# 9. QUANTUM CHROMODYNAMICS

## 9.1. The QCD Lagrangian

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Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the SU(3)×SU(2)×U(1) Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. The Lagrangian describing the interactions of quarks and gluons is (up to gauge-fixing terms)

$$L_{\rm QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu}\ F^{(a)\mu\nu} + i \sum_q \overline{\psi}^i_q\ \gamma^\mu\ (D_\mu)_{ij}\ \psi^j_q - \sum_q m_q\ \overline{\psi}^i_q\ \psi_{qi}\ , \tag{9.1}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu\ A^a_\nu - \partial_\nu\ A^a_\mu - g_s\ f_{abc}\ A^b_\mu\ A^c_\nu\ , \tag{9.2}$$

$$(D_\mu)_{ij} = \delta_{ij}\ \partial_\mu + i g_s\ \sum_a \frac{\lambda^a_{i,j}}{2} A^a_\mu\ , \tag{9.3}$$

where $g_s$ is the QCD coupling constant, and the $f_{abc}$ are the structure constants of the SU(3) algebra (the $\lambda$ matrices and values for $f_{abc}$ can be found in "SU(3) Isoscalar Factors and Representation Matrices," Sec. 32 of this *Review*). The $\psi^i_q(x)$ are the 4-component Dirac spinors associated with each quark field of (3) color $i$ and flavor $q$, and the $A^a_\mu(x)$ are the (8) Yang-Mills (gluon) fields. A complete list of the Feynman rules which derive from this Lagrangian, together with some useful color-algebra identities, can be found in Ref. 1.

The principle of "asymptotic freedom" (see below) determines that the renormalized QCD coupling is small only at high energies, and it is only in this domain that high-precision tests—similar to those in QED—can be performed using perturbation theory. Nonetheless, there has been in recent years much progress in understanding and quantifying the predictions of QCD in the nonperturbative domain, for example, in soft hadronic processes and on the lattice [2]. This short review will concentrate on QCD at short distances (large momentum transfers), where perturbation theory is the standard tool. It will discuss the processes that are used to determine the coupling constant of QCD. Other recent reviews of the coupling constant measurements may be consulted for a different perspective [3,4].

## 9.2. The QCD coupling and renormalization scheme

The renormalization scale dependence of the effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the $\beta$-function:

$$\mu\frac{\partial\alpha_s}{\partial\mu} = 2\beta(\alpha_s) = \ -\frac{\beta_0}{2\pi}\,\alpha_s^2 \ - \frac{\beta_1}{4\pi^2}\,\alpha_s^3 \ - \frac{\beta_2}{64\pi^3}\,\alpha_s^4 \ - \cdots\ , \tag{9.4a}$$

$$\beta_0 = 11 - \frac{2}{3}n_f\ , \tag{9.4b}$$

$$\beta_1 = 51 - \frac{19}{3}n_f\ , \tag{9.4c}$$

$$\beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2\ ; \tag{9.4d}$$

where $n_f$ is the number of quarks with mass less than the energy scale $\mu$. The expression for the next term in this series ($\beta_3$) can be found in Ref. 5. In solving this differential equation for $\alpha_s$, a constant of integration is introduced. This constant is the one fundamental constant of QCD that must be determined from experiment. The most sensible choice for this constant is the value of $\alpha_s$ at a fixed-reference scale $\mu_0$. It has become standard to choose $\mu_0 = M_Z$. The value at other values of $\mu$ can be obtained from $\log(\mu^2/\mu_0^2) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \dfrac{d\alpha}{\beta(\alpha)}$. It is also convenient to introduce the dimensional parameter $\Lambda$, since this provides a parameterization of the $\mu$ dependence of $\alpha_s$. The definition of $\Lambda$ is arbitrary. One way to define it (adopted here) is to write a solution of Eq. (9.4) as an expansion in inverse powers of $\ln(\mu^2)$:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0\ \ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\ \frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4\ln^2(\mu^2/\Lambda^2)}\right.$$
$$\left.\times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]\ . \tag{9.5}$$

This solution illustrates the *asymptotic freedom* property: $\alpha_s \to 0$ as $\mu \to \infty$ and shows that QCD becomes strongly coupled at $\mu \sim \Lambda$.

Consider a "typical" QCD cross section which, when calculated perturbatively, starts at $\mathcal{O}(\alpha_s)$:

$$\sigma = A_1\ \alpha_s + A_2\ \alpha_s^2 + \ \cdots\ . \tag{9.6}$$

The coefficients $A_1$, $A_2$ come from calculating the appropriate Feynman diagrams. In performing such calculations, various divergences arise, and these must be regulated in a consistent way. This requires a particular renormalization scheme (RS). The most commonly used one is the modified minimal subtraction ($\overline{\rm MS}$) scheme [6]. This involves continuing momentum integrals from 4 to 4–$2\epsilon$ dimensions, and then subtracting off the resulting $1/\epsilon$ poles and also $(\ln 4\pi - \gamma_E)$, which is an artifact of continuing the dimension. (Here $\gamma_E$ is the Euler-Mascheroni constant.) To preserve the dimensionless nature of the coupling, a mass scale $\mu$ must also be introduced: $g \to \mu^\epsilon g$. The finite coefficients $A_i$ $(i \geq 2)$ thus obtained depend implicitly on the renormalization convention used and explicitly on the scale $\mu$.

The first two coefficients $(\beta_0, \beta_1)$ in Eq. (9.4) are independent of the choice of RS's. In contrast, the coefficients of terms proportional to $\alpha_s^n$ for $n > 3$ *are* RS-dependent. The form given above for $\beta_2$ is in the $\overline{\rm MS}$ scheme.

The fundamental theorem of RS dependence is straightforward. Physical quantities, such as the cross section calculated to all orders in perturbation theory, do not depend on the RS. It follows that a truncated series *does* exhibit RS dependence. In practice, QCD cross sections are known to leading order (LO), or to next-to-leading order (NLO), or in some cases, to next-to-next-to-leading order (NNLO); and it is only the latter two cases, which have reduced RS dependence, that are useful for precision tests. At NLO the RS dependence is completely given by one condition which can be taken to be the value of the renormalization scale $\mu$. At NNLO this is not sufficient, and $\mu$ is no longer equivalent to a choice of scheme; both must now be specified. One, therefore, has to address the question of what is the "best" choice for $\mu$ within a given scheme, usually $\overline{\rm MS}$. There is no definite answer to this question—higher-order corrections do not "fix" the scale, rather they render the theoretical predictions less sensitive to its variation.

