NEUTRINO MASS

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There is now rather convincing evidence that neutrinos have nonzero masses. This evidence comes from the apparent observation of neutrino oscillation. Let us recall the physics of this phenomenon, and its relation to neutrino mass.

In the decay

$$W^+ \to \ell^+ \nu_\ell \tag{1}$$

of a W boson into a charged lepton of "flavor" $\ell(e, \mu$, or $\tau)$, the accompanying neutrino is referred to as ν_ℓ, the neutrino of flavor ℓ. Neutrinos of different flavor are different objects. When an energetic ν_ℓ undergoes a charged-current weak interaction, it produces a charged lepton ℓ of the same flavor as the neutrino [1].

If neutrinos have masses, then a neutrino of definite flavor, ν_ℓ, need not be a mass eigenstate. Indeed, if leptons behave like quarks, the ν_ℓ is a coherent linear superposition of mass eigenstates, given by

$$|\nu_\ell\rangle = \sum_m U_{\ell m} |\nu_m\rangle \quad . \tag{2}$$

Here, the ν_m are the mass eigenstates, and the coefficients $U_{\ell m}$ form a matrix U known as the leptonic mixing matrix. There are at least three ν_m, and perhaps more. However, it is most often assumed that no more than three ν_m make significant contributions to Eq. (2). Then U is a 3×3 matrix, and according to the electroweak Standard Model (SM), extended to include neutrino masses, it is unitary.

The relation Eq. (2) means that when, for example, a W^+ decays to an e^+ and a neutrino, the neutrino with probability $|U_{e1}|^2$ is a ν_1, with probability $|U_{e2}|^2$ is a ν_2, and so on. This behavior is an exact leptonic analogue of what is known to occur when a W^+ decays to quarks.

If each neutrino of definite flavor is a coherent superposition of mass eigenstates, then a neutrino of one flavor can spontaneously change into one of another flavor as it propagates [2]. This is the phenomenon referred to as neutrino oscillation.

To understand neutrino oscillation, let us consider how a neutrino born as the ν_ℓ of Eq. (2) evolves in time. First, we apply Schrödinger's equation to the ν_m component of ν_ℓ in the rest frame of that component. This tells us that [3]

$$|\nu_m(\tau_m)\rangle = e^{-iM_m\tau_m}|\nu_m(0)\rangle \ , \tag{3}$$

where M_m is the mass of ν_m, and τ_m is time in the ν_m frame. In terms of the time t and position L in the laboratory frame, the Lorentz-invariant phase factor in Eq. (3) may be written

$$e^{-iM_m\tau_m} = e^{-i(E_m t - p_m L)} \ . \tag{4}$$

Here, E_m and p_m are respectively the energy and momentum of ν_m in the laboratory frame. In practice, our neutrino will be extremely relativistic, so we will be interested in evaluating the phase factor of Eq. (4) where $t \approx L$, where it becomes $\exp[-i(E_m - p_m)L]$.

Imagine now that our ν_ℓ has been produced with a definite momentum p, so that all of its mass-eigenstate components have this common momentum. Then the ν_m component has $E_m = \sqrt{p^2 + M_m^2} \approx p + M_m^2/2p$, assuming that all neutrino masses M_m are small compared to the neutrino momentum. The phase factor of Eq. (4) is then approximately

$$e^{-i(M_m^2/2p)L} \ . \tag{5}$$

Alternatively, suppose that our ν_ℓ has been produced with a definite energy E, so that all of its mass-eigenstate components have this common energy [4]. Then the ν_m component has $p_m = \sqrt{E^2 - M_m^2} \approx E - M_m^2/2E$. The phase factor of Eq. (4) is then approximately

$$e^{-i(M_m^2/2E)L} \ . \tag{6}$$

Since highly relativistic neutrinos have $E \approx p$, the phase factors (5) and (6) are approximately equal. Thus, it doesn't matter whether our ν_ℓ is created with definite momentum or definite energy.

