# $\mu$

$J = \frac{1}{2}$

## $\mu$ MASS

The mass is known much more precisely in u (atomic mass units) than in MeV. The conversion from u to MeV, 1 u = 931.494013±0.000037 MeV/$c^2$ (MOHR 99, the 1998 CODATA value), involves the relatively poorly known electronic charge.

Where $m_\mu/m_e$ was measured, we have used the 1986 CODATA value for $m_e$ = 0.51099906 ± 0.00000015 MeV.

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|
| **105.6583568±0.0000052** | [1] MOHR | 99 | RVUE | | 1998 CODATA value |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 105.658389 ±0.000034 | [2] COHEN | 87 | RVUE | | 1986 CODATA value |
| 105.658386 ±0.000044 | [3] MARIAM | 82 | CNTR | + | |
| 105.65836 ±0.00026 | [4] CROWE | 72 | CNTR | | |
| 105.65865 ±0.00044 | [5] CRANE | 71 | CNTR | | |

[1] The mass is known much more precisely in u: 0.1134289168(34) u.
[2] The mass is known more precisely in u: $m$ = 0.113428913 ± 0.000000017 u. COHEN 87 makes use of the other entries below.
[3] MARIAM 82 gives $m_\mu/m_e$ = 206.768259(62).
[4] CROWE 72 gives $m_\mu/m_e$ = 206.7682(5).
[5] CRANE 71 gives $m_\mu/m_e$ = 206.76878(85).

## $\mu$ MEAN LIFE $\tau$

Measurements with an error > 0.001 × $10^{-6}$ s have been omitted.

| *VALUE* ($10^{-6}$ s) | *DOCUMENT ID* | | *TECN* | *CHG* |
|---|---|---|---|---|
| **2.19703 ±0.00004 OUR AVERAGE** | | | | |
| 2.197078±0.000073 | BARDIN | 84 | CNTR | + |
| 2.197025±0.000155 | BARDIN | 84 | CNTR | − |
| 2.19695 ±0.00006 | GIOVANETTI | 84 | CNTR | + |
| 2.19711 ±0.00008 | BALANDIN | 74 | CNTR | + |
| 2.1973 ±0.0003 | DUCLOS | 73 | CNTR | + |

## $\tau_{\mu^+}/\tau_{\mu^-}$ MEAN LIFE RATIO

A test of *CPT* invariance.

| *VALUE* | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **1.000024±0.000078** | BARDIN | 84 | CNTR | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 1.0008 ±0.0010 | BAILEY | 79 | CNTR | Storage ring |
| 1.000 ±0.001 | MEYER | 63 | CNTR | Mean life $\mu^+/\mu^-$ |

### $(\tau_{\mu^+} - \tau_{\mu^-}) / \tau_{\text{average}}$

A test of *CPT* invariance. Calculated from the mean-life ratio, above.

| *VALUE* | *DOCUMENT ID* |
|---|---|
| **$(2\pm8) \times 10^{-5}$ OUR EVALUATION** | |

## $\mu$ MAGNETIC MOMENT ANOMALY

The CODATA value (MOHR 99) comes from the current theoretical expression, based on the Standard Model and implicitly assuming that corrections beyond the Standard Model are negligible at the level of the quoted uncertainty. See reviews HUGHES 99 and FARLEY 90.

### $\mu_\mu/(e\hbar/2m_\mu)-1 = (g_\mu-2)/2$

| *VALUE* (units $10^{-6}$) | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|
| **1165.9160 ±0.0006 OUR EVALUATION** | From MOHR 99 (theoretical) | | | | |
| **1165.923 ±0.008 OUR AVERAGE** | Error includes scale factor of 1.1. | | | | |
| 1165.925 ±0.015 | [6] CAREY | 99 | CNTR | + | Storage ring |
| 1165.910 ±0.011 | [7] BAILEY | 79 | CNTR | + | Storage ring |
| 1165.936 ±0.012 | [7] BAILEY | 79 | CNTR | − | Storage ring |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 1165.91602±0.00064 | MOHR | 99 | RVUE | | 1998 CODATA value |
| 1165.9230 ±0.0084 | COHEN | 87 | RVUE | | 1986 CODATA value |
| 1162.0 ±5.0 | CHARPAK | 62 | CNTR | + | |

[6] CAREY 99 measure ratio $R$ to the free proton Larmor precession frequency, and then convert this to the magnetic moment anomaly using $\mu_\mu/\mu_p$= 3.18334547(47) (COHEN 87).

[7] BAILEY 79 values recalculated by HUGHES 99 using the COHEN 87 $\mu/p$ magnetic moment. The improved MOHR 99 value does not change the result.

### $(g_{\mu^+} - g_{\mu^-}) / g_{\text{average}}$

A test of *CPT* invariance.

| *VALUE* (units $10^{-8}$) | *DOCUMENT ID* | |
|---|---|---|
| **−2.6±1.6** | BAILEY | 79 |

## $\mu/p$ MAGNETIC MOMENT RATIO

This ratio is used to obtain a precise value of the muon mass and to reduce experimental muon Larmor frequency measurements to the muon magnetic moment anomaly. Measurements with an error > 0.00001 have been omitted.

| *VALUE* | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|
| **3.18334539±0.00000010** | [8] MOHR | 99 | RVUE | | 1998 CODATA value |

• • • We do not use the following data for averages, fits, limits, etc. • • •

| | | | | | |
|---|---|---|---|---|---|
| 3.18334513±0.00000039 | LIU | 99 | CNTR | + | HFS in muonium |
| 3.18334547±0.00000047 | [8] COHEN | 87 | RVUE | | 1986 CODATA value |
| 3.1833441 ±0.0000017 | KLEMPT | 82 | CNTR | + | Precession strob |
| 3.1833461 ±0.0000011 | MARIAM | 82 | CNTR | + | HFS splitting |
| 3.1833448 ±0.0000029 | CAMANI | 78 | CNTR | + | See KLEMPT 82 |
| 3.1833403 ±0.0000044 | CASPERSON | 77 | CNTR | + | HFS splitting |
| 3.1833402 ±0.0000072 | COHEN | 73 | RVUE | | 1973 CODATA value |
| 3.1833467 ±0.0000082 | CROWE | 72 | CNTR | + | Precession phase |

[8] CODATA values fitted using their selection of data, plus other data from multiparameter fits.

