CP VIOLATION IN B DECAY – STANDARD MODEL PREDICTIONS

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With the commissioning of the asymmetric B Factories at KEKB and PEP II, and of CESR III and with the completion of the main ring injector at Fermilab, we are headed into an exciting time for the study of CP violation in B meson decays. This review outlines the basic ideas of such studies. For the most part, we follow the discussions given in Refs. [1–3].

Time evolution of neutral B meson states

Neutral B mesons, like neutral K mesons, have mass eigenstates which are not flavor eigenstates. This subject is reviewed separately [4]. Here we give some formulae to establish the notation used in this review. The mass eigenstates are given by:

$$\begin{aligned} |B_1\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \ , \\ |B_2\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle \ , \end{aligned} \tag{1}$$

where B^0 and $\overline{B}^0$ are flavor eigenstates containing the $\overline{b}$ and b quarks respectively. The ratio

$$\frac{q}{p} = +\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \ . \tag{2}$$

Here, the CP operator is defined so that $CP|B^0\rangle = |\overline{B}^0\rangle$, and CPT symmetry is assumed. We define $M_{12} = \overline{M}_{12}e^{i\xi}$, where the phase ξ is restricted to $-\frac{1}{2}\pi < \xi < \frac{1}{2}\pi$, and $\overline{M}_{12}$ is taken to be real but not necessarily positive; and similarly (with a different phase) for Γ_{12}. The convention used here is that the real part of q/p is positive.

The differences in the eigenvalues $\Delta M = M_2 - M_1$ and $\Delta\Gamma = \Gamma_1 - \Gamma_2$ are given by

$$\begin{aligned} \Delta M &= -2\mathrm{Re}\left(\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right) \\ &\simeq -2\overline{M}_{12} \end{aligned}$$

$$\begin{aligned}\Delta\Gamma &= -4\mathrm{Im}\left(\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right)\\ &\simeq 2\overline{\Gamma}_{12}\cos\zeta\ .\end{aligned} \tag{3}$$

Here we denoted $\dfrac{\Gamma_{12}}{M_{12}} = re^{i\zeta}$. As we expect $r \sim 10^{-3}$ in the Standard Model for B_d, we kept only the leading order term in r. In the Standard Model, with these conventions and given that all models give a positive value for the parameter B_B, ΔM is positive, so that B_2 is heavier than B_1; this is unlikely to be tested soon. (Note that a common alternative convention is to name the two states B_L and B_H for light and heavy respectively; then the sign of q/p becomes the quantity to be tested.)

This review focuses on the B_d system, but also mentions some possibly interesting studies for CP violation in B_s decays, which may be pursued at hadron colliders. Much of the discussion here can be applied directly for B_s decays with the appropriate replacement of the spectator quark type.

The time evolution of states starting out at time $t = 0$ as pure B^0 or $\overline{B}^0$ is given by:

$$\begin{aligned}|B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}\,g_-(t)|\overline{B}^0\rangle\\ |\overline{B}^0(t)\rangle &= g_+(t)|\overline{B}^0\rangle + \frac{p}{q}\,g_-(t)|B^0\rangle,\end{aligned} \tag{4}$$

where

$$g_\pm(t) = \frac{1}{2}e^{-iM_1t}e^{-\frac{1}{2}\Gamma_1 t}\left[1 \pm e^{-i\Delta Mt}e^{\frac{1}{2}\Delta\Gamma t}\right]\ . \tag{5}$$

We define

$$\begin{aligned}A(f) &= \langle f|H|B^0\rangle\ ,\\ \overline{A}(f) &= \langle f|H|\overline{B}^0\rangle\ ,\\ \overline{\rho}(f) &= \frac{\overline{A}(f)}{A(f)} = \rho(f)^{-1}\ ,\end{aligned} \tag{6}$$

where f is a final state that is possible for both B^0 and $\overline{B}^0$ decays. The time-dependent decay rates are thus given by