One should expect that choosing a scale $\mu$ characteristic of the typical energy scale $(E)$ in the process would be most appropriate. In general, a poor choice of scale generates terms of order $\ln(E/\mu)$ in the $A_i$'s. Various methods have been proposed including choosing the scale for which the next-to-leading-order correction vanishes ("Fastest Apparent Convergence [7]"); the scale for which the next-to-leading-order prediction is stationary [8], (*i.e.*, the value of $\mu$ where $d\sigma/d\mu = 0$); or the scale dictated by the effective charge scheme [9] or by the BLM scheme [10]. By comparing the values of $\alpha_s$ that different reasonable schemes give, an estimate of theoretical errors can be obtained. It has also been suggested to replace the perturbation series by its Pade approximant [11]. Results obtained using this method have, in certain cases, a reduced scale dependence [12,13]. One can also attempt to determine the scale from data by allowing it to vary and using a fit to determine it. This method can allow a determination of the error due to the scale choice and can give more confidence in the end result [14]. In many of the cases discussed below this scale uncertainty is the dominant error.

An important corollary is that if the higher-order corrections are naturally small, then the additional uncertainties introduced by the $\mu$ dependence are likely to be small. There are some processes, however, for which the choice of scheme *can* influence the extracted value of $\alpha_s(M_Z)$. There is no resolution to this problem other than to try to calculate even more terms in the perturbation series. It is important to note that, since the perturbation series is an asymptotic expansion, there is a limit to the precision with which any theoretical quantity can be calculated. In some processes, the highest-order perturbative terms may be comparable in size to nonperturbative corrections (sometimes called higher-twist or renormalon effects, for a discussion see [15]); an estimate of these terms and their uncertainties is required if a value of $\alpha_s$ is to be extracted.

Cases occur where there is more than one large scale, say $\mu_1$ and $\mu_2$. In these cases, terms appear of the form $\log(\mu_1/\mu_2)$. If the ratio $\mu_1/\mu_2$ is large, these logarithms can render naive perturbation theory unreliable and a modified perturbation expansion that takes these terms into account must be used. A few examples are discussed below.

In the cases where the higher-order corrections to a process are known and are large, some caution should be exercised when quoting the value of $\alpha_s$. In what follows, we will attempt to indicate the size of the theoretical uncertainties on the extracted value of $\alpha_s$. There are two simple ways to determine this error. First, we can estimate it by comparing the value of $\alpha_s(\mu)$ obtained by fitting data using the QCD formula to highest known order in $\alpha_s$, and then comparing it with the value obtained using the next-to-highest-order formula ($\mu$ is chosen as the typical energy scale in the process). The corresponding $\Lambda$'s are then obtained by evolving $\alpha_s(\mu)$ to $\mu = M_Z$ using Eq. (9.4) to the same order in $\alpha_s$ as the fit. Alternatively, we can vary the value of $\mu$ over a reasonable range, extracting a value of $\Lambda$ for each choice of $\mu$. This method is by its nature imprecise, since "reasonable" involves a subjective judgment. In either case, if the perturbation series is well behaved, the resulting error on $\alpha_s(M_Z)$ will be small.

In the above discussion we have ignored quark-mass effects, *i.e.*, we have assumed an idealized situation where quarks of mass greater than $\mu$ are neglected completely. In this picture, the $\beta$-function coefficients change by discrete amounts as flavor thresholds (a quark of mass $M$) are crossed when integrating the differential equation for $\alpha_s$. Now imagine an experiment at energy scale $\mu$; for example, this could be $e^+e^- \to$ hadrons at center-of-mass energy $\mu$. If $\mu \gg M$, the mass $M$ is negligible and the process is well described by QCD with $n_f$ massless flavors and its parameter $\alpha_{(n_f)}$ up to terms of order $M^2/\mu^2$. Conversely if $\mu \ll M$, the heavy quark plays no role and the process is well described by QCD with $n_f - 1$ massless flavors and its parameter $\alpha_{(n_f-1)}$ up to terms of order $\mu^2/M^2$. If $\mu \sim M$, the effects of the quark mass are process-dependent and cannot be absorbed into the running coupling. The values of $\alpha_{(n_f)}$ and $\alpha_{(n_f-1)}$ are related so that a physical quantity calculated in both "theories" gives the same result [16]. This implies, for $\mu = M$

$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2}\alpha^3_{(n_f-1)}(M) + \mathcal{O}\left(\alpha^4_{(n_f-1)}\right) \tag{9.7}$$

which is almost identical to the naive result $\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M)$. Here $M$ is the mass of the value of the running quark mass defined in the $\overline{\rm MS}$ scheme (see the note on "Quark Masses" in the Particle Listings for more details), *i.e.*, where $M_{\overline{\rm MS}}(M) = M$.

It also follows that, for a relationship such as Eq. (9.5) to remain valid for all values of $\mu$, $\Lambda$ must also change as flavor thresholds are crossed, the value corresponds to an effective number of massless quarks: $\Lambda \to \Lambda^{(n_f)}$ [16,17]. The formulae are given in the previous edition of this review.

Data from deep-inelastic scattering are in a range of energy where the bottom quark is not readily excited, and hence, these experiments quote $\Lambda^{(4)}_{\overline{\rm MS}}$. Most data from PEP, PETRA, TRISTAN, LEP, and SLC quote a value of $\Lambda^{(5)}_{\overline{\rm MS}}$ since these data are in an energy range where the bottom quark is light compared to the available energy. We have converted it to $\Lambda^{(4)}_{\overline{\rm MS}}$ as required. A few measurements, including the lattice gauge theory values from the $J/\psi$ system, and from $\tau$ decay are at sufficiently low energy that $\Lambda^{(3)}_{\overline{\rm MS}}$ is appropriate.