## $\mu$ ELECTRIC DIPOLE MOMENT

A nonzero value is forbidden by both $T$ invariance and $P$ invariance.

| *VALUE* ($10^{-19}$ $e$ cm) | *DOCUMENT ID* | | *TECN* | *CHG* | *COMMENT* |
|---|---|---|---|---|---|
| **3.7±3.4** | [9] BAILEY | 78 | CNTR | ± | Storage ring |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 8.6±4.5 | BAILEY | 78 | CNTR | + | Storage rings |
| 0.8±4.3 | BAILEY | 78 | CNTR | − | Storage rings |

[9] This is the combination of the two BAILEY 78 results given below.

## $\mu^-$ DECAY MODES

$\mu^+$ modes are charge conjugates of the modes below.

| | Mode | | Fraction ($\Gamma_i/\Gamma$) | Confidence level |
|---|---|---|---|---|
| $\Gamma_1$ | $e^- \overline{\nu}_e \nu_\mu$ | | $\approx 100\%$ | |
| $\Gamma_2$ | $e^- \overline{\nu}_e \nu_\mu \gamma$ | | [a] $(1.4\pm0.4)$ % | |
| $\Gamma_3$ | $e^- \overline{\nu}_e \nu_\mu e^+ e^-$ | | [b] $(3.4\pm0.4) \times 10^{-5}$ | |
| | **Lepton Family number (*LF*) violating modes** | | | |
| $\Gamma_4$ | $e^- \nu_e \overline{\nu}_\mu$ | *LF* | [c] $< 1.2$ % | 90% |
| $\Gamma_5$ | $e^- \gamma$ | *LF* | $< 1.2 \times 10^{-11}$ | 90% |
| $\Gamma_6$ | $e^- e^+ e^-$ | *LF* | $< 1.0 \times 10^{-12}$ | 90% |
| $\Gamma_7$ | $e^- 2\gamma$ | *LF* | $< 7.2 \times 10^{-11}$ | 90% |

[a] This only includes events with the $\gamma$ energy $> 10$ MeV. Since the $e^- \overline{\nu}_e \nu_\mu$ and $e^- \overline{\nu}_e \nu_\mu \gamma$ modes cannot be clearly separated, we regard the latter mode as a subset of the former.

[b] See the Particle Listings below for the energy limits used in this measurement.

[c] A test of additive vs. multiplicative lepton family number conservation.

# $\mu^-$ BRANCHING RATIOS

## $\Gamma(e^- \overline{\nu}_e \nu_\mu \gamma)/\Gamma_{total}$ — $\Gamma_2/\Gamma$

| VALUE | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **0.014 ±0.004** | | CRITTENDEN | 61 | CNTR | $\gamma$ KE > 10 MeV |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| | 862 | BOGART | 67 | CNTR | $\gamma$ KE > 14.5 MeV |
| 0.0033±0.0013 | | CRITTENDEN | 61 | CNTR | $\gamma$ KE > 20 MeV |
| | 27 | ASHKIN | 59 | CNTR | |

## $\Gamma(e^- \overline{\nu}_e \nu_\mu e^+ e^-)/\Gamma_{total}$ — $\Gamma_3/\Gamma$

| VALUE (units $10^{-5}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **3.4±0.2±0.3** | 7443 | [10] BERTL | 85 | SPEC | + | SINDRUM |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 2.2±1.5 | 7 | [11] CRITTENDEN | 61 | HLBC | + | $E(e^+ e^-)$>10 MeV |
| 2 | 1 | [12] GUREVICH | 60 | EMUL | + | |
| 1.5±1.0 | 3 | [13] LEE | 59 | HBC | + | |

[10] BERTL 85 has transverse momentum cut $p_T$ > 17 MeV/$c$. Systematic error was increased by us.

[11] CRITTENDEN 61 count only those decays where total energy of either ($e^+$, $e^-$) combination is >10 MeV.

[12] GUREVICH 60 interpret their event as either virtual or real photon conversion. $e^+$ and $e^-$ energies not measured.

[13] In the three LEE 59 events, the sum of energies E($e^+$) + E($e^-$) + E($e^+$) was 51 MeV, 55 MeV, and 33 MeV.

## $\Gamma(e^- \nu_e \overline{\nu}_\mu)/\Gamma_{total}$ — $\Gamma_4/\Gamma$

Forbidden by the additive conservation law for lepton family number. A multiplicative law predicts this branching ratio to be 1/2. For a review see NEMETHY 81.

| VALUE | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **< 0.012** | 90 | [14] FREEDMAN | 93 | CNTR | + | $\nu$ oscillation search |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| < 0.018 | 90 | KRAKAUER | 91B | CALO | + | |
| < 0.05 | 90 | [15] BERGSMA | 83 | CALO | | $\overline{\nu}_\mu e \rightarrow \mu^- \overline{\nu}_e$ |
| < 0.09 | 90 | JONKER | 80 | CALO | | See BERGSMA 83 |
| −0.001±0.061 | | WILLIS | 80 | CNTR | + | |
| 0.13 ±0.15 | | BLIETSCHAU | 78 | HLBC | ± | Avg. of 4 values |
| < 0.25 | 90 | EICHTEN | 73 | HLBC | + | |

[14] FREEDMAN 93 limit on $\overline{\nu}_e$ observation is here interpreted as a limit on lepton family number violation.

[15] BERGSMA 83 gives a limit on the inverse muon decay cross-section ratio $\sigma(\overline{\nu}_\mu e^- \rightarrow \mu^- \overline{\nu}_e)/\sigma(\nu_\mu e^- \rightarrow \mu^- \nu_e)$, which is essentially equivalent to $\Gamma(e^- \nu_e \overline{\nu}_\mu)/\Gamma_{total}$ for small values like that quoted.