$$
\begin{aligned}
&\Gamma(B^0(t) \to f) \\
&\propto e^{-\Gamma_1 t}|A(f)|^2\Bigg[K_+(t) + K_-(t)\left|\frac{q}{p}\right|^2 |\overline{\rho}(f)|^2 \\
&+ 2\mathrm{Re}\Big[L^*(t)\left(\frac{q}{p}\right)\overline{\rho}(f)\Big]\Bigg] \ ,
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\Gamma(\overline{B}^0(t) \to f) \\
&\propto e^{-\Gamma_1 t}|\overline{A}(f)|^2\Bigg[K_+(t) + K_-(t)\left|\frac{p}{q}\right|^2 |\rho(f)|^2 \\
&+ 2\mathrm{Re}\left[L^*(t)\left(\frac{p}{q}\right)\rho(f)\right]\Bigg],
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
|g_\pm(t)|^2 &= \frac{1}{4}e^{-\Gamma_1 t}K_\pm(t) \ , \\
g_-(t)g_+^*(t) &= \frac{1}{4}e^{-\Gamma_1 t}L^*(t) \ , \\
K_\pm(t) &= 1 + e^{\Delta\Gamma t} \pm 2e^{\frac{1}{2}\Delta\Gamma t}\cos\Delta M t \ , \\
L^*(t) &= 1 - e^{\Delta\Gamma t} + 2ie^{\frac{1}{2}\Delta\Gamma t}\sin\ \Delta M t \ .
\end{aligned} \tag{9}
$$

For the case of B_d decays the quantity $\Delta\Gamma/\Gamma$ is small and is usually dropped, for B_s decays it may be significant [6] and hence is retained in Eqs. 4–8.

Three classes of CP violation in B decays

When two amplitudes with different phase-structure contribute to a B decay, they may interfere and produce CP-violating effects [5]. There are three distinct types of CP violation: (1) CP violation from nonvanishing relative phase between the mass and the width parts of the mixing matrix which gives $|q/p| \neq 1$, often called "indirect;" (2) Direct CP violation, which is any effect that indicates two decay amplitudes have different weak phases (those arising from Lagrangian couplings), in particular it occurs whenever $|\rho(f)| \neq 1$; (3) Interference between a decays with and without mixing which can occur for decays to CP eigenstates whenever $\mathrm{Arg}((q/p)\overline{\rho}(f)) \neq 0$. This can occur even for modes where both the other types do not, *i.e.* $|q/p|, \ |\rho(f)| = 1$.

(1) Indirect CP violation

In the next few years, experiments will accumulate a large number of semileptonic B decays. Any asymmetry in the wrong-sign semileptonic decays (or in any other wrong-flavor decays) is a clean sign of indirect CP violation.

The semileptonic asymmetry for the wrong sign B_q decay, where $q = d$ or s, is given by

$$a_{SL}(B_q) = \frac{\Gamma(\overline{B}_q(t) \to \ell^+ X) - \Gamma(B_q(t) \to \ell^- X)}{\Gamma(\overline{B}_q(t) \to \ell^+ X) + \Gamma(B_q(t) \to \ell^- X)}$$
$$= \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = r_{B_q} \sin \zeta_{B_q} \ , \qquad (10)$$

where we kept only the leading order term in r_{B_q}. Within the context of the Standard Model, if hadronic rescattering effects are small then $\sin \zeta_{B_q}$ is small because M_{12} and Γ_{12} acquire their phases from the same combination of CKM matrix elements. Since this asymmetry is tiny in the Standard Model, this may be a fruitful area to search for physics beyond the Standard Model.

(2) Direct CP violation

Direct CP violation is the name given to CP violation that arises because there is a difference between the weak phases of any two decay amplitudes for a single decay. Weak phases are those that arise because of a complex coupling constant in the Lagrangian. Note that a single weak phase from a complex coupling constant is never physically meaningful because it can generally be removed by redefining some field by a phase. Only the differences between the phases of couplings which cannot be changed by such redefinitions are physically meaningful. The strong and electromagnetic couplings can always be defined to be real but, as Kobayashi and Maskawa first observed, in the three generation Standard Model one cannot remove all the phases from the CKM matrix by any choice of field redefinitions [7].