In order to compare the values of $\alpha_s$ from various experiments, they must be evolved using the renormalization group to a common scale. For convenience, this is taken to be the mass of the $Z$ boson. This evolution uses third-order perturbation theory and can introduce additional errors particularly if extrapolation from very small scales is used. The variation in the charm and bottom quark masses ($M_b = 4.3 \pm 0.2$ GeV and $M_c = 1.3 \pm 0.3$ GeV are used [18]) can also introduce errors. These result in a fixed value of $\alpha_s(2\ \text{GeV})$ giving an uncertainty in $\alpha_s(M_Z) = \pm 0.001$ if only perturbative evolution is used. There could be additional errors from nonperturbative effects that enter at low energy.

## 9.3. QCD in deep-inelastic scattering

The original and still one of the most powerful quantitative tests of perturbative QCD is the breaking of Bjorken scaling in deep-inelastic lepton-hadron scattering. The review 'Structure Functions," (Sec. 36 of this *Review*) describes the basic formalism and reviews the data. $\alpha_s$ is obtained together with the structure functions. The global fit of (Sec. 36) gives $\alpha_s(M_Z) = 0.119 \pm 0.004$.

Nonsinglet structure function offers in principle the most precise test of the theory, since the $Q^2$ evolution is independent of the unmeasured gluon distribution. The CCFR collaboration fit to the Gross-Llewellyn Smith sum rule [19] which is known to order $\alpha_s^3$ [20,21] which is one order larger than used in the structure function fits (estimates of the order $\alpha_s^4$ term are available [22]).

$$\int_0^1 dx \left(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)\right) = 3\left[1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0\left(\frac{\alpha_s}{\pi}\right)^2) - \Delta HT\right] \quad , \tag{9.8}$$

where the higher-twist contribution $\Delta HT$ is estimated to be $(0.09 \pm 0.045)/Q^2$ in Refs. 20,23 and to be somewhat smaller by Ref. 24. The CCFR collaboration [25],

combines their data with that from other experiments [26] and gives $\alpha_s$ ($\sqrt{3}$ GeV) = $0.28 \pm 0.035$ (expt.) $\pm 0.05$ (sys)$^{+0.035}_{-0.03}$ (theory). The error from higher-twist terms (assumed to be $\Delta HT = 0.05 \pm 0.05$) dominates the theoretical error. If the higher twist result of Ref. 24 is used, the central value increases to 0.31 in agreement with the fit of [27]. This value corresponds to $\alpha_s(M_Z) = 0.118 \pm 0.011$. Fits of the $Q^2$ evolution [28] of $xF_3$ using the CCFR data using NNLO and estimates of NNNLO QCD and higher twist terms enables the effect of these terms to be studied.

The spin-dependent structure functions, measured in polarized lepton-nucleon scattering, can also be used to test QCD and to determine $\alpha_s$. Here the values of $Q^2 \sim 2.5$ GeV$^2$ are small, particularly for the E143 data [29], and higher-twist corrections are important. A fit [30] by an experimental group using the measured spin dependent structure functions for several experiments Refs. 29,31 as well as their own data has been made. When data from HERMES [32] and [33] are included [34] $\alpha_s(M_Z) = 0.114^{+0.004}_{-0.005}(\text{expt.}) \pm 0.009(\text{theory})$.

$\alpha_s$ can also be determined from the Bjorken sum rule [35]; a fit gives [36] $\alpha_s(M_Z) = 0.118^{+0.010}_{-0.024}$; consistent with an earlier determination [37], the larger error being due to the extrapolation into the (unmeasured) small $x$ region. Theoretically, the sum rule is preferable as the perturbative QCD result is known to higher order and these terms are important at the low $Q^2$ involved. It has been shown that the theoretical errors associated with the choice of scale are considerably reduced by the use of Pade approximants [12] which results in $\alpha_s(1.7 \text{ GeV}) = 0.328 \pm 0.03(\text{expt.}) \pm 0.025(\text{theory})$ corresponding to $\alpha_s(M_Z) = 0.116^{+0.003}_{-0.005}(\text{expt.}) \pm 0.003(\text{theory})$. No error is included from the extrapolation into the region of $x$ that is unmeasured. Should data become available at smaller values of $x$ so that this extrapolation could be more tightly constrained, the sum rule method could provide the best determination of $\alpha_s$; the result from the structure functions themselves is used in the average.

## 9.4. QCD in decays of the $\tau$ lepton

The semi-leptonic branching ratio of the tau ($\tau \to \nu_\tau + \text{hadrons}$, $R_\tau$) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the $W$ self energy $\big(\Pi(s)\big)$. It is sensitive to a range of energies since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1 - \frac{s}{m_\tau^2})^2 \ \text{Im}\,(\Pi(s)) \ . \tag{9.9}$$

Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the $\tau$ mass.

$$\begin{aligned} R_\tau =& 3.058\Bigg[1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.2\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^2 + 26.4\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^3 \\ &+ a\frac{m^2}{m_\tau^2} + b\frac{m\psi\overline{\psi}}{m_\tau^4} + c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6} + \cdots\Bigg] \ . \end{aligned} \tag{9.10}$$

Here $a, b$, and $c$ are dimensionless constants and $m$ is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light quark masses and is consequently very

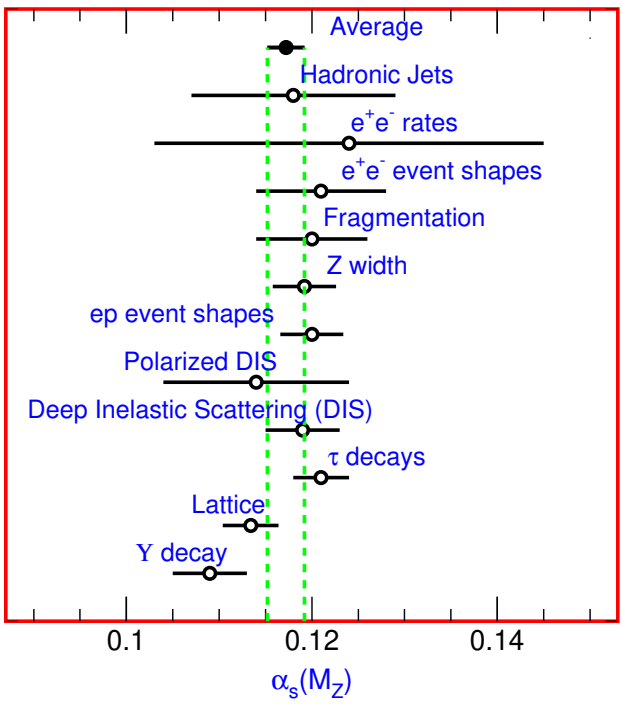


**Figure 9.1:** Summary of the value of $\alpha_s(M_Z)$ from various processes. The values shown indicate the process and the measured value of $\alpha_s$ extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties.

