# $K^0$

$I(J^P) = \frac{1}{2}(0^-)$

## $K^0$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **497.672±0.031 OUR FIT** | | | | | |
| **497.672±0.031 OUR AVERAGE** | | | | | |
| 497.661±0.033 | 3713 | BARKOV | 87B | CMD | $e^+ e^- \to K^0_L K^0_S$ |
| 497.742±0.085 | 780 | BARKOV | 85B | CMD | $e^+ e^- \to K^0_L K^0_S$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| 497.44 ±0.50 | | FITCH | 67 | OSPK | |
| 498.9 ±0.5 | 4500 | BALTAY | 66 | HBC | $K^0$ from $\overline{p}p$ |
| 497.44 ±0.33 | 2223 | KIM | 65B | HBC | $K^0$ from $\overline{p}p$ |
| 498.1 ±0.4 | | CHRISTENS... | 64 | OSPK | |

## $m_{K^0} - m_{K^\pm}$

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | CHG | COMMENT |
|---|---|---|---|---|---|---|
| **3.995±0.034 OUR FIT** Error includes scale factor of 1.1. | | | | | | |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | | |
| 3.95 ±0.21 | 417 | HILL | 68B | DBC | + | $K^+ d \to K^0 p p$ |
| 3.90 ±0.25 | 9 | BURNSTEIN | 65 | HBC | − | |
| 3.71 ±0.35 | 7 | KIM | 65B | HBC | − | $K^- p \to n\overline{K}^0$ |
| 5.4 ±1.1 | | CRAWFORD | 59 | HBC | + | |
| 3.9 ±0.6 | | ROSENFELD | 59 | HBC | − | |

## $|m_{K^0} - m_{\overline{K}^0}| / m_{\text{average}}$

A test of $CPT$ invariance.

| VALUE | DOCUMENT ID |
|---|---|
| **<$10^{-18}$ OUR EVALUATION** | |

## $T$-VIOLATION PARAMETER IN $K^0$-$\overline{K}^0$ MIXING

The asymmetry $A_T = \frac{\Gamma(\overline{K}^0 \to K^0) - \Gamma(K^0 \to \overline{K}^0)}{\Gamma(\overline{K}^0 \to K^0) + \Gamma(K^0 \to \overline{K}^0)}$ must vanish if $T$ invariance holds.

## ASYMMETRY $A_T$ IN $K^0$-$\overline{K}^0$ MIXING

| VALUE (units $10^{-3}$) | EVTS | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **6.6±1.3±1.0** | 640k | [1] ANGELOPO... | 98E | CPLR |

[1] ANGELOPOULOS 98E measures the asymmetry $A_T = [\Gamma(\overline{K}^0_{t=0} \to e^+ \pi^- \nu_{t=\tau}) - \Gamma(K^0_{t=0} \to e^- \pi^+ \overline{\nu}_{t=\tau})]/[\Gamma(\overline{K}^0_{t=0} \to e^+ \pi^- \nu_{t=\tau}) + \Gamma(K^0_{t=0} \to e^- \pi^+ \overline{\nu}_{t=\tau})]$ as a function of the neutral-kaon eigentime $\tau$. The initial strangeness of the neutral kaon is tagged by the charge of the accompanying charged kaon in the reactions $p\overline{p} \to K^- \pi^+ K^0$ and $p\overline{p} \to K^+ \pi^- \overline{K}^0$. The strangeness at the time of the decay is tagged

by the lepton charge. The reported result is the average value of $A_T$ over the interval $1\tau_S < \tau < 20\tau_S$.

## *CPT* INVARIANCE TESTS IN NEUTRAL KAON DECAY

Written September 1999 by P. Bloch, CERN.

