \nu_e$	$(1.69 {}^{+0.34}_{-0.29}) \times 10^{-5}$	

$K^\pm$ DECAY RATES

$\Gamma(\mu^+ \nu_\mu)$ — Γ_1

VALUE (10^6 s^{-1})	*DOCUMENT ID*		*TECN*	*CHG*
51.28±0.18 OUR FIT Error includes scale factor of 1.5.				
51.2 ±0.8	FORD	67	CNTR	±

$\Gamma(\pi^+\pi^+\pi^-)$ Γ_4

VALUE (10^6 s^{-1})	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
4.52 ±0.04 OUR FIT Error includes scale factor of 1.8.					
4.511±0.024		[6] FORD	70	ASPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
4.529±0.032	3.2M	[6] FORD	70	ASPK	
4.496±0.030		[6] FORD	67	CNTR	±

[6] First FORD 70 value is second FORD 70 combined with FORD 67.

$(\Gamma(K^+) - \Gamma(K^-)) / \Gamma(K)$

$K^\pm \to \mu^\pm \nu_\mu$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
−0.54±0.41	FORD	67	CNTR

$K^\pm \to \pi^\pm\pi^+\pi^-$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.07±0.12 OUR AVERAGE					
0.08±0.12		[7] FORD	70	ASPK	
−0.50±0.90		FLETCHER	67	OSPK	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.02±0.16		[8] SMITH	73	ASPK	±
0.10±0.14	3.2M	[7] FORD	70	ASPK	
−0.04±0.21		[7] FORD	67	CNTR	

[7] First FORD 70 value is second FORD 70 combined with FORD 67.
[8] SMITH 73 value of $K^\pm \to \pi^\pm\pi^+\pi^-$ rate difference is derived from SMITH 73 value of $K^\pm \to \pi^\pm 2\pi^0$ rate difference.

$K^\pm \to \pi^\pm\pi^0\pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*
0.0 ±0.6 OUR AVERAGE					
0.08±0.58		SMITH	73	ASPK	±
−1.1 ±1.8	1802	HERZO	69	OSPK	

$K^\pm \to \pi^\pm\pi^0$ RATE DIFFERENCE/AVERAGE

Test of *CPT* conservation.

VALUE (%)	*DOCUMENT ID*		*TECN*
0.8±1.2	HERZO	69	OSPK

$K^\pm \to \pi^\pm\pi^0\gamma$ RATE DIFFERENCE/AVERAGE

Test of *CP* conservation.

VALUE (%)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
0.9± 3.3 OUR AVERAGE						
0.8± 5.8	2461	SMITH	76	WIRE	±	E_π 55–90 MeV
1.0± 4.0	4000	ABRAMS	73B	ASPK	±	E_π 51–100 MeV
0.0±24.0	24	EDWARDS	72	OSPK		E_π 58–90 MeV