### $\Gamma(e^- \gamma)/\Gamma_{total}$ — $\Gamma_5/\Gamma$

Forbidden by lepton family number conservation.

| VALUE (units $10^{-11}$) | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **< 1.2** | 90 | BROOKS | 99 | SPEC | + | LAMPF |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| < 4.9 | 90 | BOLTON | 88 | CBOX | + | LAMPF |
| <100 | 90 | AZUELOS | 83 | CNTR | + | TRIUMF |
| < 17 | 90 | KINNISON | 82 | SPEC | + | LAMPF |
| <100 | 90 | SCHAAF | 80 | ELEC | + | SIN |

### $\Gamma(e^- e^+ e^-)/\Gamma_{total}$ — $\Gamma_6/\Gamma$

Forbidden by lepton family number conservation.

| VALUE (units $10^{-12}$) | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **< 1.0** | 90 | [16] BELLGARDT | 88 | SPEC | + | SINDRUM |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| < 36 | 90 | BARANOV | 91 | SPEC | + | ARES |
| < 35 | 90 | BOLTON | 88 | CBOX | + | LAMPF |
| < 2.4 | 90 | [16] BERTL | 85 | SPEC | + | SINDRUM |
| <160 | 90 | [16] BERTL | 84 | SPEC | + | SINDRUM |
| <130 | 90 | [16] BOLTON | 84 | CNTR | | LAMPF |

[16] These experiments assume a constant matrix element.

### $\Gamma(e^- 2\gamma)/\Gamma_{total}$ — $\Gamma_7/\Gamma$

Forbidden by lepton family number conservation.

| VALUE (units $10^{-11}$) | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **< 7.2** | 90 | BOLTON | 88 | CBOX | + | LAMPF |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| < 840 | 90 | [17] AZUELOS | 83 | CNTR | + | TRIUMF |
| <5000 | 90 | [18] BOWMAN | 78 | CNTR | | DEPOMMIER 77 data |

[17] AZUELOS 83 uses the phase space distribution of BOWMAN 78.

[18] BOWMAN 78 assumes an interaction Lagrangian local on the scale of the inverse $\mu$ mass.

## LIMIT ON $\mu^- \rightarrow e^-$ CONVERSION

Forbidden by lepton family number conservation.

### $\sigma(\mu^- {}^{32}S \rightarrow e^- {}^{32}S) / \sigma(\mu^- {}^{32}S \rightarrow \nu_\mu {}^{32}P^*)$

| VALUE | CL% | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **$<7 \times 10^{-11}$** | 90 | BADERT... | 80 | STRC | SIN |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| $<4 \times 10^{-10}$ | 90 | BADERT... | 77 | STRC | SIN |

### $\sigma(\mu^- Cu \rightarrow e^- Cu) / \sigma(\mu^- Cu \rightarrow capture)$

| VALUE | CL% | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $<1.6 \times 10^{-8}$ | 90 | BRYMAN | 72 | SPEC |

### $\sigma(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$

| VALUE | CL% | | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|---|
| **$<4.3 \times 10^{-12}$** | 90 | [19] | DOHMEN | 93 | SPEC | SINDRUM II |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| $<4.6 \times 10^{-12}$ | 90 | | AHMAD | 88 | TPC | TRIUMF |
| $<1.6 \times 10^{-11}$ | 90 | | BRYMAN | 85 | TPC | TRIUMF |

[19] DOHMEN 93 assumes $\mu^- \rightarrow e^-$ conversion leaves the nucleus in its ground state, a process enhanced by coherence and expected to dominate.

### $\sigma(\mu^- \text{Pb} \rightarrow e^- \text{Pb}) / \sigma(\mu^- \text{Pb} \rightarrow \text{capture})$

| VALUE | CL% | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **$<4.6 \times 10^{-11}$** | 90 | HONECKER | 96 | SPEC | SINDRUM II |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| $<4.9 \times 10^{-10}$ | 90 | AHMAD | 88 | TPC | TRIUMF |

## LIMIT ON $\mu^- \rightarrow e^+$ CONVERSION

Forbidden by total lepton number conservation.

### $\sigma(\mu^- {}^{32}\text{S} \rightarrow e^+ {}^{32}\text{Si}^*) / \sigma(\mu^- {}^{32}\text{S} \rightarrow \nu_\mu {}^{32}\text{P}^*)$

| VALUE | CL% | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **$<9 \times 10^{-10}$** | 90 | BADERT... | 80 | STRC | SIN |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| $<1.5 \times 10^{-9}$ | 90 | BADERT... | 78 | STRC | SIN |

### $\sigma(\mu^- {}^{127}\text{I} \rightarrow e^+ {}^{127}\text{Sb}^*) / \sigma(\mu^- {}^{127}\text{I} \rightarrow \text{anything})$

| VALUE | CL% | | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|---|
| **$<3 \times 10^{-10}$** | 90 | [20] | ABELA | 80 | CNTR | Radiochemical tech. |

[20] ABELA 80 is upper limit for $\mu^- e^+$ conversion leading to particle-stable states of ${}^{127}$Sb. Limit for total conversion rate is higher by a factor less than 4 (G. Backenstoss, private communication).

### $\sigma(\mu^- \text{Cu} \rightarrow e^+ \text{Co}) / \sigma(\mu^- \text{Cu} \rightarrow \nu_\mu \text{Ni})$

| VALUE | CL% | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $<2.6 \times 10^{-8}$ | 90 | BRYMAN | 72 | SPEC |
| $<2.2 \times 10^{-7}$ | 90 | CONFORTO | 62 | OSPK |

### $\sigma(\mu^- \text{Ti} \rightarrow e^+ \text{Ca}) / \sigma(\mu^- \text{Ti} \rightarrow \text{capture})$

| VALUE | CL% | EVTS | | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|---|
| **$<3.6 \times 10^{-11}$** | 90 | 1 | [21,22] | KAULARD | 98 | SPEC | – | SINDRUM II |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | | | |
| $<1.7 \times 10^{-12}$ | 90 | 1 | [22,23] | KAULARD | 98 | SPEC | – | SINDRUM II |
| $<4.3 \times 10^{-12}$ | 90 | | [23] | DOHMEN | 93 | SPEC | | SINDRUM II |
| $<8.9 \times 10^{-11}$ | 90 | | [21] | DOHMEN | 93 | SPEC | | SINDRUM II |
| $<1.7 \times 10^{-10}$ | 90 | | [24] | AHMAD | 88 | TPC | | TRIUMF |

[21] This limit assumes a giant resonance excitation of the daughter Ca nucleus (mean energy and width both 20 MeV).