The time evolution of a neutral kaon state state is described by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi \ , \qquad \Lambda \equiv M - \frac{i}{2}\Gamma \tag{1}$$

where $M$ and $\Gamma$ are Hermitian $2\times 2$ matrices known as the mass and decay matrices. The corresponding eigenvalues are $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\gamma_{L,S}$. *CPT* invariance requires the diagonal elements of $\Lambda$ to be equal. The *CPT*-violation complex parameter $\Delta$ is defined as

$$\begin{aligned}\Delta &= \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{2(\lambda_L - \lambda_S)} \\ &= \Delta_{\parallel}\exp\Big(i\phi_{SW}\Big) + \Delta_{\perp}\exp\Big(i(\phi_{SW} + \frac{\pi}{2})\Big)\end{aligned} \tag{2}$$

where we have introduced the projections $\Delta_{\parallel}$ and $\Delta_{\perp}$ respectively parallel and perpendicular to the superweak direction $\phi_{SW} = \tan^{-1}(2\Delta m/\Delta\gamma)$. These projections are linked to the mass and width difference between $K^0$ and $\overline{K}^0$:

$$\Delta_{\parallel} = \frac{1}{4}\frac{\gamma_{K^0} - \gamma_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ , \ \Delta_{\perp} = \frac{1}{2}\frac{m_{K^0} - m_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ . \tag{3}$$

Re($\Delta$) can be directly measured by studying the time evolution of the strangeness content of initially pure $K^0$ and $\overline{K}^0$ states, for example through the asymmetry

$$A_{CPT} = \frac{P[\overline{K}^0 \to \overline{K}^0(t)] - P[K^0 \to K^0(t)]}{P[\overline{K}^0 \to \overline{K}^0(t)] + P[K^0 \to K^0(t)]} = 4\mathrm{Re}(\Delta) \tag{4}$$

where $P[a \to b(t)]$ is the probability that the pure initial state $a$ is seen as state $b$ at proper time $t$. This method has been used by tagging the initial strangeness with strong interactions and the final strangeness with the semileptonic decay (a more appropriate combination of semileptonic rates allows to be independent of any direct $CPT$ violation in the decay itself) and yields today's best value of $\mathrm{Re}(\Delta)$, compatible with zero with an error of $\sim 3 \times 10^{-4}$.

As an alternative it has been proposed to compare the semileptonic charge asymmetries for $K_L$ and $K_S$

$$\delta_{L,S} = \frac{R(K_{L,S} \to \pi^- \ell^+ \nu) - R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})}{R(K_{L,S} \to \pi^- \ell^+ \nu) + R(K_{L,S} \to \pi^+ \ell^- \overline{\nu})} \ ,$$

$$\delta_S - \delta_L = 4\mathrm{Re}(\Delta) \ . \tag{5}$$

$\delta_L$ has been accurately measured and $\delta_S$ should be measured in the near future with tagged $K_S$ at $\phi$ factories. Note however that Eq. (5) assumes $CPT$ invariance in the $\Delta S = -\Delta Q$ semileptonic decay amplitude.

$\Delta_\perp$ can be obtained from the measurement of the $\pi\pi$ decays $CP$-violation parameters $\eta_{+-}$ and $\eta_{00}$. Figure 1 shows the various contributions to $\eta_{\pi\pi}$ [1]. The $T$-violation parameter $\epsilon_T$

$$\epsilon_T = i \frac{|\Lambda_{K^0 \overline{K}^0}|^2 - |\Lambda_{\overline{K}^0 K^0}|^2}{\Delta\gamma(\lambda_L - \lambda_S)} \tag{6}$$

has been defined in such a way that it is exactly aligned along the superweak direction [‡]. $A_I$ (resp. $B_I$) is the $CPT$-conserving (resp. violating) decay amplitude for the $\pi\pi$ Isospin $I$ state, $\varepsilon'$ is the direct $CP/CPT$-violation parameter $[\varepsilon' = 1/3(\eta_{+-} - \eta_{00})]$ and $\delta\phi = \frac{1}{2}\left[\varphi_\Gamma - \arg(A_0^* \overline{A}_0)\right]$ is the phase difference between