From Eq. (2) and either Eq. (5) or Eq. (6), it follows that after a neutrino born as a ν_ℓ has propagated a distance L, its state vector has become

$$|\nu_\ell(L)\rangle \approx \sum_m U_{\ell m} e^{-i(M_m^2/2E)L} |\nu_m\rangle \ . \tag{7}$$

Using the unitarity of U to invert Eq. (2), and inserting the result in Eq. (7), we find that

$$|\nu_\ell(L)\rangle \approx \sum_{\ell'} \left[\sum_m U_{\ell m} e^{-i(M_m^2/2E)L} U^*_{\ell' m} \right] |\nu_{\ell'}\rangle \ . \tag{8}$$

We see that our ν_ℓ, in traveling the distance L, has turned into a superposition of all the flavors. The probability that it has flavor ℓ', $P(\nu_\ell \to \nu_{\ell'}; L)$, is obviously given by

$$P(\nu_\ell \to \nu_{\ell'};\ L) = |\langle \nu_{\ell'} | \nu_\ell(L)\rangle|^2 = \left| \sum_m U_{\ell m} e^{-i(M_m^2/2E)L} U^*_{\ell' m} \right|^2 \ . \tag{9}$$

If it should turn out that the number of neutrino flavors, N, is greater than three, and that the N neutrinos of definite flavor are made up out of N light neutrino mass eigenstates, then the neutrino oscillation probability will still be given by this equation, but with U an $N \times N$, rather than 3×3, unitary matrix.

The mixing matrix U is often called the "Maki-Nakagawa-Sakata matrix" in recognition of the very insightful early work of these three authors on neutrino mixing and oscillation [2].

The quantum mechanics of neutrino oscillation leading to the result Eq. (9) is somewhat subtle. It has been analyzed using wave packets [5], treating a propagating neutrino as a virtual particle [6], evaluating the phase acquired by a propagating mass eigenstate in terms of the proper time of propagation [3], requiring that a neutrino's flavor cannot change unless the neutrino travels [4], and taking different neutrino mass eigenstates to have both different momenta and different energies [7]. The subtleties of oscillation are still being explored.

Frequently, a neutrino oscillation experiment is analyzed assuming that only two neutrino flavors, ν_e and ν_μ for example, mix appreciably. Then the mixing matrix U takes the form

$$U = \begin{pmatrix} \cos\theta_{e\mu} & \sin\theta_{e\mu} \\ -\sin\theta_{e\mu} & \cos\theta_{e\mu} \end{pmatrix} \quad , \tag{10}$$

where $\theta_{e\mu}$ is the ν_e–ν_μ mixing angle. Inserting this matrix into Eq. (9), we find that

$$P(\nu_e \to \nu_\mu;\ L) = \sin^2 2\theta_{e\mu} \sin^2\left(\Delta M^2_{21} L/4E\right) \quad . \tag{11}$$

Here, $\Delta M^2_{21} \equiv M_2^2 - M_1^2$, where ν_1 and ν_2 are the mass eigenstates which make up ν_e and ν_μ. If the omitted factors of $\hbar$ and c are inserted into the argument $\Delta M^2_{21} L/4E$ of the oscillatory sine function, it becomes $1.27\ \Delta M^2_{21}\ (\mathrm{eV}^2) L\ (\mathrm{km})/E\ (\mathrm{GeV})$. The probability that a ν_e will retain its original flavor during propagation over a distance L is simply

$$P(\nu_e \to \nu_e;\ L) = 1 - P(\nu_e \to \nu_\mu;\ L) \quad . \tag{12}$$

When ν_e, ν_μ, and ν_τ all mix, but two of the three corresponding mass eigenstate neutrinos ν_m are nearly degenerate, neutrino oscillation is described by an expression nearly identical to the "two-neutrino formula" of Eq. (11). To be more precise, suppose that $|\Delta M^2_{21}| \ll |\Delta M^2_{31}| \cong |\Delta M^2_{32}|$, where $\Delta M^2_{mm'} \equiv M^2_m - M^2_{m'}$ is the splitting between the squared masses of mass eigenstates ν_m and $\nu_{m'}$. That is, ν_2 and ν_1 form a pair with a much smaller splitting than that between ν_3 and this pair. Now, suppose an oscillation experiment has L/E such that $|\Delta M^2_{31}| L/E$ is of order unity, so that $|\Delta M^2_{21}| L/E \ll 1$. For this experiment, it follows from Eq. (9) and the unitarity of U that [8]

$$P(\nu_\ell \to \nu_{\ell' \neq \ell};\ L) \cong |2U_{\ell 3} U_{\ell' 3}|^2 \sin^2\left(\Delta M^2_{31} L/4E\right) \quad . \tag{13}$$

Because $|\Delta M^2_{21}| L/E \ll 1$, this experiment cannot "see" the splitting between ν_2 and ν_1, so these two mass eigenstates behave as if they were a single one. Thus, in this experiment there appear to be only two mass eigenstates altogether, so it is no surprise that the expression for neutrino oscillation, Eq. (13), is very similar to the "two-neutrino" result of Eq. (11).