[22] KAULARD 98 obtained these same limits using the unified classical analysis of FELDMAN 98.
[23] This limit assumes the daughter Ca nucleus is left in the ground state. However, the probability of this is unknown.
[24] Assuming a giant-resonance-excitation model.

## LIMIT ON MUONIUM → ANTIMUONIUM CONVERSION

Forbidden by lepton family number conservation.

### $R_g = G_C / G_F$

The effective Lagrangian for the $\mu^+ e^- \rightarrow \mu^- e^+$ conversion is assumed to be

$$\mathcal{L} = 2^{-1/2}\ G_C\ [\overline{\psi}_\mu \gamma_\lambda\ (1 - \gamma_5)\ \psi_e]\ [\overline{\psi}_\mu \gamma_\lambda\ (1 - \gamma_5)\ \psi_e] + \text{h.c.}$$

The experimental result is then an upper limit on $G_C/G_F$, where $G_F$ is the Fermi coupling constant.

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|---|
| **< 0.0030** | 90 | 1 | [25] WILLMANN | 99 | SPEC | + | $\mu^+$ at 26 GeV/$c$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | | |
| < 0.14 | 90 | 1 | [26] GORDEEV | 97 | SPEC | + | JINR phasotron |
| < 0.018 | 90 | 0 | [27] ABELA | 96 | SPEC | + | $\mu^+$ at 24 MeV |
| < 6.9 | 90 | | NI | 93 | CBOX | | LAMPF |
| < 0.16 | 90 | | MATTHIAS | 91 | SPEC | | LAMPF |
| < 0.29 | 90 | | HUBER | 90B | CNTR | | TRIUMF |
| <20 | 95 | | BEER | 86 | CNTR | | TRIUMF |
| <42 | 95 | | MARSHALL | 82 | CNTR | | |

[25] WILLMANN 99 quote both probability $P_{M\overline{M}} < 8.3 \times 10^{-11}$ at 90%CL in a 0.1 T field and $R_g = G_C/G_F$.
[26] GORDEEV 97 quote limits on both $f = G_{MM}/GF$ and the probability $W_{MM} < 4.7 \times 10^{-7}$ (90%CL).
[27] ABELA 96 quote both probability $P_{M\overline{M}} < 8 \times 10^{-9}$ at 90% CL and $R_g = G_C/G_F$.

## MUON DECAY PARAMETERS

Revised October 1997 by W. Fetscher and H.-J. Gerber (ETH Zürich).

***Introduction:*** All measurements in direct muon decay, $\mu^- \to e^- + 2$ neutrals, and its inverse, $\nu_\mu + e^- \to \mu^- +$ neutral, are successfully described by the "$V$-$A$ interaction", which is a particular case of a local, derivative-free, lepton-number-conserving, four fermion interaction [1]. As shown below, within this framework, the Standard Model assumptions, such as the $V$-$A$ form and the nature of the neutrals ($\nu_\mu$ and $\bar{\nu}_e$), and hence the doublet assignments $(\nu_e\ e^-)_L$ and $(\nu_\mu\ \mu^-)_L$, have been determined from experiments [2,3]. All considerations on muon decay are valid for the leptonic tau decays $\tau \to \ell + \nu_\tau + \bar{\nu}_e$ with the replacements $m_\mu \to m_\tau$, $m_e \to m_\ell$.

***Parameters:*** The differential decay probability to obtain an $e^\pm$ with (reduced) energy between $x$ and $x + dx$, emitted in the direction $\hat{z}$ at an angle between $\vartheta$ and $\vartheta + d\vartheta$ with respect to the muon polarization vector $\vec{P}_\mu$, and with its spin pointing in the arbitrary direction $\hat{\zeta}$, neglecting radiative corrections, is given by

$$\begin{aligned}\frac{d^2\Gamma}{dx\ d\cos\vartheta} =& \frac{m_\mu}{4\pi^3}\ W_{e\mu}^4\ G_F^2\ \sqrt{x^2 - x_0^2} \\ & \times (F_{IS}(x) \pm P_\mu \cos\vartheta\ F_{AS}(x)) \\ & \times \left[1 + \vec{P}_e(x,\vartheta)\cdot\hat{\zeta}\right]\ . \end{aligned} \tag{1}$$

Here, $W_{e\mu} = \max(E_e) = (m_\mu^2 + m_e^2)/2m_\mu$ is the maximum $e^\pm$ energy, $x = E_e/W_{e\mu}$ is the reduced energy, $x_0 = m_e/W_{e\mu} = 9.67 \times 10^{-3}$, and $P_\mu = |\vec{P}_\mu|$ is the degree of muon polarization. $\hat{\zeta}$ is the direction in which a perfect polarization-sensitive electron detector is most sensitive. The isotropic part of the

spectrum, $F_{IS}(x)$, the anisotropic part $F_{AS}(x)$ and the electron polarization, $\vec{P}_e(x,\vartheta)$, may be parametrized by the Michel parameters [1,4] $\rho$, $\eta$, $\xi$, $\delta$, *etc.* These are bilinear combinations of the coupling constants $g^{\gamma}_{\varepsilon\mu}$, which occur in the matrix element (given below).

If the masses of the neutrinos as well as $x_0^2$ are neglected, the energy and angular distribution of the electron in the rest frame of a muon ($\mu^{\pm}$) measured by a polarization insensitive detector, is given by

$$\frac{d^2\Gamma}{dx\ d\cos\vartheta} \sim x^2 \cdot \left\{ 3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta\ x_0(1-x)/x \right.$$
$$\left. \pm\ P_\mu \cdot \xi \cdot \cos\vartheta \left[ 1 - x + \frac{2\delta}{3}(4x-3) \right] \right\} \ . \qquad (2)$$

Here, $\vartheta$ is the angle between the electron momentum and the muon spin, and $x \equiv 2E_e/m_\mu$. For the Standard Model coupling, we obtain $\rho = \xi\delta = 3/4$, $\xi = 1$, $\eta = 0$ and the differential decay rate is

$$\frac{d^2\Gamma}{dx\ \ d\cos\vartheta} \ = \ \frac{G_F^2 m_\mu^5}{192\pi^3}\ \left[3 - 2x \pm P_\mu \cos\vartheta(2x-1)\right]\ x^2 \quad . \quad (3)$$

The coefficient in front of the square bracket is the total decay rate.