There are two distinct ways to observe direct CP-violation effects in B decays:

• $|\overline{A}_{\overline{f}}/A_f| \neq 1$ leading to rate asymmetries for CP-conjugate decays. Here, two amplitudes with different weak phases must contribute to the same decay; they must also have different strong phases, that is, the phases that arise because of absorptive parts (often called final-state interaction effects). When the final state f has different flavor content than its CP conjugate, this gives a rate asymmetry that is directly observable. The asymmetry is given by

$$a = \frac{2A_1A_2\sin(\xi_1-\xi_2)\sin(\delta_1-\delta_2)}{A_1^2+A_2^2+2A_1A_2\cos(\xi_1-\xi_2)\cos(\delta_1-\delta_2)} \ , \tag{11}$$

where the A_i are the magnitudes, the ξ_i are the weak phases, and the δ_i are the strong phases of the two amplitudes contributing to A_f. The impact of direct CP violation of this type in decays of neutral B's to flavor eigenstates is discussed below.

• Any difference (other than an overall sign) between the CP asymmetries for decays of B_d mesons to flavor eigenstates, or between those of neutral B_s mesons, is an evidence of direct CP violation. As is shown below, such asymmetries arise whenever the decay weak phase is not canceled by the mixing weak phase, hence any two different results imply that there is a difference between the weak phases of the amplitudes for the two decays. Only if the asymmetries are the same can one choose a phase convention which ascribes all CP-violating phases to the mixing amplitude. For example, the expected asymmetries for the $B \to J/\psi K_S$ and $B \to \pi\pi$ decays are different (whether or not penguin graphs add additional direct CP-violating effects of the type $|\overline{A}_{\overline{f}}/A_f| \neq 1$ in the latter channel) because the dominant decay amplitudes have different weak phases in the Standard Model.

(3) Decays of B^0 and $\overline{B}^0$ to CP eigenstates

In decays to CP eigenstates, the time-dependent asymmetry is given by

$$a_f(t) = \frac{\Gamma(\overline{B}^0(t)\to f) - \Gamma(B^0(t)\to f)}{\Gamma(\overline{B}^0(t)\to f) + \Gamma(B^0(t)\to f)} \ . \tag{12}$$

Asymmetry is generated if: (i) both $A(B \to f)$ and $A(\overline{B} \to f)$ are nonzero; and (ii) the mixing weak phase in $\frac{q}{p}$ is different from the weak decay phase in $\overline{\rho}(f)$. To the leading order in r, the Standard Model predicts

$$q/p = \frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}} = e^{-i2\phi_{\rm mixing}} \ . \tag{13}$$

If there is only one amplitude (or two with the same weak phase) contributing to $A(B \to f)$ and $A(\overline{B} \to f)$ then $|\overline{\rho}(f)| = 1$ and the relationship between the measured asymmetry and the Kobayshi-Maskawa phases is cleanly predicted by

$$\begin{aligned} a_f(t) &= {\rm Im}\left(\frac{q}{p}\overline{\rho}(f)\right)\sin \varDelta M t \\ &= -\eta_f \sin 2(\phi_{\rm mixing} + \phi_{\rm decay}) \sin \varDelta M t \ . \end{aligned} \tag{14}$$

Here we have used the fact that in such cases we can write $\overline{\rho}(f) = \eta_f e^{-i2\phi_{\rm decay}}$ where $\eta_f = \pm$ is the CP eigenvalue of the state f. The weak phases $\phi_{\rm mixing}$ and $\phi_{\rm decay}$ are parameterization dependent quantities, but the combination $\phi_{\rm mixing} + \phi_{\rm decay}$ is parameterization independent. This is CP violation due to the interference between decays with and without mixing. Note that a single measurement of $\sin(2\phi)$ yields four ambiguous solutions for ϕ.

When more than one amplitude with different weak phases contribute to a decay to a CP eigenstate there can also be direct CP violation effects $|\lambda_f = (q/p)\,\rho(f)| \neq 1$ and the asymmetry takes the more complicated form

$$a_f(t) = \frac{(|\lambda_f|^2 - 1)\cos(\varDelta M t) + 2{\rm Im}\lambda_f \sin(\varDelta M t)}{(1 + |\lambda_f|^2)} \ . \tag{15}$$

The quantity λ_f involves the ratio of the two amplitudes that contribute to A_f as well as their relative strong phases and hence introduces the uncertainties of hadronic physics into the relationship between the measured asymmetry and the K–M phases. However in certain cases such channels can be useful in resolving the ambiguities mentioned above. If $\cos(2\phi)$ can be measured as well as $\sin(\phi)$ only a two-fold ambiguity remains.