In a beam of neutrinos born with flavor ℓ_a, neutrino oscillation can be sought in two ways: First, one may seek the *appearance* in the beam of neutrinos of a different flavor, ℓ_b. Secondly, one may seek a *disappearance* of some of the original ν_{ℓ_a} flux, or an L- or E-dependence of this flux.

Clearly, no oscillation is expected unless L/E of the experiment is sufficiently large that the phase factors $\exp(-iM_m^2\, L/2E)$ in Eq. (9) differ appreciably from one another. Otherwise, $\mathrm{P}(\nu_\ell \to \nu_{\ell'}; L) = |\sum_m U_{\ell m} U^*_{\ell' m}|^2 = \delta_{\ell\ell'}$. Now, with omitted factors of $\hbar$ and c inserted, the relative phase of $\exp(-iM_m^2\, L/2E)$ and $\exp(-iM_{m'}^2 L/2E)$ is 2.54 $\Delta M^2_{mm'}(\mathrm{eV}^2)$ $L(\mathrm{km})/E(\mathrm{GeV})$. Thus, for example, an experiment in which neutrinos with $E \approx$ 1 GeV travel 1 km between production and detection will be sensitive to $\Delta M^2 \gtrsim 1$ eV^2.

A more direct way than neutrino oscillation experiments to search for neutrino mass is to look for its kinematical effects in decays which produce a neutrino. In the decay $X \to Y\ell^+\nu_\ell$, where X is a hadron and Y is zero or more hadrons, the momenta of ℓ^+ and the particles in Y will obviously be modified if ν_ℓ has a mass. If ν_ℓ is a superposition of mass eigenstates ν_m, then $X \to Y\ell^+\nu_\ell$ is actually the sum of the decays $X \to Y\ell^+\nu_m$ yielding every ν_m light enough to be emitted. Thus, if, for example, one ν_m is much heavier than the others, the energy spectrum of ℓ^+ may show a threshold rise where the ℓ^+ energy becomes low enough for the heavy ν_m to be emitted [9]. However, if neutrino mixing is small, then the decays $X \to Y\ell^+\nu_m$ yield almost always the neutrino mass eigenstate which is the dominant component of ν_ℓ. The kinematics of ℓ^+ and Y then reflect the mass of this mass eigenstate.

From kinematical studies of the particles produced in $^3\mathrm{H} \to {}^3\mathrm{He}\; e^-\,\overline{\nu}_e, \pi \to \mu\nu_\mu$, and $\tau \to n\pi\nu_\tau$, various upper bounds on neutrino mass have been obtained. In the case of the decay $^3\mathrm{H} \to {}^3\mathrm{He}\; e^-\,\overline{\nu}_e$, the upper bound on the neutrino mass is derived from study of the e^- energy spectrum. It should be noted that in several experiments, the observed spectrum is not well fit by the standard theoretical expression, either with vanishing or nonvanishing neutrino mass. However, progress is being made in understanding the spectral anomalies [10].

Neutrinos carry neither electric charge nor, as far as we know, any other charge-like quantum numbers. To be sure, it may be that the reason an interacting "neutrino" creates an ℓ^-, while an "antineutrino" creates an ℓ^+, is that neutrinos and antineutrinos carry opposite values of a conserved "lepton number." However, there may be no lepton number. Even then, the fact that "neutrinos" and "antineutrinos" interact differently can be easily understood. One need only note that, in practice, the particles we call "neutrinos" are always left-handed, while the ones we call "antineutrinos" are right-handed. Since the weak interactions are not invariant under parity, it is then possible to attribute the difference between the interactions of "neutrinos" and "antineutrinos" to the fact that these particles are oppositely polarized.