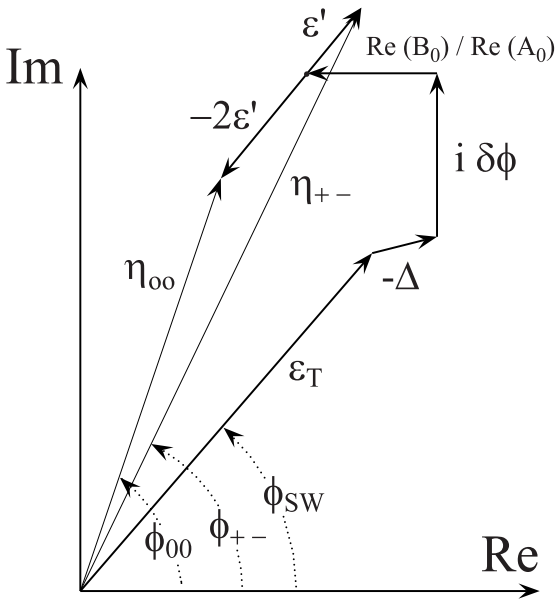


**Figure 1:** *CP*- and *CPT*-violation parameters in $2\pi$ decay.

the $I = 0$ component of the decay amplitude and the matrix element $\Gamma_{K^0\overline{K}^0}$. From Fig. 1 one obtains

$$\Delta_\perp = |\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}) - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\sin(\phi_{SW}) + \delta\phi \ \cos(\phi_{SW}) \ . \quad (7)$$

The present accuracy on the term $|\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00})$ is $2.6 \times 10^{-5}$. $\delta\phi$ gets contributions from *CP* violation in semileptonic and $3\pi$ decays [2,3] and can only be neglected at the present time if one assumes that $\eta_{000}$ is not significantly larger than $\eta_{+-0}$. Furthermore, $B_0$ is not directly measured, so additional assumptions (for example, *CPT* conservation in the

decay which implies $B_0 = 0$) or a combination with other measurements are necessary to obtain $\Delta_\perp$.

If one assumes unitarity, one can measure $\mathrm{Im}(\Delta)$ using the Bell-Steinberger relation which relates $K_S$ and $K_L$ decay amplitudes into all final states $f$:

$$\mathrm{Re}(\epsilon_T) - i\mathrm{Im}(\Delta) = \frac{1}{2(i\Delta m + \frac{1}{2}(\gamma_L + \gamma_S))} \times \sum A_{f_L} A^*_{f_S} \ . \quad (8)$$

Since the $\pi\pi$ amplitudes dominate, the result relies also strongly on the $\phi_{\pi\pi}$ phase measurements. The advantage is that $B_0$ does not enter. Using all available data, one obtains a value of $\mathrm{Im}(\Delta)$ compatible with zero with a precision of $5 \times 10^{-5}$. The precision here is also limited by the poor measurement of $\eta_{000}$.

The results on $\mathrm{Re}(\Delta)$ and $\mathrm{Im}(\Delta)$ can be combined to obtain $\Delta_\parallel$ and $\Delta_\perp$ and therefore the $K^0$–$\overline{K}^0$ mass and width difference shown in Fig. 2. The current accuracy is a few $10^{-18}$ GeV for both.

If one assumes that $CPT$ is conserved in the decays ($\gamma_{K^0} = \gamma_{\overline{K}^0}$, $\Delta_\parallel = 0$, $B_I = 0$), the phase of $\Delta$ is known, and the $\Delta_\perp$ and Bell-Steinberger methods are identical. Assuming in addition $\eta_{+-0} = \eta_{000}$, one in this case obtains a limit for $|m_{K^0} - m_{\overline{K}^0}|$ of $4.4 \times 10^{-19}$ GeV (90%CL).

## Footnotes and References

[‡] Many authors have a different definition of the $T$-violation parameter, $\epsilon = (\Lambda_{\overline{K}^0 K^0} - \Lambda_{K^0 \overline{K}^0})/(2(\lambda_L - \lambda_S))$. $\epsilon$ is not exactly aligned with the superweak direction. The two definitions can be related through $\epsilon = \epsilon_T + i\delta\phi$.

1. See for instance, C.D. Buchanan *et al.*, Phys. Rev. **D45**, 4088 (1992). See also the Second Daphne Handbook, Ed. L.Maiani *et al.*, INFN Frascati (1995).
2. V.V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).
3. L. Lavoura, Mod. Phys. Lett. **A7**, 1367 (1992).