small. The nonperturbative terms are estimated using sum rules [38]. In total, they are estimated to be $-0.014 \pm 0.005$ [39,40]. This estimate relies on there being no term of order $\Lambda^2/m_\tau^2$ $\left(\text{note that } \frac{\alpha_s(m_\tau)}{\pi} \sim (\frac{0.5 \text{ GeV}}{m_\tau})^2\right)$. The $a$, $b$, and $c$ can be determined from the data [41] by fitting to moments of the $\Pi(s)$ and separately to the final states accessed by the vector and axial parts of the $W$ coupling. The values so extracted [42,43] are consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by $\sim 0.02$.

For $\alpha_s(m_\tau) = 0.35$ the perturbative series for $R_\tau$ is $R_\tau \sim 3.058(1+0.112+0.064+0.036)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order $\alpha_s^3$ term. The perturbation series is not very well convergent; if the order $\alpha_s^3$ term is omitted, the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. The order $\alpha_s^4$ term has been estimated [44] and attempts made to resum the entire series [45,46]. These estimates can be used to obtain an estimate of the errors due to these unknown terms [47,48]. We assign an uncertainty of $\pm 0.02$ to $\alpha_s(m_\tau)$ from these sources.

$R_\tau$ can be extracted from the semi-leptonic branching ratio from the relation $R_\tau = 1/(\mathrm{B}(\tau \to e\nu\overline{\nu}) - 1.97256$; where $\mathrm{B}(\tau \to e\nu\overline{\nu})$ is measured directly or extracted from the lifetime, the muon mass, and the muon lifetime assuming universality of lepton couplings. Using the average lifetime of $290.6 \pm 1.1$ fs and a $\tau$ mass of $1776.99 \pm 0.29$ MeV from the PDG fit gives $R_\tau = 3.645 \pm 0.020$. The direct measurement of $\mathrm{B}(\tau \to e\nu\overline{\nu})$ can be combined with $\mathrm{B}(\tau \to \mu\nu\overline{\nu})$ to give $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1785 \pm 0.0005$ which gives $R_\tau = 3.629 \pm 0.015$. Averaging these yields $\alpha_s(m_\tau) = 0.353 \pm 0.007$ using the experimental error alone. We assign a theoretical error equal to 40% of the contribution from the order $\alpha^3$ term and all of the nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.35 \pm 0.03$ for the final result. This corresponds to $\alpha_s(M_Z) = 0.121 \pm 0.003$. This result is consistent with that obtained by using the moments [49].

## 9.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadronic jets with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* Higher–order QCD calculations of the jet rates [50] and shapes are in impressive agreement with data [51]. This agreement has led to the proposal that these data could be used to provide a determination of $\alpha_s$ [52]. A set of structure functions is assumed and jet data are fitted over a very large range of transverse momenta to the QCD prediction for the underlying scattering process that depends on $\alpha_s$. The evolution of the coupling over this energy range (40 to 250 GeV) is therefore tested in the analysis. CDF obtains $\alpha_s(M_Z) = 0.1178 \pm 0.0001$ (stat.) $\pm 0.0085$ (syst.) [53]. Estimation of the theoretical errors is not straightforward. The structure functions used depend implicitly on $\alpha_s$ and an iteration procedure must be used to obtain a consistent result; different sets of structure functions yield different correlations between the two values of $\alpha_s$. CDF includes a scale error of 4% and a structure function error of 5% in the determination of $\alpha_s$. Ref. 52 estimates the error from unknown higher order QCD corrections to be $\pm 0.005$. Combining these then gives $\alpha_s(M_Z) = 0.118 \pm 0.011$. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [54,55].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass $Q$ from virtual photons, or of real $W$ or $Z$ bosons), are known [56]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of $Q$. The correction to $W$ and $Z$ production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8$ TeV, is of order 30%. The NNLO corrections to this process are known [57].

The production of $W$ and $Z$ bosons and photons at large transverse momentum can also be used to test QCD. The leading-order QCD subprocesses are $q\overline{q} \to Vg$ and $qg \to Vq$ ($V = W, Z, \gamma$). If the parton distributions are taken from other processes and a value of $\alpha_s$ assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of $\alpha_s$. The next-to-leading-order QCD corrections are known [58,59] (for photons), and for

$W/Z$ production [60], and so a precision test is possible. Data exist on photon production from the CDF and DØ collaborations [61,62] and from fixed target experiments [63]. Detailed comparisons with QCD predictions [64] may indicate an excess of the data over the theoretical prediction at low value of transverse momenta, although other authors [65] find smaller excesses.

The UA2 collaboration [66] extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018(\text{stat.}) \pm 0.017(\text{syst.})$ from the measured ratio $R_W = \dfrac{\sigma(W + 1\text{jet})}{\sigma(W + 0\text{jet})}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events (the former causes jet energy to be lost, the latter causes it to be increased) are connected to the algorithm. There is also dependence on the parton distribution functions, and hence, $\alpha_s$ appears explicitly in the formula for $R_W$, and implicitly in the distribution functions. The UA2 result is not used in the final average. Data from CDF and DØ on the $W$ $p_t$ distribution [67] are in agreement with QCD but are not able to determine $\alpha_s$ with sufficient precision to have any weight in a global average.

In the region of low $p_t$, fixed order perturbation theory is not applicable; one must sum terms of order $\alpha_s^n \ln^n(p_t/M_W)$ [68]. Data from DØ [69] on the $p_t$ distribution of $Z$ bosons agree well with these predictions.