If the neutrino mass eigenstates do not carry any charge-like attributes, they may be their own antiparticles. A neutrino which is its own antiparticle is called a Majorana neutrino, while one which is not is called a Dirac neutrino.

If neutrinos are of Majorana character, we can have neutrinoless double beta-decay ($\beta\beta_{0\nu}$), in which one nucleus decays to another by emitting two electrons and nothing else. This process can be initiated through the emission of two virtual W bosons by the parent nucleus. One of these W bosons then emits an electron and an accompanying virtual "antineutrino." In the Majorana case, this "antineutrino" is no different from a "neutrino," except for its right-handed helicity. If the virtual neutrino has a mass, then (like the e^+ in nuclear β-decay), it is not fully right-handed, but has a small amplitude, proportional to its mass, for being left-handed. Its left-handed component is precisely what we call a "neutrino," and can be absorbed by the second virtual W boson to create the second outgoing electron. This mechanism yields for $\beta\beta_{0\nu}$ an amplitude proportional to an effective neutrino mass $\langle M\rangle$, given in a common phase convention by [11]

$$\langle M\rangle = \sum_m U_{em}^2 M_m \quad . \tag{14}$$

Experimental upper bounds on the $\beta\beta_{0\nu}$ rate are used to derive upper bounds on $\langle M \rangle$. Note that, owing to possible phases in the mixing matrix elements U_{em}, the relation between $\langle M \rangle$ and the actual masses M_m of the neutrino mass eigenstates can be somewhat complicated. The process $\beta\beta_{0\nu}$ is discussed further by P. Vogel in this *Review*.

If neutrinos are their own antiparticles, then their magnetic and electric dipole moments must vanish. To see why, recall that CPT invariance requires that the dipole moments of the electron and its antiparticle be equal and opposite. Similarly, CPT invariance would require that the dipole moments of a neutrino and its antiparticle be equal and opposite. But, if the antiparticle of the neutrino is the neutrino itself, this means that the dipole moments must vanish [12].

If neutrinos are not their own antiparticles, then they can have dipole moments. However, for a Dirac neutrino mass eigenstate ν_m, the magnetic dipole moment μ_m predicted by the Standard Model (extended to include neutrino masses) is only [13]

$$\mu_m = 3.2 \times 10^{-19} M_m(\mathrm{eV}) \mu_B \ , \tag{15}$$

where μ_B is the Bohr magneton.

Whether neutrinos are their own antiparticles or not, there may be *transition* magnetic and electric dipole moments. These induce the transitions $\nu_m \to \nu_{m' \neq m} \gamma$.

A Majorana neutrino, being its own antiparticle, obviously consists of just two states: spin up and spin down. In contrast, a Dirac neutrino, together with its antiparticle, consists of four states: the spin-up and spin-down neutrino states, plus the spin-up and spin-down antineutrino states. A four-state Dirac neutrino may be pictured as comprised of two degenerate two-state Majorana neutrinos. Conversely, in the field-theory description of neutrinos, by introducing so-called Majorana mass terms, one can split a Dirac neutrino, D, into two nondegenerate Majorana neutrinos, ν and N. In some extensions of the SM, it is natural for the D, ν, and N masses, M_D, M_ν, and M_N, to be related by

$$M_\nu M_N \approx M_D^2 \ . \tag{16}$$

In these extensions, it is also natural for M_D to be of the order of $M_{\ell \text{ or } q}$, the mass of a typical charged lepton or quark. Then we have [14]

$$M_\nu M_N \sim M^2_{\ell \text{ or } q} \ . \tag{17}$$

Suppose now that $M_N \gg M_{\ell \text{ or } q}$, so that N is a very heavy neutrino which has not yet been observed. Then relation Eq. (17), known as the seesaw relation, implies that $M_\nu \ll M_{\ell \text{ or } q}$. Thus, ν is a candidate for one of the light neutrino mass eigenstates which make up ν_e, ν_μ, and ν_τ. So long as N is heavy, the seesaw relation explains, without fine tuning, why a mass eigenstate component of ν_e, ν_μ, or ν_τ will be light. Interestingly, the picture from which the seesaw relation arises predicts that the mass eigenstate components of ν_e, ν_μ, and ν_τ are Majorana neutrinos.