This can be resolved only by knowledge of the sign of certain strong phase shifts [8].

When a B meson decays to a CP self-conjugate set of quarks the final state is in general a mixture of CP even and CP odd states, which contribute opposite sign and hence partially canceling asymmetries. In two special cases, namely the decay to two spin zero particles, or one spin zero and one non-zero spin particle there is a unique CP eigenvalue because there is only one possible relative angular momentum between the two final state particles. Quasi-two-body modes involving two particles with non-zero spin can sometimes be resolved into contributions of definite CP by angular analysis of the decays of the "final-state" particles [9].

There can also be a direct CP violation in these channels from the interference of two contributions to the same decay amplitude, $|\rho(f)| \neq 1$. This introduces dependence on the relative strengths of the two amplitude contributions and on their relative strong phases. Since these cannot be reliably calculated at present, this complicates the attempt to relate the measured asymmetry to the phases of CKM matrix elements.

Standard Model predictions for CP-violating asymmetries

• ***Unitarity Triangles***

The requirement that the CKM matrix be unitary leads to a number of relationships among its entries. The constraints that the product of row i with the complex conjugate of row j is zero are generically referred to as "unitarity triangles" because they each take the form of a sum of three complex numbers equal to zero and hence can be represented by triangles in the complex plane. There are six such relationships, (see for example Ref. 10); the most commonly studied is that with all angles of the same order of magnitude, given by the relationship

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \tag{16}$$

This relation can be represented as a triangle on the complex plane, as shown in Fig. 1, where the signs of all three angles are also defined. When the sides are scaled by $|V_{cd}V_{cb}^*|$, the apex of the triangle is the point ρ, η, where these parameters are defined by the Wolfenstein parameterization of the CKM matrix [11]. If $\eta = 0$, the CKM matrix is real and there is no CP violation in the Standard Model.

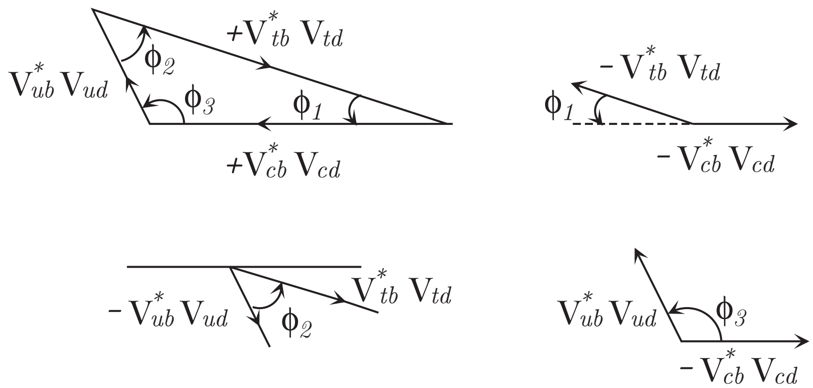

Figure 1: Angles of the unitarity triangle are related to the Kobayashi-Maskawa phases of the CKM matrix. The right-hand rule gives the positive direction of the angle between two vectors. This figure was reproduced from Ref. 1 with permission from Cambridge University Press.

The angles of the triangle are

$$\phi_1 = \pi - \arg\left(\frac{-\mathbf{V}^*_{tb}V_{td}}{-V^*_{cb}V_{cd}}\right) = \beta \ ,$$

$$\phi_2 = \arg\left(\frac{V^*_{tb}V_{td}}{-V^*_{ub}V_{ud}}\right) = \alpha \ ,$$

$$\phi_3 = \arg\left(\frac{V^*_{ub}V_{ud}}{-V^*_{cb}V_{cd}}\right) = \gamma \ . \tag{17}$$

Two naming conventions for these angles are commonly used in the literature [12,13]; we provide the translation dictionary in Eq. (17), but use the ϕ_i notation in the remainder of this review, where ϕ_i is the angle opposite the side $V^*_{ib}V_{id}$ of the unitarity triangle and i represents the i-th up-type quark. As defined here, for consistency with the measured value of ϵ_K, these angles are all positive in the Standard Model, thus a determination of the sign of these angles constitutes a test of the Standard Model [14].