If only the neutrino masses are neglected, and if the $e^{\pm}$ polarization is detected, then the functions in Eq. (1) become

$$F_{IS}(x) = x(1-x) + \tfrac{2}{9}\ \rho(4x^2 - 3x - x_0^2) + \eta \cdot x_0(1-x)$$
$$F_{AS}(x) = \tfrac{1}{3}\xi\ \sqrt{x^2 - x_0^2}$$
$$\times \left[ 1 - x + \tfrac{2}{3}\delta\Big(4x - 3 + \Big(\sqrt{1-x_0^2} - 1\Big)\Big) \right]$$
$$\vec{P}_e(x,\vartheta) = P_{T_1}\ \widehat{x} + P_{T_2}\widehat{y} + P_L\ \widehat{z}\ .$$

Here $\widehat{x}$, $\widehat{y}$, and $\widehat{z}$ are orthogonal unit vectors defined as follows:

| | |
|---|---|
| $\widehat{z}$ | is along the $e$ momentum |
| $\widehat{y} = [\widehat{z} \times \vec{P}_\mu]/|[\widehat{z} \times \vec{P}_\mu]|$ | is transverse to the $e$ momentum and perpendicular to the "decay plane" |
| $\widehat{x} = \widehat{y} \times \widehat{z}$ | is transverse to the $e$ momentum and in the "decay plane." |

The components of $\vec{P}_e$ then are given by

$$
\begin{aligned}
P_{T_1}(x,\vartheta) &= P_\mu \sin\vartheta \; F_{T_1}(x)/\left(F_{IS}(x) \pm P_\mu \cos\vartheta \; F_{AS}(x)\right) \\
P_{T_2}(x,\vartheta) &= P_\mu \sin\vartheta \; F_{T_2}(x)/\left(F_{IS}(x) \pm P_\mu \; cos\vartheta \; F_{AS}(x)\right) \\
P_L(x,\vartheta) &= \pm F_{IP}(x) + P_\mu \cos\vartheta \\
&\quad \times F_{AP}(x)/\left(F_{IS}(x) \pm P_\mu \cos\vartheta \; F_{AS}(x)\right) \;,
\end{aligned}
$$

where

$$
\begin{aligned}
F_{T_1}(x) &= \tfrac{1}{12}\left\{-2\left[\xi'' + 12(\rho - \tfrac{3}{4})\right](1-x)x_0 \right. \\
&\quad \left. -3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2)\right\} \\
F_{T_2}(x) &= \tfrac{1}{3}\sqrt{x^2 - x_0^2}\left\{3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}\sqrt{1-x_0^2}\right\} \\
F_{IP}(x) &= \tfrac{1}{54}\sqrt{x^2 - x_0^2}\left\{9\xi'\left(-2x + 2 + \sqrt{1-x_0^2}\right)\right. \\
&\quad \left. + 4\xi(\delta - \tfrac{3}{4})(4x - 4 + \sqrt{1-x_0^2})\right\} \\
F_{AP}(x) &= \tfrac{1}{6}\left\{\xi''(2x^2 - x - x_0^2) + 4(\rho - \tfrac{3}{4})\left(4x^2 - 3x - x_0^2\right)\right. \\
&\quad \left. +2\eta''(1-x)x_0\right\} \;.
\end{aligned}
$$

For the experimental values of the parameters $\rho$, $\xi$, $\xi'$, $\xi''$, $\delta$, $\eta$, $\eta''$, $\alpha/A$, $\beta/A$, $\alpha'/A$, $\beta'/A$, which are not all independent, see the Data Listings below. Experiments in the past have also

been analyzed using the parameters $a$, $b$, $c$, $a'$, $b'$, $c'$, $\alpha/A$, $\beta/A$, $\alpha'/A$, $\beta'/A$ (and $\eta = (\alpha - 2\beta)/2A$), as defined by Kinoshita and Sirlin [5]. They serve as a model-independent summary of all possible measurements on the decay electron (see Listings below). The relations between the two sets of parameters are

$$\begin{aligned}
\rho - \tfrac{3}{4} &= \tfrac{3}{4}(-a + 2c)/A \ , \\
\eta &= (\alpha - 2\beta)/A \ , \\
\eta'' &= (3\alpha + 2\beta)/A \ , \\
\delta - \tfrac{3}{4} &= \tfrac{9}{4} \cdot \frac{(a' - 2c')/A}{1 - [a + 3a' + 4(b + b') + 6c - 14c']/A} \ , \\
1 - \xi\frac{\delta}{\rho} &= 4\,\frac{[(b + b') + 2(c - c')]/A}{1 - (a - 2c)/A} \ , \\
1 - \xi' &= [(a + a') + 4(b + b') + 6(c + c')]/A \ , \\
1 - \xi'' &= (-2a + 20c)/A \ ,
\end{aligned}$$

where

$$A = a + 4b + 6c \ .$$

The differential decay probability to obtain a *left-handed* $\nu_e$ with (reduced) energy between $y$ and $y + dy$, neglecting radiative corrections as well as the masses of the electron and of the neutrinos, is given by [6]

$$\frac{d\Gamma}{dy} = \frac{m_\mu^5\, G_F^2}{16\pi^3} \cdot Q_L^{\nu_e} \cdot y^2 \Big\{ (1 - y) - \omega_L \cdot (y - \tfrac{3}{4}) \Big\} \ . \qquad (4)$$

Here, $y = 2\ E_{\nu_e}/m_\mu$. $Q_L^{\nu_e}$ and $\omega_L$ are parameters. $\omega_L$ is the neutrino analog of the spectral shape parameter $\rho$ of Michel. Since in the Standard Model, $Q_L^{\nu_e} = 1$, $\omega_L = 0$, the measurement of $d\Gamma/dy$ has allowed a null-test of the Standard Model (see Listings below).