There are three reported indications that neutrinos actually oscillate in nature, and thus have mass. There is rather convincing evidence that the atmospheric neutrinos oscillate, fairly strong evidence that the solar neutrinos do, and so-far unconfirmed evidence that the neutrinos studied by the LSND experiment do as well.

The atmospheric neutrinos are produced in the earth's atmosphere by cosmic rays, and then detected in an underground detector. Incident on this detector are neutrinos coming from all directions, created in the atmosphere all around the earth. The most compelling evidence that something very interesting happens to these atmospheric neutrinos en route to the detector is the fact that the detected upward-going atmospheric ν_μ flux U (coming from all directions below the horizontal at the detector) differs from the corresponding downward-going flux D. Suppose that neither neutrino oscillation nor any other mechanism decreases or increases the ν_μ flux as the neutrinos travel from their points of origin to the detector. Then, as illustrated in Fig. 1, any ν_μ that enters the sphere S defined in the figure caption will later exit this sphere. Thus, since we are dealing with a steady-state situation, the total ν_μ fluxes entering and exiting S per unit time must be equal. Now, for neutrino energies above a few GeV, the flux of cosmic rays which produce the atmospheric neutrinos is isotropic. Consequently, these neutrinos are being created at the same rate all around the earth. Owing to this spherical symmetry, the equality between the ν_μ fluxes entering and exiting S must hold at any point of S, such as the location of the detector. Now, as shown in Fig. 1, a ν_μ *entering* S through the detector must be part of the downward-going flux D. One *exiting* S through the detector must be part of the upward-going flux U. Thus, the equality of the ν_μ fluxes entering and exiting S at the detector implies that $D = U$. (It is easily shown that this equality must hold not only for the integrated downward and upward fluxes, but angle by angle. That is, the flux coming down from zenith angle θ_Z must equal that coming up from angle $\pi - \theta_Z$.)

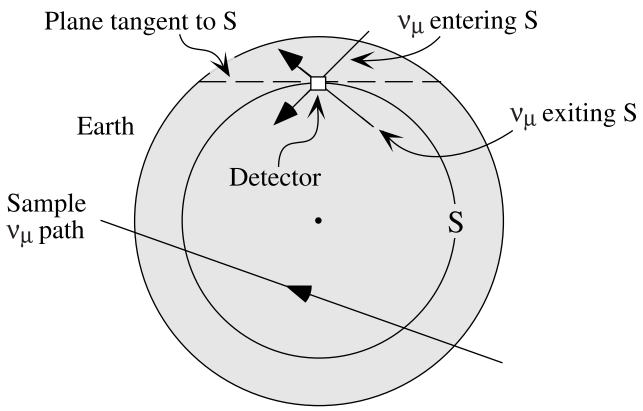

Figure 1: Atmospheric muon neutrino fluxes at an underground detector. S is a sphere centered at the center of the earth and passing through the detector.

The underground Super-Kamiokande detector (Super-K) finds that for multi-GeV atmospheric muon neutrinos [15],

$$\frac{\text{Flux Up}(-1.0 < \cos\theta_Z < -0.2)}{\text{Flux Down}(+0.2 < \cos\theta_Z < +1.0)} = 0.52 \pm 0.05 \ , \qquad (18)$$

in strong disagreement with the requirement that the upward and downward fluxes be equal. Thus, some mechanism must be changing the ν_μ flux as the neutrinos travel to the detector.

The most attractive candidate for this mechanism is neutrino oscillation. Since the atmospheric ν_e flux is compatible with up-down symmetry, the electron neutrinos do not seem to be involved significantly in this oscillation. All of the detailed Super-K atmospheric neutrino data are well described by the hypothesis that $\nu_\mu \to \nu_\tau$ oscillation is occurring, with [16]

$$2 \times 10^{-3}\mathrm{eV}^2 \lesssim \Delta M^2 \lesssim 6 \times 10^{-3}\mathrm{eV}^2 \tag{19}$$

and

$$\sin^2 2\theta \approx 1 \ . \tag{20}$$

Other experiments favor roughly similar regions of parameter space [17].