There are two other independent angles of the Standard Model which appear in other triangles. These are denoted

$$\chi = \arg\left(\frac{-V_{cs}^* V_{cb}}{V_{ts}^* V_{tb}}\right) = \beta_s$$
$$\chi' = \arg\left(\frac{-V_{ud}^* V_{us}}{V_{cd}^* V_{cs}}\right) = -\beta_K \ . \quad (18)$$

Again there are two naming conventions in common usage so we give both. These angles are of order λ^2 and λ^4 respectively [15], where $\lambda = V_{us}$. The first of them is the phase of the B_s mixing and thus is in principle measurable, though it will not be easy to achieve a result significantly different from zero for such a small angle. The angle χ' will be even more difficult to measure. Meaningful standard model tests can be defined which use the measured value of λ coupled with χ and any two of the three ϕ_i [16].

A major aim of CP-violation studies of B decays is to make enough independent measurements of the sides and angles that this unitarity triangle is overdetermined, and thereby check the validity of the Standard Model predictions that relate various measurements to aspects of this triangle. Constraints can be made on the basis of present data on the B-meson mixing and lifetime, and on the ratio of charmless decays to decays with charm (V_{ub}/V_{cb}), and on ϵ in K decays [17]. These constraints have been discussed in many places in the literature; for a recent summary of the measurements involved, see Ref. [18]. Note, however, that any given "Standard Model allowed range" cannot be interpreted as a statistically-based error range. The ranges of allowed values depend on matrix element estimates. Improved methods to calculate such quantities, and understand the uncertainties in them, are needed to further sharpen tests of the Standard Model. Recent progress in lattice simulation using dynamical fermions seems encouraging [19]. It can be hoped that reliable computations of f_B, B_B, and B_K will be completed in the next few years. This will reduce the theoretical uncertainties in the relationships between measured mixing effects and the magnitudes of CKM parameters.

In the Standard Model there are only two independent phases in this triangle since, by definition, the three angles add up to π. The literature often discusses tests of whether the angles add up to π; but this really means tests of whether relationships between different measurements, predicted in terms of the two independent parameters in the Standard Model, hold true. For example, many models that go beyond the Standard Model predict an additional contribution to the mixing matrix. Any change in phase of M_{12} will change the measured asymmetries so that ϕ_1(measured) $\rightarrow \phi_1 - \phi_{\rm new}$ and ϕ_2(measured) $\rightarrow \phi_2 + \phi_{\rm new}$. Thus the requirement that the sum of the three angles must add up to π is not sensitive to $\phi_{\rm new}$ [20]. However, the angles as determined from the sides of the triangle would, in general, no longer coincide with those measured from asymmetries. It is equally important to check the asymmetries in channels for which the Standard model predicts very small or vanishing asymmetries. A new mixing contribution which changes the phase of M_{12} will generate significant asymmetries in such channels. In the Standard Model the CKM matrix must be unitary, this leads to relationships among its entries.

• ***Standard Model decay amplitudes***

In the Standard Model, there are two classes of quark-level diagrams that contribute to hadronic B decays, as shown in Fig. 2. Tree diagrams are those where the W produces an additional quark-antiquark pair. Penguin diagrams are loop diagrams where the W reconnects to the same quark line. Penguin diagrams can further be classified by the nature of the particle emitted from the loop: gluonic or QCD penguins if it is a gluon, and electroweak penguins if it is a photon or a Z boson. In addition, one can label penguin diagrams by the flavor of the up-type quark in the loop; for any process all three flavor types contribute. For some processes, there are additional annihilation-type diagrams; these always contribute to the same CKM structure as the corresponding trees. For a detailed discussion of the status of calculations based on these diagrams, or rather on the more complete operator product approach which also includes higher order QCD corrections see, for example, Ref. 21. Note that the distinction between tree and penguin contributions is a heuristic one, the separation of contributions by the operator that enters is more precise.