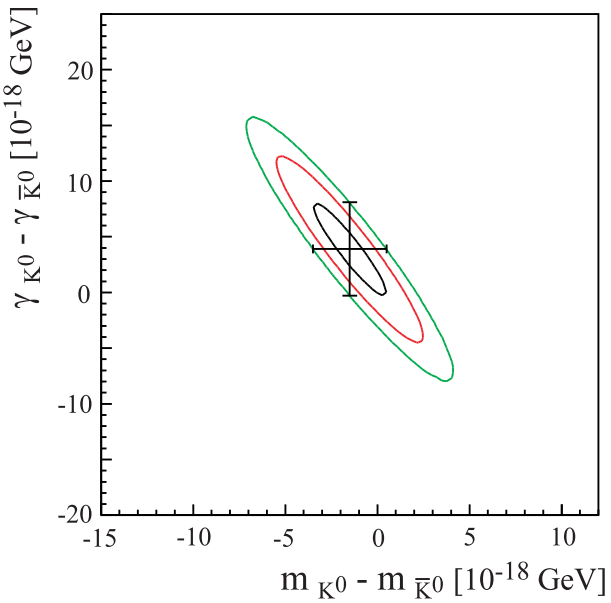


**Figure 2:** $K^0$–$\overline{K}^0$ mass vs width difference.

## *CPT*-VIOLATION PARAMETERS IN $K^0$-$\overline{K}^0$ MIXING

If *CP*-violating interactions include a *T* conserving part then

$$|K_S\rangle = [|K_1\rangle + (\epsilon + \Delta)|K_2\rangle]/\sqrt{1+|\epsilon+\Delta|^2}$$
$$|K_L\rangle = [|K_2\rangle + (\epsilon - \Delta)|K_1\rangle]/\sqrt{1+|\epsilon-\Delta|^2}$$

where

$$|K_1\rangle = [|K^0\rangle + |\overline{K}^0\rangle]/\sqrt{2}$$
$$|K_2\rangle = [|K^0\rangle - |\overline{K}^0\rangle]/\sqrt{2}$$

and

$$|\overline{K}^0\rangle = CP|K^0\rangle.$$

The parameter $\Delta$ specifies the *CPT*-violating part.

Estimates of $\Delta$ are given below assuming the validity of the $\Delta S = \Delta Q$ rule. See also THOMSON 95 for a test of *CPT*-symmetry conservation in $K^0$ decays using the Bell-Steinberger relation.

### REAL PART OF $\Delta$

A nonzero value violates *CPT* invariance.

| *VALUE* (units $10^{-4}$) | *EVTS* | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|---|
| **2.9± 2.7 OUR AVERAGE** | | | | | |
| 2.9± 2.6±0.6 | 1.3M | [2] ANGELOPO... | 98F | CPLR | |
| 180 ±200 | 6481 | [3] DEMIDOV | 95 | | $K_{\ell 3}$ reanalysis |

[2] If $\Delta S = \Delta Q$ is not assumed, ANGELOPOULOS 98F finds $\mathrm{Re}\Delta = (3.0 \pm 3.3 \pm 0.6) \times 10^{-4}$.
[3] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

## IMAGINARY PART OF Δ

A nonzero value violates *CPT* invariance.

| *VALUE* (units $10^{-3}$) | *EVTS* | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|---|
| **= 0.8± 3.1 OUR AVERAGE** | | | | | |
| − 0.9± 2.9±1.0 | 1.3M | [4] ANGELOPO... | 98F | CPLR | |
| 21 ±37 | 6481 | [5] DEMIDOV | 95 | | $K_{\ell 3}$ reanalysis |

[4] If $\Delta S = \Delta Q$ is not assumed, ANGELOPOULOS 98F finds $\mathrm{Im}\Delta = (-15 \pm 23 \pm 3) \times 10^{-3}$.
[5] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

## $K^0$ REFERENCES


| | | | | |
|---|---|---|---|---|
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