***Matrix element:*** All results in direct muon decay (energy spectra of the electron and of the neutrinos, polarizations, and angular distributions) and in inverse muon decay (the reaction cross section) at energies well below $m_W c^2$ may be parametrized in terms of amplitudes $g^{\gamma}_{\varepsilon\mu}$ and the Fermi coupling constant $G_F$, using the matrix element

$$\frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g^{\gamma}_{\varepsilon\mu} \langle \bar{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle \bar{\nu}_{\mu})_m | \Gamma_{\gamma} | \mu_{\mu} \rangle . \tag{5}$$

We use the notation of Fetscher *et al.* [2], who in turn use the sign conventions and definitions of Scheck [7]. Here, $\gamma = S, V, T$ indicates a scalar, vector, or tensor interaction; and $\varepsilon, \mu = R, L$ indicate a right- or left-handed chirality of the electron or muon. The chiralities $n$ and $m$ of the $\nu_e$ and $\bar{\nu}_{\mu}$ are then determined by the values of $\gamma, \varepsilon$ and $\mu$. The particles are represented by fields of definite chirality [8].

As shown by Langacker and London [9], explicit lepton-number nonconservation still leads to a matrix element equivalent to Eq. (5). They conclude that it is not possible, even in principle, to test lepton-number conservation in (leptonic) muon decay if the final neutrinos are massless and are not observed.

The ten complex amplitudes $g^{\gamma}_{\varepsilon\mu}$ ($g^{T}_{RR}$ and $g^{T}_{LL}$ are identically zero) and $G_F$ constitute 19 independent (real) parameters to be determined by experiment. The Standard Model interaction corresponds to one single amplitude $g^{V}_{LL}$ being unity and all the others being zero.

The (direct) muon decay experiments are compatible with an arbitrary mix of the scalar and vector amplitudes $g^{S}_{LL}$ and $g^{V}_{LL}$ – in the extreme even with purely scalar $g^{S}_{LL} = 2$, $g^{V}_{LL} = 0$. The decision in favour of the Standard Model comes from the

quantitative observation of inverse muon decay, which would be forbidden for pure $g^S_{LL}$ [2].

***Experimental determination of V−A:*** In order to determine the amplitudes $g^\gamma_{\varepsilon\mu}$ uniquely from experiment, the following set of equations, where the left-hand sides represent experimental results, has to be solved.

$$
\begin{aligned}
a &= 16(|g^V_{RL}|^2 + |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 + |g^S_{LR} + 6g^T_{LR}|^2 \\
a' &= 16(|g^V_{RL}|^2 - |g^V_{LR}|^2) + |g^S_{RL} + 6g^T_{RL}|^2 - |g^S_{LR} + 6g^T_{LR}|^2 \\
\alpha &= 8\mathrm{Re}\left\{g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR}) + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL})\right\} \\
\alpha' &= 8\mathrm{Im}\left\{g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) - g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})\right\} \\
b &= 4(|g^V_{RR}|^2 + |g^V_{LL}|^2) + |g^S_{RR}|^2 + |g^S_{LL}|^2 \\
b' &= 4(|g^V_{RR}|^2 - |g^V_{LL}|^2) + |g^S_{RR}|^2 - |g^S_{LL}|^2 \\
\beta &= -4\mathrm{Re}\left\{g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}\right\} \\
\beta' &= 4\mathrm{Im}\left\{g^V_{RR}g^{S*}_{LL} - g^V_{LL}g^{S*}_{RR}\right\} \\
c &= \tfrac{1}{2}\left\{|g^S_{RL} - 2g^T_{RL}|^2 + |g^S_{LR} - 2g^T_{LR}|^2\right\} \\
c' &= \tfrac{1}{2}\left\{|g^S_{RL} - 2g^T_{RL}|^2 - |g^S_{LR} - 2g^T_{LR}|^2\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
Q^{\nu_e}_L &= 1 - \left\{\tfrac{1}{4}|g^S_{LR}|^2 + \tfrac{1}{4}|g^S_{LL}|^2 + |g^V_{RR}|^2 + |g^V_{RL}|^2 + 3|g^T_{LR}|^2\right\} \\
\omega_L &= \tfrac{3}{4}\,\frac{\{|g^S_{RR}|^2 + 4|g^V_{LR}|^2 + |g^S_{RL} + 2g^T_{RL}|^2\}}{|g^S_{RL}|^2 + |g^S_{RR}|^2 + 4|g^V_{LL}|^2 + 4|g^V_{LR}|^2 + 12|g^T_{RL}|^2\}}\ .
\end{aligned}
$$

It has been noted earlier by C. Jarlskog [10], that certain experiments observing the decay electron are especially informative if the yield the $V$-$A$ values. The complete solution is now found as follows. Fetscher *et al.* [2] introduced four probabilities

$Q_{\varepsilon\mu}(\varepsilon, \mu = R, L)$ for the decay of a $\mu$-handed muon into an $\varepsilon$-handed electron and showed that there exist upper bounds on $Q_{RR}$, $Q_{LR}$, and $Q_{RL}$, and a lower bound on $Q_{LL}$. These probabilities are given in terms of the $g^{\gamma}_{\varepsilon\mu}$'s by

$$Q_{\varepsilon\mu} = \tfrac{1}{4}|g^{S}_{\varepsilon\mu}|^2 + |g^{V}_{\varepsilon\mu}|^2 + 3(1-\delta_{\varepsilon\mu})|g^{T}_{\varepsilon\mu}|^2 \ , \tag{6}$$

where $\delta_{\varepsilon\mu} = 1$ for $\varepsilon = \mu$, and $\delta_{\varepsilon\mu} = 0$ for $\varepsilon \neq \mu$. They are related to the parameters $a$, $b$, $c$, $a', b'$, and $c'$ by

$$\begin{aligned}
Q_{RR} &= 2(b+b')/A \ , \\
Q_{LR} &= [(a-a') + 6(c-c')]/2A \ , \\
Q_{RL} &= [(a+a') + 6(c+c')]/2A \ , \\
Q_{LL} &= 2(b-b')/A \ ,
\end{aligned}$$

with $A = 16$. In the Standard Model, $Q_{LL} = 1$ and the others are zero.

