DALITZ PLOT PARAMETERS FOR 
\( K \rightarrow 3\pi \) DECAYS

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The Dalitz plot distribution for \( K^\pm \rightarrow \pi^\pm \pi^\mp \pi^\mp \), \( K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm \), and \( K_L^0 \rightarrow \pi^+ \pi^- \pi^0 \) can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

\[
\left|M\right|^2 \propto 1 + \frac{g(s_3 - s_0)}{am_{\pi^+}^2} + h \left[ \frac{s_3 - s_0}{m_{\pi^+}^2} \right]^2 \\
+ j \left[ \frac{(s_2 - s_1)}{m_{\pi^+}^2} \right]^2 \left[ \frac{s_2 - s_1}{m_{\pi^+}^2} \right]^2 + f \left[ \frac{(s_2 - s_1)(s_3 - s_0)}{m_{\pi^+}^2 m_{\pi^+}^2} \right] + \cdots ,
\]

(1)

where \( m_{\pi^+}^2 \) has been introduced to make the coefficients \( g, h, j, \) and \( k \) dimensionless, and

\[
s_i = (P_i - P) = (m_K - m_i)^2 - 2m_KT_i , \quad i = 1, 2, 3,
\]

\[
s_0 = \frac{1}{3} \sum_i s_i = \frac{1}{3}(m_K^2 + m_1^2 + m_2^2 + m_3^2)
\]

Here the \( P_i \) are four-vectors, \( m_i \) and \( T_i \) are the mass and kinetic energy of the \( i^{th} \) pion, and the index 3 is used for the odd pion.

The coefficient \( g \) is a measure of the slope in the variable \( s_3 \) (or \( T_3 \)) of the Dalitz plot, while \( h \) and \( k \) measure the quadratic dependence on \( s_3 \) and \( (s_2 - s_1) \), respectively. The coefficient \( j \) is related to the asymmetry of the plot and must be zero if CP invariance holds. Note also that if CP is good, \( g, h, \) and \( k \) must be the same for \( K^+ \rightarrow \pi^+ \pi^+ \pi^- \) as for \( K^- \rightarrow \pi^- \pi^- \pi^+ \).

Since different experiments use different forms for \( \left|M\right|^2 \), in order to compare the experiments we have converted to \( g, h, j, \) and \( k \) whatever coefficients have been measured. Where such conversions have been done, the measured coefficient \( a_g, a_t, a_u, \) or \( a_v \) is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References