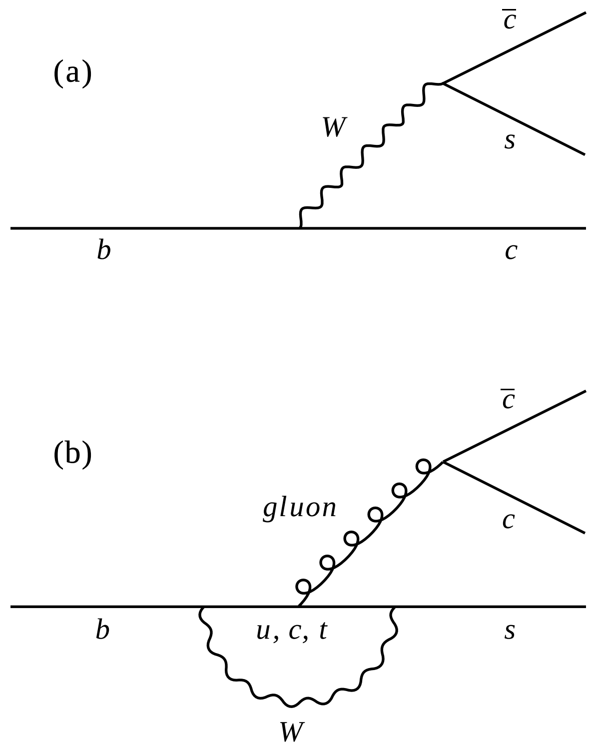

Figure 2: Quark level processes for the example of $b \to c\bar{c}s$. (a) Tree diagram; (b) Penguin diagram. In the case of electroweak penguin contributions, the gluon is replaced by a Z or a γ.

To explore possible CP violations, it is useful to tabulate all possible decays by the CKM structure of the various amplitudes. Let us first consider decays $b \to q\overline{q}'s$. The CKM factors for the diagrams for such decays are given in Table 1. Here we have used the fact that, for all such decays, the contribution to the amplitude from penguin graphs has the structure

$$A_P(q\overline{q}s) = V_{tb}V_{ts}^*P_t + V_{cb}V_{cs}^*P_c + V_{ub}V_{us}^*P_u \ , \qquad (19)$$

where the P_i quantities are the amplitudes described by the loop diagram with a flavor i quark apart from the explicitly shown CKM factor (*i.e.*, including strong phases). These are actually divergent quantities, so it is convenient to use a Standard Model unitarity relationship, $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$, to regroup them in the following way

$$A_P(q\overline{q}s) = V_{cb}V_{cs}^*(P_c - P_t) + V_{ub}V_{us}^*(P_u - P_t) \ , \tag{20}$$

or, equivalently,

$$A_P(q\overline{q}s) = V_{tb}V_{ts}^*(P_t - P_c) + V_{ub}V_{us}^*(P_u - P_c) \ . \tag{21}$$

The first term is of order λ^2, whereas the second is of order λ^4, and can be ignored in most instances. For modes with $q' \neq q$, there are no penguin contributions. Note also that for the $q\overline{q} = u\overline{u}, d\overline{d}$ cases, the QCD penguin graphs contribute only to the isospin zero combinations, whereas tree graphs contribute only for $u\overline{u}$ and hence have both $\Delta I = 0$ and $\Delta I = 1$ parts, as do electroweak penguins.

The CKM coefficients for $b \to q\overline{q}'d$ are listed in Table 2. A similar exercise to that described above for the penguins yields

$$A_P(q\overline{q}d) = V_{tb}V_{td}^*(P_t - P_c) + V_{ub}V_{ud}^*(P_u - P_c) \ . \tag{22}$$

Here the two CKM contributions are of the same order of magnitude λ^3, so both must be considered. This grouping is generally preferred over the alternative, because the second term here is somewhat smaller than the first term; it has no top-quark contribution and would vanish if the up and charm quarks were degenerate. In early literature it was often dropped, but, particularly for modes where there is no tree contribution, its effect in generating direct CP violation may be important [22]. Here the $q\overline{q} = u\overline{u}, d\overline{d}$ cases in the penguin graph contribute only to the isospin zero combinations, yielding $\Delta I = 1/2$ for the three-quark combination, whereas tree graphs and electroweak penguins have both $\Delta I = 1/2$ and $\Delta I = 3/2$ parts. For $q\overline{q} = c\overline{c}$, isospin does not distinguish between tree and penguin contributions.