Since the upper bounds on $Q_{RR}$, $Q_{LR}$, and $Q_{RL}$ are found to be small, and since the helicity of the $\nu_\mu$ in pion decay is known from experiment [11,12] to very high precision to be $-1$ [13], the cross section $S$ of *inverse* muon decay, normalized to the $V$-$A$ value, yields [2]

$$|g^{S}_{LL}|^2 \leq 4(1-S) \tag{7}$$

and

$$|g^{V}_{LL}|^2 \ = \ S \ . \tag{8}$$

Thus the Standard Model assumption of a pure $V$-$A$ leptonic charged weak interaction of $e$ and $\mu$ is derived (within errors) from experiments at energies far below mass of the $W^{\pm}$: Eq. (8) gives a lower limit for $V$-$A$, and Eqs. (6) and (7) give upper

limits for the other four-fermion interactions. The existence of such upper limits may also be seen from $Q_{RR}+Q_{RL} = (1-\xi')/2$ and $Q_{RR}+Q_{LR} = \frac{1}{2}(1+\xi/3-16\ \xi\delta/9)$. Table 1 gives the current experimental limits on the magnitudes of the $g^{\gamma}_{\varepsilon\mu}$'s.

Limits on the "charge retention" coordinates, as used in the older literature (*e.g.*, Ref. 16), are given by Burkard *et al.* [17].

**Table 1.** Coupling constants $g^{\gamma}_{\varepsilon\mu}$. Ninety-percent confidence level experimental limits. The limits on $|g^S_{LL}|$ and $|g^V_{LL}|$ are from Ref. 14, and the others are from Ref. 15. The experimental uncertainty on the muon polarization in pion decay is included.

| | | |
|---|---|---|
| $\lvert g^S_{RR}\rvert < 0.066$ | $\lvert g^V_{RR}\rvert < 0.033$ | $\lvert g^T_{RR}\rvert \equiv 0$ |
| $\lvert g^S_{LR}\rvert < 0.125$ | $\lvert g^V_{LR}\rvert < 0.060$ | $\lvert g^T_{LR}\rvert < 0.036$ |
| $\lvert g^S_{RL}\rvert < 0.424$ | $\lvert g^V_{RL}\rvert < 0.110$ | $\lvert g^T_{RL}\rvert < 0.122$ |
| $\lvert g^S_{LL}\rvert < 0.550$ | $\lvert g^V_{LL}\rvert > 0.960$ | $\lvert g^T_{LL}\rvert \equiv 0$ |

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## $\mu$ DECAY PARAMETERS

### $\rho$ PARAMETER

$(V-A)$ theory predicts $\rho = 0.75$.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.7518±0.0026** | | DERENZO | 69 | RVUE | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.762 ±0.008 | 170k | [28] FRYBERGER | 68 | ASPK | + | 25–53 MeV $e^+$ |
| 0.760 ±0.009 | 280k | [28] SHERWOOD | 67 | ASPK | + | 25–53 MeV $e^+$ |
| 0.7503±0.0026 | 800k | [28] PEOPLES | 66 | ASPK | + | 20–53 MeV $e^+$ |

[28] $\eta$ constrained = 0. These values incorporated into a two parameter fit to $\rho$ and $\eta$ by DERENZO 69.

### $\eta$ PARAMETER

$(V-A)$ theory predicts $\eta = 0$.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **−0.007±0.013 OUR AVERAGE** | | | | | | |
| −0.007±0.013 | 5.3M | [29] BURKARD | 85B | FIT | + | 9–53 MeV $e^+$ |
| −0.12 ±0.21 | 6346 | DERENZO | 69 | HBC | + | 1.6–6.8 MeV $e^+$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| −0.012±0.015±0.003 | 5.3M | [30] BURKARD | 85B | CNTR | + | 9–53 MeV $e^+$ |
| 0.011±0.081±0.026 | 5.3M | BURKARD | 85B | CNTR | + | 9–53 MeV $e^+$ |
| −0.7 ±0.5 | 170k | [31] FRYBERGER | 68 | ASPK | + | 25–53 MeV $e^+$ |
| −0.7 ±0.6 | 280k | [31] SHERWOOD | 67 | ASPK | + | 25–53 MeV $e^+$ |
| 0.05 ±0.5 | 800k | [31] PEOPLES | 66 | ASPK | + | 20–53 MeV $e^+$ |
| −2.0 ±0.9 | 9213 | [32] PLANO | 60 | HBC | + | Whole spectrum |

[29] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.
[30] $\alpha = \alpha' = 0$ assumed.
[31] $\rho$ constrained = 0.75.
[32] Two parameter fit to $\rho$ and $\eta$; PLANO 60 discounts value for $\eta$.

## $\delta$ PARAMETER

$(V-A)$ theory predicts $\delta = 0.75$.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.7486±0.0026±0.0028** | | [33] BALKE | 88 | SPEC | + | Surface $\mu^+$'s |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| | | [34] VOSSLER | 69 | | | |
| 0.752 ±0.009 | 490k | FRYBERGER | 68 | ASPK | + | 25–53 MeV $e^+$ |
| 0.782 ±0.031 | | KRUGER | 61 | | | |
| 0.78 ±0.05 | 8354 | PLANO | 60 | HBC | + | Whole spectrum |

[33] BALKE 88 uses $\rho = 0.752 \pm 0.003$.

[34] VOSSLER 69 has measured the asymmetry below 10 MeV. See comments about radiative corrections in VOSSLER 69.

## |($\xi$ PARAMETER)×($\mu$ LONGITUDINAL POLARIZATION)|

$(V-A)$ theory predicts $\xi = 1$, longitudinal polarization $= 1$.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **1.0027±0.0079±0.0030** | | BELTRAMI | 87 | CNTR | | SIN, $\pi$ decay in flight |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 1.0013±0.0030±0.0053 | | [35] IMAZATO | 92 | SPEC | + | $K^+ \rightarrow \mu^+ \nu_\mu$ |
| 0.975 ±0.015 | | AKHMANOV | 68 | EMUL | | 140 kG |
| 0.975 ±0.030 | 66k | GUREVICH | 64 | EMUL | | See AKHMANOV 68 |
| 0.903 ±0.027 | | [36] ALI-ZADE | 61 | EMUL | + | 27 kG |
| 0.93 ±0.06 | 8354 | PLANO | 60 | HBC | + | 8.8 kG |
| 0.97 ±0.05 | 9k | BARDON | 59 | CNTR | | Bromoform target |

[35] The corresponding 90% confidence limit from IMAZATO 92 is $|\xi P_\mu| > 0.990$. This measurement is of $K^+$ decay, not $\pi^+$ decay, so we do not include it in an average, nor do we yet set up a separate data block for $K$ results.