The production rates of $b$ quarks in $p\overline{p}$ have been used to determine $\alpha_s$ [70]. The next-to-leading-order QCD production processes [71] have been used. By selecting events where the $b$ quarks are back-to-back in azimuth, the next-to-leading-order calculation can be used to compare rates to the measured value and a value of $\alpha_s$ extracted. The errors are dominated by the measurement errors, the choice of $\mu$ and $M$, and uncertainties in the choice of structure functions. The last were estimated by varying the structure functions used. The result is $\alpha_s(M_Z) = 0.113^{+0.009}_{-0.013}$, which is not included in the final average, as the measured $b\overline{b}$ cross section is in poor agreement with perturbative QCD.

## 9.6. QCD in heavy-quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of $\alpha_s$ at the heavy-quark mass scale. The most precise data come from the decay widths of the $1^{--}$ $J/\psi(1S)$ and $\Upsilon$ resonances. The total decay width of the $\Upsilon$ is predicted by perturbative QCD [72,73]

$$\begin{aligned} R_\mu(\Upsilon) &= \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)} \\ &= \frac{10(\pi^2-9)\alpha_s^3(M_b)}{9\pi\alpha_{\text{em}}^2} \\ &\quad \times \left[1 + \frac{\alpha_s}{\pi}\left(-19.36 + \frac{3\beta_0}{2}\left(1.161 + \ln\left(\frac{2M}{M_\Upsilon}\right)\right)\right)\right] . \end{aligned} \tag{9.11}$$

Data are available for the $\Upsilon$, $\Upsilon'$, $\Upsilon''$, and $J/\psi$. The result is very sensitive to $\alpha_s$ and the data are sufficiently precise ($R_\mu(\Upsilon) = 39.11 \pm 0.4$) [74] that the theoretical errors will dominate. There are theoretical corrections to this formula due to the relativistic nature of the $Q\overline{Q}$ system which have been calculated [73] to order $v^2/c^2$. These corrections are more severe for the $J/\psi$. There are also nonperturbative corrections arising from annihilation from higher Fock states ("color octet" contribution) which can only be estimated [75]; again these are more severe for the $J/\psi$. The $\Upsilon$ gives $\alpha_s(M_b) = 0.177 \pm 0.01$, where the error includes that from the "color octet" term and the choice of scale which together dominate. The ratio of widths $\dfrac{\Upsilon \to \gamma gg}{\Upsilon \to ggg}$ has been measured by the CLEO collaboration and can be used to determine $\alpha_s(M_b) = 0.189 \pm 0.01 \pm 0.01$. The error is dominated by theoretical uncertainties associated with the scale choice; the uncertainty due to the "color octet" piece is not present in this case [76]. The theoretical uncertainties due to the production of photons in fragmentation [77] are small [78]. Higher order QCD calculations of the photon energy distribution are available [79]; this distribution could now be used to further test the theory. The width $\Gamma(\Upsilon \to e^+e^-)$ can also be used to determine $\alpha_s$ by using moments of the quantity $R_b(s) = \dfrac{\sigma(e^+e^- \to b\overline{b})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ defined by $M_n = \int_0^\infty \dfrac{R_b(s)}{s^{n+1}}$ [80]. At large values of $n$, $M_n$ is dominated by $\Gamma(\Upsilon \to e^+e^-)$. Higher order corrections are available and the method gives [81] $\alpha_s(M_b) = 0.220 \pm 0.027$. The dominant error is theoretical and is dominated by the choice of scale and by uncertainties in Coulomb corrections. These various $\Upsilon$ measurements can be combined and give $\alpha_s(M_b) = 0.185 \pm 0.01$ corresponding to $\alpha_s(M_Z) = 0.109 \pm 0.004$.

## 9.7. Perturbative QCD in $e^+e^-$ collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)} \left[ 1 + \frac{\alpha_s}{\pi} + C_2 \left(\frac{\alpha_s}{\pi}\right)^2 + C_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \cdots \right] , \tag{9.12}$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [82].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over $\Omega$. The formula is given in Sec. 35.2 of this *Review*. This result is only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [83]. The principal advantage of determining $\alpha_s$ from $R$ in $e^+e^-$ annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

A measurement by CLEO [84] at $\sqrt{s} = 10.52$ GeV yields $\alpha_s(10.52\ \text{GeV}) = 0.20 \pm 0.01 \pm 0.06$, which corresponds to $\alpha_s(M_Z) = 0.13 \pm 0.005 \pm 0.03$. A comparison of the theoretical prediction of Eq. (9.12) (corrected for the $b$-quark mass), with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV, gives [85] $\alpha_s(35\ \text{GeV}) = 0.146 \pm 0.030$ . The size of the order $\alpha_s^3$ term is of order 40% of that of the order $\alpha_s^2$ and 3% of the order $\alpha_s$. If the order $\alpha_s^3$ term is not included, a fit to the data yields $\alpha_s\ (35\ \text{GeV}) = 0.142 \pm 0.03$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the $Z$ at LEP and SLC, $\Gamma_h/\Gamma_\mu$ probe, the same quantity as $R$. Using the average of $\Gamma_h/\Gamma_\mu = 20.783 \pm 0.029$ gives $\alpha_s(M_Z) = 0.123 \pm 0.004$ [86]. There are theoretical errors arising from the values of top-quark and Higgs masses which enter due to electroweak corrections to the $Z$ width and from the choice of scale.

While this method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the $Z$ to quarks [87]. The presence of new physics which changes these couplings via electroweak radiative corrections would invalidate the value of $\alpha_s(M_Z)$. However, given the excellent agreement [88] of the many measurements at the $Z$, there is no reason not to use the value of $\alpha_s(M_Z) = 0.1200 \pm 0.0028 \pm 0.002(\text{scale})$ from the global fits of the various precision measurements at LEP/SLC and the $W$ and top masses in the world average (see the section on "Electroweak model and constraints on new physics," Sec. 10 of this *Review*).