The order of magnitude of the splitting ΔM^2 in Eq. (19) may be understood by noting that for $E \sim 1$ GeV, upward-going neutrinos have $L/E \sim 10^4$km/1 GeV, while downward-going ones have $L/E \sim 10$ km/1 GeV. Thus, if $\Delta M^2 \sim 10^{-3}$ eV^2, the argument $[1.27\Delta M^2(\mathrm{eV}^2)L(\mathrm{km})/E(\mathrm{GeV})]$ of the oscillatory factor in Eq. (11) (applied to the relevant observation channel) exceeds unity for the upward-going neutrinos, but is quite small for the downward-going ones. As a result, the upward-going muon neutrinos oscillate away into neutrinos of another flavor, but the downward-going ones do not. This explains why the flux ratio of Eq. (18) is less than unity.

Conceivably, upward-going muon neutrinos are disappearing, not as a result of neutrino oscillation, but through neutrino decay. This possibility is theoretically less likely than oscillation. However, it is interesting to note that it is not at all excluded by the present data [18]. Of course, neutrino decay, like neutrino oscillation, implies neutrino mass.

The flux of solar neutrinos has been detected on earth by several experiments [19] with different neutrino energy thresholds. In every experiment, the flux is found to be below the corresponding prediction of the Standard Solar Model (SSM) [20]. The discrepancies between the observed fluxes and the SSM predictions have proven very difficult to explain by simply modifying the SSM, without invoking neutrino mass [21]. Indeed, we know of no attempt which has succeeded despite very serious and clever attempts [22–24]. By contrast, all the existing observations can successfully and elegantly be explained if one does invoke neutrino mass. The most popular explanation of this type is based on the Mikheyev-Smirnov-Wolfenstein (MSW) effect—a matter-enhanced neutrino oscillation [25].

The neutrinos produced by the nuclear processes that power the sun are electron neutrinos ν_e. With some probability, the MSW effect converts a ν_e into a neutrino ν_x of another flavor. Depending on the specific version of the effect, ν_x is a ν_μ, a ν_τ, a ν_μ–ν_τ mixture, or perhaps a sterile neutrino ν_s. Since present solar neutrino detectors are sensitive to a ν_e, but wholly, or at least largely, insensitive to a ν_μ, ν_τ, or ν_s, the flavor conversion accounts for the low observed fluxes.

The MSW $\nu_e \to \nu_x$ conversion results from interaction between neutrinos and solar electrons as the neutrinos travel outward from the solar core, where they were produced. When, for example, the neutrino mixing is small, the conversion requires that, somewhere in the sun, the total energy of a ν_e of given momentum, including the energy of its interaction with the solar electrons, equal the total energy of the ν_x of the same momentum, so that we have an energy level crossing. Given the typical density of solar electrons, and the typical momenta of solar neutrinos, the condition that there be a level crossing requires that

$$M_{\nu_x}^2 - M_{\nu_e}^2 \equiv \Delta M_{\nu_x \nu_e}^2 \sim 10^{-5}\mathrm{eV}^2 \ , \tag{21}$$

where M_{ν_e}, continuing to assume small mixing, is the mass of the dominant mass eigenstate component of ν_e, and similarly for M_{ν_x}.

The observed solar neutrino fluxes can also be explained by supposing that on their way from the sun to the earth, the electron neutrinos produced in the solar core undergo vacuum oscillation into neutrinos of another flavor [26]. Assuming that only two neutrino flavors are important to this oscillation, the oscillation probability is described by an expression of the form given by Eq. (11). To explain the observed suppression of the solar ν_e flux to less than half the predicted value at some energies, and to accommodate the observation that the suppression is energy-dependent, the argument $[1.27\Delta M^2(\mathrm{eV}^2)L(\mathrm{km})/E(\mathrm{GeV})]$ of the oscillatory factor in Eq. (11) must be of order unity when L is the distance from the sun to the earth, and $E \simeq 1$ MeV is the typical energy of a solar neutrino. Perhaps this apparent coincidence makes the vacuum oscillation explanation of the solar neutrino observations less likely than the MSW explanation. To have $[1.27\Delta M^2(\mathrm{eV}^2)L(\mathrm{km})/E(\mathrm{GeV})]\sim 1$, we require that $\Delta M^2 \sim 10^{-10}$ eV^2.

In addition to measuring the solar neutrino fluxes, one can explore the physics of the solar neutrinos by studying the solar ν_e energy spectrum [27], by probing the dependence of the solar ν_e flux on whether it is day or night, and on the time of night [28], and by measuring the solar ν_e flux as a function of the season of the year. The Super-K experiment is doing all of these things [29].