[36] Depolarization by medium not known sufficiently well.

## $\xi$ × ($\mu$ LONGITUDINAL POLARIZATION) × $\delta$ / $\rho$

| VALUE | CL% | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **>0.99682** | 90 | [37] JODIDIO | 86 | SPEC | + | TRIUMF |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| >0.9966 | 90 | [38] STOKER | 85 | SPEC | + | $\mu$-spin rotation |
| >0.9959 | 90 | CARR | 83 | SPEC | + | 11 kG |

[37] JODIDIO 86 includes data from CARR 83 and STOKER 85. The value here is from the erratum.

[38] STOKER 85 find $(\xi P_\mu \delta/\rho)$ >0.9955 and >0.9966, where the first limit is from new $\mu$ spin-rotation data and the second is from combination with CARR 83 data. In $V-A$ theory, $(\delta/\rho) = 1.0$.

## $\xi'$ = LONGITUDINAL POLARIZATION OF $e^+$

($V-A$) theory predicts the longitudinal polarization = $\pm 1$ for $e^\pm$, respectively. We have flipped the sign for $e^-$ so our programs can average.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **1.00 ±0.04 OUR AVERAGE** | | | | | | |
| 0.998±0.045 | 1M | BURKARD | 85 | CNTR | + | Bhabha + annihil |
| 0.89 ±0.28 | 29k | SCHWARTZ | 67 | OSPK | − | Moller scattering |
| 0.94 ±0.38 | | BLOOM | 64 | CNTR | + | Brems. transmiss. |
| 1.04 ±0.18 | | DUCLOS | 64 | CNTR | + | Bhabha scattering |
| 1.05 ±0.30 | | BUHLER | 63 | CNTR | + | Annihilation |

## $\xi''$ PARAMETER

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.65±0.36** | 326k | [39] BURKARD | 85 | CNTR | + | Bhabha + annihil |

[39] BURKARD 85 measure $(\xi''-\xi\xi')/\xi$ and $\xi'$ and set $\xi = 1$.

## TRANSVERSE $e^+$ POLARIZATION IN PLANE OF $\mu$ SPIN, $e^+$ MOMENTUM

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 0.016±0.021±0.01 | 5.3M | BURKARD | 85B | CNTR | + | Annihil 9–53 MeV |

## TRANSVERSE $e^+$ POLARIZATION NORMAL TO PLANE OF $\mu$ SPIN, $e^+$ MOMENTUM

Zero if $T$ invariance holds.

| VALUE | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.007±0.022±0.007** | 5.3M | BURKARD | 85B | CNTR | + | Annihil 9–53 MeV |

## $\alpha/A$

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **0.4± 4.3** | | [40] BURKARD | 85B | FIT | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 15 ±50 ±14 | 5.3M | BURKARD | 85B | CNTR | + | 9–53 MeV $e^+$ |

[40] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

## $\alpha'/A$

Zero if $T$ invariance holds.

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| = **0.2± 4.3** | | [41] BURKARD | 85B | FIT | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| −47 ±50 ±14 | 5.3M | [42] BURKARD | 85B | CNTR | + | 9–53 MeV $e^+$ |

[41] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

[42] BURKARD 85B measure $e^+$ polarizations $P_{T_1}$ and $P_{T_2}$ versus $e^+$ energy.

### $\beta/A$

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|---|---|---|---|---|---|
| **3.9± 6.2** | | [43] BURKARD | 85B FIT | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 2 ±17 ±6 | 5.3M | BURKARD | 85B CNTR | + | 9–53 MeV $e^+$ |

[43] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

### $\beta'/A$

Zero if $T$ invariance holds.

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|---|---|---|---|---|---|
| **1.5± 6.3** | | [44] BURKARD | 85B FIT | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 17 ±17 ±6 | 5.3M | [45] BURKARD | 85B CNTR | + | 9–53 MeV $e^+$ |

[44] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

[45] BURKARD 85B measure $e^+$ polarizations $P_{T_1}$ and $P_{T_2}$ versus $e^+$ energy.

### $a/A$

This comes from an alternative parameterization to that used in the Summary Table (see the "Note on Muon Decay Parameters" above).

| VALUE (units $10^{-3}$) | CL% | DOCUMENT ID | TECN |
|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| <15.9 | 90 | [46] BURKARD | 85B FIT |

[46] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

### $a'/A$

This comes from an alternative parameterization to that used in the Summary Table (see the "Note on Muon Decay Parameters" above).

| VALUE (units $10^{-3}$) | DOCUMENT ID | TECN |
|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | |
| 5.3±4.1 | [47] BURKARD | 85B FIT |

[47] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

### $(b'+b)/A$

This comes from an alternative parameterization to that used in the Summary Table (see the "Note on Muon Decay Parameters" above).

| VALUE (units $10^{-3}$) | CL% | DOCUMENT ID | TECN |
|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| <1.04 | 90 | [48] BURKARD | 85B FIT |

[48] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

### c/A

This comes from an alternative parameterization to that used in the Summary Table (see the "Note on Muon Decay Parameters" above).

| VALUE (units $10^{-3}$) | CL% | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| <6.4 | 90 | [49] BURKARD | 85B | FIT |

[49] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

### c'/A

This comes from an alternative parameterization to that used in the Summary Table (see the "Note on Muon Decay Parameters" above).

| VALUE (units $10^{-3}$) | DOCUMENT ID | | TECN |
|---|---|---|---|
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | |
| 3.5±2.0 | [50] BURKARD | 85B | FIT |

[50] Global fit to all measured parameters. Correlation coefficients are given in BURKARD 85B.

## $\overline{\eta}$ PARAMETER

$(V-A)$ theory predicts $\overline{\eta} = 0$. $\overline{\eta}$ affects spectrum of radiative muon decay.

| VALUE | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|
| **0.02 ±0.08 OUR AVERAGE** | | | | | |
| −0.014±0.090 | EICHENBER... | 84 | ELEC | + | $\rho$ free |
| +0.09 ±0.14 | BOGART | 67 | CNTR | + | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| −0.035±0.098 | EICHENBER... | 84 | ELEC | + | $\rho$=0.75 assumed |

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