An alternative method of determining $\alpha_s$ in $e^+e^-$ annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. A review should be consulted for more details [89] of the issues mentioned briefly here. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [90], energy-energy correlations [91], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at

order $\alpha_s$, for the process $e^+e^- \to qqg$: [92]

$$\frac{1}{\sigma}\,\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\ , \tag{9.13}$$

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{9.14}$$

where $x_i$ are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order $\alpha_s^2$ corrections to this process have been computed, as well as the 4-jet final states such as $e^+e^- \to qqgg$ [93].

There are many methods used by the $e^+e^-$ experimental groups to determine $\alpha_s$ from the event topology. The jet-counting algorithm, originally introduced by the JADE collaboration [94], has been used by many other groups. Here, particles of momenta $p_i$ and $p_j$ are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is iterated until all pairs of particles or pseudoparticles have a mass-measure that exceeds $y_0\sqrt{s}$; the remaining number is then defined to be the jet multiplicity. The remaining number is then defined to be the number of jets in the event, and can be compared to the QCD prediction. The Durham algorithm is slightly different: in computing the mass of a pair of partons, it uses $M^2 = 2\mathrm{min}(E_i^2, E_j^2)(1-\cos\theta_{ij})$ for partons of energies $E_i$ and $E_j$ separated by angle $\theta_{ij}$ [95].

There are theoretical ambiguities in the way this process is carried out. Quarks and gluons are massless, whereas the observed hadrons are not, so that the massive jets that result from this scheme cannot be compared directly to the massless jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless. These schemes result in the same data giving slightly different values [96,97] of $\alpha_s$. These differences can be used to determine a systematic error. In addition, since what is observed are hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons, so that detector corrections can be applied. The QCD matrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [98–101] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP/SLD experiments.

The scale $M$ at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the $e^+e^-$ center-of-mass energy. While there is no justification for doing so, if the value is allowed to float in the

fit to the data, the fit improves and the data tend to prefer values of order $\sqrt{s}/10$ GeV for some variables [97,102]; the exact value depends on the variable that is fitted.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust ($T$) distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contain terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events), the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in $\alpha_s$ [103]. If the jet recombination methods are used higher-order terms involve $\alpha_s^n \ln^m(y_0)$, these too can be resummed [104]. The resummed results give better agreement with the data at large values of $T$. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading-log corrections. Different schemes for combining the order $\alpha_s^2$ and the resummations are available [105]. These different schemes result in shifts in $\alpha_s$ of order $\pm 0.002$. The use of the resummed results improves the agreement between the data and the theory. An average of results at the $Z$ resonance from SLD [97], OPAL [106], L3 [107], ALEPH [108], and DELPHI [109], using the combined $\alpha_s^2$ and resummation fitting to a large set of shape variables, gives $\alpha_s(M_Z) = 0.122 \pm 0.007$. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale, and the effects of hadronization Monte Carlos on the different quantities fitted.

Estimates are available for the nonperturbative corrections to the mean value of $1-T$ [110]. These are of order $1/E$ and involve a single parameter to be determined from experiment. These corrections can then be used as an alternative to those modeled by the fragmentation Monte-Carlos. The DELPHI collaboration [111] uses data up to the $Z$ mass from many experiments and determines $\alpha_s(M_Z) = 0.119 \pm 0.006$, the error being dominated by the choice of scale. The value is also determined by a fit to a second variable (the mean jet mass); while the extracted values of $\alpha_s(M_Z)$ are consistent with each other, the values of the non perturbative parameter are not. The analysis is useful as one can directly determine the size of the $1/E$ corrections; they are approximately 20% (50%) of the perturbative result at $\sqrt{s} = 91(11)$ GeV.

Similar studies on event shapes have been undertaken at lower energies at TRISTAN, PEP/PETRA, and CLEO. A combined result from various shape parameters by the TOPAZ collaboration gives $\alpha_s(58 \text{ GeV}) = 0.125 \pm 0.009$, using the fixed order QCD result, and $\alpha_s(58 \text{ GeV}) = 0.132 \pm 0.008$ (corresponding to $\alpha_s(M_Z) = 0.123 \pm 0.007$), using the same method as in the SLD and LEP average [112]. The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. A recent reevaluation of the JADE data [113] obtained using resummed QCD results and by averaging over several shape variables gives $\alpha_s(35 \text{ GeV}) = 0.145^{+0.012}_{-0.007}$. An analysis by the TPC group [114] gives $\alpha_s(29 \text{ GeV}) = 0.160 \pm 0.012$, using the same method as TOPAZ. This value corresponds to $\alpha_s(M_Z) = 0.131 \pm 0.010$

The CLEO collaboration fits to the order $\alpha_s^2$ results for the two jet fraction at $\sqrt{s} = 10.53$ GeV, and obtains $\alpha_s(10.93 \text{ GeV}) = 0.164 \pm 0.004$ (expt.) $\pm 0.014$ (theory) [115]. The dominant systematic error arises from the choice of scale ($\mu$), and is determined from

the range of $\alpha_s$ that results from fit with $\mu = 10.53$ GeV, and a fit where $\mu$ is allowed to vary to get the lowest $\chi^2$. The latter results in $\mu = 1.2$ GeV. Since the quoted result corresponds to $\alpha_s(1.2\ \text{GeV}) = 0.35$, it is by no means clear that the perturbative QCD expression is reliable and the resulting error should, therefore, be treated with caution. A fit to many different variables as is done in the LEP/SLC analyses would give added confidence to the quoted error.

Studies have been carried out at energies between ~130 GeV [116] and ~200 GeV [117]. These can be combined to give $\alpha_s(130\ \text{GeV}) = 0.114 \pm 0.008$ and $\alpha_s(189\ \text{GeV}) = 0.1104 \pm 0.005$. The dominant errors are theoretical and systematic and, as most of these are in common at the two energies. These data and those at the $Z$ resonance and below provide clear confirmation of the expected decrease in $\alpha_s$ as the energy is increased.

Since the errors in the event shape measurements are dominantly systematic, and are common to the experiments, the results from PEP/PETRA, TRISTAN, LEP, SLC, and CLEO are combined to give $\alpha_s(M_Z) = 0.121 \pm 0.007$. All of the experiments are consistent with this average and, taken together, provide verification of the running of the coupling constant with energy.