The solar neutrino experiments, and the comparison between their results and theoretical predictions, are discussed in some detail by K. Nakamura in this *Review*.

The LSND experiment [30] has studied neutrinos from stopped positively-charged pions, which decay via the chain

$$\begin{aligned} \pi^+ &\to \mu^+\nu_\mu \\ &\quad\ \hookrightarrow e^+\nu_e\overline{\nu}_\mu \end{aligned} \tag{22}$$

We note that this chain does not produce $\overline{\nu}_e$, but an excess of $\overline{\nu}_e$ over expected background is reported by the experiment. This excess is interpreted as arising from oscillation of the $\overline{\nu}_\mu$ which the chain does produce into $\overline{\nu}_e$. Since the experiment has $L(\mathrm{km})/E(\mathrm{GeV}) \sim 1$, the implied mass splitting is $\Delta M^2 \gtrsim 1$ eV^2. LSND finds supporting evidence for its reported oscillation in a study of the neutrinos from the decay

$$\pi^+ \to \mu^+\nu_\mu \tag{23}$$

of positively-charged pions in flight [31].

Other experiments do not observe the oscillation seen by LSND, and allow only some of the $(\Delta M^2,\ \sin^2 2\theta)$ region favored by LSND. However, these other experiments do not exclude the oscillation interpretation of the LSND data [32,33].

Suppose we assume that the behavior of the atmospheric, solar, and LSND neutrinos are all to be understood in terms of neutrino oscillation. What neutrino masses are then suggested?

If there are only three neutrinos of definite flavor, ν_e, ν_μ, and ν_τ, made up out of just three neutrinos of definite mass, ν_1, ν_2, and ν_3, then there are only three mass splittings $\Delta M^2_{mm'}$, and they obviously satisfy

$$\begin{aligned} &\Delta M^2_{32} + \Delta M^2_{21} + \Delta M^2_{13} = \\ &(M_3^2 - M_2^2) + (M_2^2 - M_1^2) + (M_1^2 - M_3^2) = 0 \; . \end{aligned} \tag{24}$$

Now, as we have seen, the ΔM^2 values required to explain the atmospheric, solar, and LSND oscillations are of three different orders of magnitude. Thus, they cannot possibly obey the constraint of Eq. (24). Hence, to explain all three of the reported neutrino oscillations, one must introduce a fourth neutrino. Since this neutrino is known to make no contribution to the width of the Z^0 [34], it must be a neutrino which does not participate in the normal weak interactions—a "sterile" neutrino.

One four-neutrino scheme which accounts for all three reported oscillations contains the following neutrino mass eigenstates: A nearly degenerate pair, ν_3, ν_2, with $M_3 \approx M_2 \sim 1$ eV, and a much lighter pair, ν_1, ν_0, with the mass of ν_0, M_0, roughly 3×10^{-3} eV, and $M_1 \ll M_0$. The mass splitting $M_3^2 - M_2^2$ is chosen to be $\sim 4 \times 10^{-3}$ eV^2 to explain the oscillation of the atmospheric neutrinos. Interpreting that oscillation as $\nu_\mu \to \nu_\tau$ with near maximal mixing, we take ν_2 and ν_3 to be approximately 50–50 mixtures of ν_μ and ν_τ. The splitting $M_0^2 - M_1^2 \approx M_0^2 \sim 10^{-5}$ eV^2 allows us to interpret the solar neutrino observations in terms of the MSW effect. We take ν_1 to be largely ν_e, and ν_0 to be largely a sterile neutrino ν_s, so that the MSW effect converts ν_e to a sterile neutrino. Finally, the mass-squared splitting of ~ 1 eV^2 between the heavier pair and the lighter one enables us to explain the oscillation reported by LSND [35].

The existing indications of neutrino oscillation, and the possible neutrino-mass scenarios which they suggest, will be probed in future neutrino experiments.

In addition to the ν_e, ν_μ, and ν_τ sections, the *Review of Particle Physics* includes sections on "Number of Light Neutrino Types," "Heavy Lepton Searches," and "Searches for Massive Neutrinos and Lepton Mixing." Also see other recent reviews [36].

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