## 9.8. Scaling violations in fragmentation functions

Measurements of the fragmentation function $d_i(z, E)$, (the probability that a hadron of type $i$ be produced with energy $zE$ in $e^+e^-$ collisions at $\sqrt{s} = 2E$) can be used to determine $\alpha_s$. (Detailed definitions and a discussion of the properties of fragmentation functions can be found in Sec. 37 of this *Review*). As in the case of scaling violations in structure functions, QCD predicts only the $E$ dependence. Hence, measurements at different energies are needed to extract a value of $\alpha_s$. Because the QCD evolution mixes the fragmentation functions for each quark flavor with the gluon fragmentation function, it is necessary to determine each of these before $\alpha_s$ can be extracted.

The ALEPH collaboration has used data from energies ranging from $\sqrt{s} = 22$ GeV to $\sqrt{s} = 91$ GeV. A flavor tag is used to discriminate between different quark species, and the longitudinal and transverse cross sections are used to extract the gluon fragmentation function [118]. The result obtained is $\alpha_s(M_Z) = 0.126 \pm 0.007$ (expt.) $\pm$ 0.006 (theory) [119]. The theory error is due mainly to the choice of scale. The OPAL collaboration [120] has also extracted the separate fragmentation functions. DELPHI [121] has also performed a similar analysis using data from other experiments at lower energy with the result $\alpha_s(M_Z) = 0.124 \pm 0.007 \pm 0.009$ (theory). The larger theoretical error is due to the larger range of scales that were used in the fit. These results can be combined to give $\alpha_s(M_Z) = 0.125 \pm 0.005 \pm 0.008$ (theory).

A global analysis [122] uses data on the production of $\pi, K, p$, and $\overline{p}$ from SLC [123], DELPHI [124], OPAL [125], ALEPH [126], and lower-energy data from the TPC collaboration [127]. A flavor tag and a three-jet analysis is used to disentangle the quark and gluon fragmentation functions. The value $\alpha_s(M_Z) = 0.117^{+0.0055+0.0017}_{-0.0069-0.0025}$ is obtained. The second error is a theoretical one arising from the choice of scale. The fragmentation functions resulting from this fit are consistent with a recent fit of Ref. 128.

It is unclear how to combine the measurements discussed in the two previous paragraphs as much of the data used are common to both. If the theoretical errors dominate then a simple average is appropriate as the methods are different. For want of a better solution, the naive average of $\alpha_s(M_Z) = 0.120 \pm 0.006$ is used.

## 9.9. Photon structure functions

$e^+e^-$ can also be used to study photon-photon interactions, which can be used to measure the structure function of a photon [129], by selecting events of the type $e^+e^- \to e^+e^- + hadrons$ which proceeds via two photon scattering. If events are selected where one of the photons is almost on mass shell and the other has a large invariant mass $Q$, then the latter probes the photon structure function at scale $Q$; the process is analogous to deep inelastic scattering where a highly virtual photon is used to probe the proton structure. This process was included in earlier versions of this *Review* which can be consulted for details on older measurements [130–133]. A recent review of the data can be found in Ref. 134. Data are available from LEP [135–139] and from TRISTAN [140,141] which extend the range of $Q^2$ to of order 300 GeV$^2$ and $x$ as low as $2 \times 10^{-3}$and show $Q^2$ dependence of the structure function that is consistent with QCD expectations. There is evidence for a hadronic (non-perturbative) component to the photon structure function that complicates attempts to extract a value of $\alpha_s$ from the data.

Experiments at HERA can also probe the photon structure function by looking at jet production in $\gamma p$ collisions; this is analogous to the jet production in hadron-hadron collisions which is sensitive to hadron structure functions. The data [142] are consistent with theoretical models [143].

## 9.10. Jet rates in *ep* collisions

At lowest order in $\alpha_s$, the $ep$ scattering process produces a final state of (1+1) jets, one from the proton fragment and the other from the quark knocked out by the process $e + quark \to e + quark$. At next order in $\alpha_s$, a gluon can be radiated, and hence a (2+1) jet final state produced. By comparing the rates for these (1+1) and (2+1) jet processes, a value of $\alpha_s$ can be obtained. A NLO QCD calculation is available [144]. The basic methodology is similar to that used in the jet counting experiments in $e^+e^-$ annihilation discussed above. Unlike those measurements, the ones in $ep$ scattering are not at a fixed value of $Q^2$. In addition to the systematic errors associated with the jet definitions, there are additional ones since the structure functions enter into the rate calculations. Results from H1 [145] and ZEUS [146] can be combined to give $\alpha_s(M_Z) = 0.1179 \pm 0.0006$ (stat.) $\pm 0.0059$ (syst.). The contributions to the systematic errors from experimental effects (mainly the hadronic energy scale) are slightly smaller than the theoretical ones arising from scale choice, structure functions, and jet definitions. The theoretical errors are common to the two measurements; therefore, we have not reduced the systematic error after forming the average.

## 9.11. QCD in diffractive events

In approximately 10% of the deep-inelastic scattering events at HERA a rapidity gap is observed [147]; that is events are seen where there are almost no hadrons produced in the direction of the incident proton. This was unexpected; QCD based models of the final state predicted that the rapidity interval between the quark that is hit by the electron and the proton remnant should be populated approximately evenly by the hadrons. Similar phenomena have been observed at the Tevatron in $W$ and jet production. For a review see Ref. 148.

## 9.12. Lattice QCD

Lattice gauge theory calculations can be used to calculate, using non-perturbative methods, a physical quantity that can be measured experimentally. The value of this quantity can then be used to determine the QCD coupling that enters in the calculation. For a review of the methodology see Ref. 149. For example, the energy levels of a $Q\overline{Q}$ system can be determined and then used to extract $\alpha_s$. The masses of the $Q\overline{Q}$ states depend only on the quark mass and on $\alpha_s$. A limitation is that calculations have not been performed for three light quark flavors. Results are available for zero ($n_f = 0$, quenched approximation) and two light flavors, which allow extrapolation to three. The coupling constant so extracted is in a lattice renormalization scheme, and must be converted to the $\overline{\rm MS}$ scheme for comparison with other results. Using the mass differences of $\Upsilon$ and $\Upsilon'$ and $\Upsilon''$ and $\chi_b$, Davies *et al.* [150] extract a value of $\alpha_s(M_Z) = 0.1174 \pm 0.0024$. A similar result with larger errors is reported by [151], whose results are consistent with $\alpha_s(M_Z) = 0.111 \pm 0.006$. The SESAM collaboration [152] uses the $\Upsilon$ and $\Upsilon'$ and $\chi_b$ masses to obtain $\alpha_s(M_Z) = 0.1118 \pm 0.0017$ using Wilson fermions. These authors point out that their result is more than $3\sigma$ from that of Davies *et al.* which uses Kogut-Susskind fermions. A combination of the results from quenched [153] and ($n_f = 2$) [154] gives $\alpha_s(M_Z) = 0.116 \pm 0.003$ [155]. The ALPHA collaboration [156] who use the strength of the QCD potential from the Upsilon system inferred by Ref. 157 have begun to probe the systematic errors in detail for simulations involving two flavors of massless quarks. Ref. 158 sets the physical scale using the method of Ref. 152, uses Wilson fermions with two massless flavors and quotes $\alpha_s(M_Z) = 0.113 \pm 0.002$. Ref. 159 uses a similar method and results from quenched and two massless flavors to get $\alpha_s(M_Z) = 0.1076 \pm 0.0038$.

There have also been investigations of the running of $\alpha_s$ [160]. These show remarkable agreement with the two loop perturbative result of Eq. (9.5).

There are several sources of error in these estimates of $\alpha_s(M_Z)$. The experimental error associated with the measurements of the particle masses is negligible. The conversion from the lattice coupling constant to the $\overline{\rm MS}$ constant is obtained using a perturbative expansion where one coupling expanded as a power series in the other. The series is known to third order [161]. The effect of the third-order term is a shift in the extracted value of $\alpha_s(M_Z)$ of $+0.002$. Other theoretical errors arising from the limited statistics of the Monte-Carlo calculation, extrapolation in $n_f$, and corrections for light quark masses are smaller than this.

The result of averaging Refs. [150,152,153,158,159] gives with a more conservative error $\alpha_s(M_Z) = 0.1134 \pm 0.003$. This will be used in the average.

In addition to the strong coupling constant other quantities can be determined. Of particular interest are the decay constants of charmed and bottom mesons. These are required, for example, to facilitate the extraction of CKM elements from measurements of charm and bottom decay rates. See Ref. 162 for a recent review.

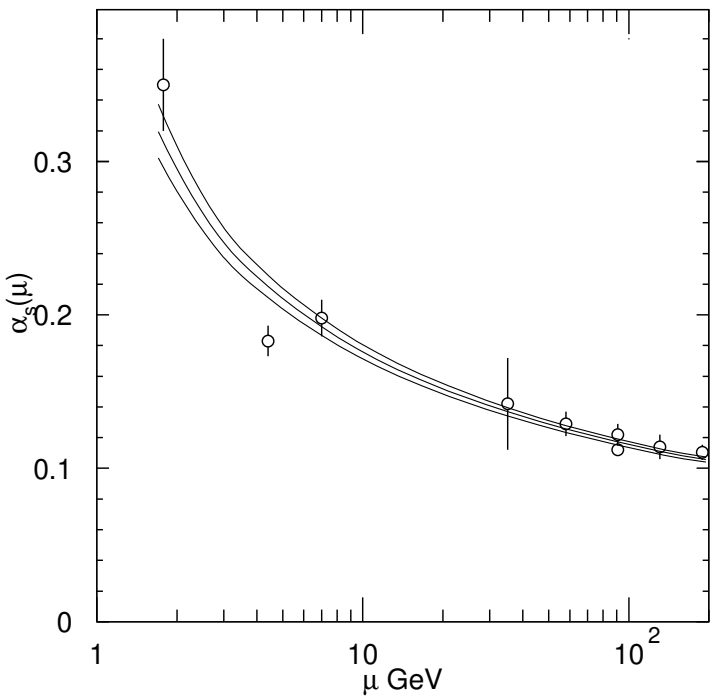


**Figure 9.2:** Summary of the values of $\alpha_s(\mu)$ at the values of $\mu$ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing $\mu$. The data are, in increasing order of $\mu$, $\tau$ width, $\Upsilon$ decays, deep inelastic scattering, $e^+e^-$ event rate at 25 GeV, event shapes at TRISTAN, $Z$ width, $e^+e^-$ event shapes of $M_Z$, 135, and 189 GeV.

## 9.13. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory, and QCD effects in hadron spectroscopy.

We have focused on those high-energy processes which currently offer the most quantitative tests of perturbative QCD. Figure 9.1 shows the values of $\alpha_s(M_Z)$ deduced from the various experiments. Figure 9.2 shows the values and the values of $Q$ where they are measured. This figure clearly shows the experimental evidence for the variation of $\alpha_s(Q)$ with $Q$.

An average of the values in Fig. 9.1 gives $\alpha_s(M_Z) = 0.1171$, with a total $\chi^2$ of 9 for eleven fitted points, showing good consistency among the data. The error on the average, assuming that all of the errors in the contributing results are uncorrelated, is $\pm 0.0014$, and may be an underestimate. Almost all of the values used in the average are dominated by systematic, usually theoretical, errors. Only some of these, notably from the choice of scale, are correlated. The average is not dominated by a single measurement; there are several results with comparable small errors: these are the ones from $\tau$ decay, lattice gauge theory, deep inelastic scattering, upsilon decay and the $Z^0$ width. We quote our average value as $\alpha_s(M_Z) = 0.1172 \pm 0.002$, which corresponds to $\Lambda^{(5)} = 216^{+25}_{-24}$ MeV using Eq. (9.5). Future experiments can be expected to improve the measurements of $\alpha_s$ somewhat. Precision at the 1% level may be achievable if the systematic and theoretical errors can be reduced [163].

The value of $\alpha_s$ at any scale corresponding to our average can be obtained from `http://www-theory.lbl.gov/~ianh/alpha/alpha.html` which uses Eq. (9.5) to interpolate.

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