Modes with direct CP violation

The largest direct CP violation is expected when there are two comparable magnitude contributions with different weak phases. Modes where the tree graphs are Cabibbo suppressed, compared to the penguins or modes with two comparable penguin contributions, are thus the best candidates. As can be seen from the tables and expressions for penguin contributions above, there are many possible modes to study. Because strong phases cannot usually be predicted, there is no clean prediction as to which modes will show the largest direct CP-violation effects. One interesting suggestion is to study three-body modes with more than one resonance in the same kinematic region. Then the different amplitudes can have very different, possibly known, strong phase structure because of the resonance (Breit-Wigner) phases [23].

Over the past two years, new information has become available from the CLEO Collaboration which suggests that penguin contributions, at least for some modes, are larger than initial estimates suggested. This is seen by using SU(3) and comparing $B \to K\pi$ and $B \to \pi\pi$ decays. To get an order of magnitude picture, we ignore such details as Clebsch-Gordan coefficients and assume that top penguins dominate the penguin contributions. Thus, we identify the tree and penguin contributions, minus their CKM coefficients, as T and P, the same for both modes. Writing $A_{T,P}(K\pi)$ for the tree and penguin contributions to the $K\pi$ amplitude, and similarly for $\pi\pi$ from the Tables, we see that $|A^T(K\pi)/A^T(\pi\pi)| = \mathcal{O}(\lambda)$. Thus, if the tree graph matrix elements were to dominate both decays, we would expect $\mathrm{Br}(B \to K\pi)/\mathrm{Br}(B \to \pi\pi) \sim \mathcal{O}(\lambda^2)$. Naively, this was expected, since the ratio of tree to penguin contribution was estimated to be $\dfrac{P}{T} = \dfrac{\alpha_S}{12\pi}\log\dfrac{m_t^2}{m_b^2} \sim \mathcal{O}(0.02)$. Experimentally, this is not so [24]; in fact, the $K\pi$ branching ratio is larger. This indicates that $A^P(K\pi) \sim A^T(\pi\pi)$, which suggests that $\dfrac{P}{T} = \mathcal{O}(\lambda)$ or larger, considerably bigger than expected. Note that this is one way that new physics could be hidden in modes with $|\rho(f)| \neq 1$; any new physics contribution can always be written as a sum of two terms with the weak phases of the two Standard Model terms (for example in Eq. (22)), and thus, when added to the Standard Model contributions, appears only as a change in the sizes of P and T from that expected in the Standard Model. However, we cannot calculate these relative sizes well enough to identify such an effect with confidence.

From the point of view of looking for direct CP-violation effects, a large P/T is good news. The largest asymmetry is expected when the interfering amplitudes have comparable magnitudes. This may be so in $B \to K\pi$ decay (or the penguin contribution may even be larger than the tree). There is no reason for the strong phases to be equal (although they could both be small). Therefore, $B^{\pm} \to K^{\pm}\pi$ is a likely hunting ground for direct CP violation. (Note there is no gluonic penguin contribution to charged $B \to \pi\pi$, and hence, no significant CP violation expected in the Standard Model.) However, as we will see below, a large P/T complicates the relationship between the measured asymmetry in neutral B decays to $\pi^+\pi^-$ and KM phases.

Studies of CP eigenstates

• $\boldsymbol{f = J/\psi K_S}$

The asymmetry in the "golden mode" $B \to J/\psi K_S$ has now been measured by both the BaBar and Belle experiments [25]. The Standard Model prediction for this mode is very clean. Since, using Eq. (20), the dominant penguin contribution has the same weak phase as the tree graph, and the remaining term is tiny, there is effectively only one weak phase in the decay amplitude. Hence, in the asymmetry, all dependence on the amplitudes cancel. With about 1% uncertainty,

$$\frac{q}{p}\overline{\rho}(J/\psi K_S) \simeq -\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \cdot \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \cdot \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \equiv -e^{-2i\phi_1} \ , \qquad (23)$$

where the last factor arises from the K^0–$\overline{K}^0$ mixing amplitude and appears because of the K_S in the final state. The asymmetry is thus given by

$$a_{J/\psi K_S} = \sin(2\phi_1)\sin \Delta M t \ , \qquad (24)$$

where the angle ϕ_1 is defined in Fig. 1. The result is consistent within errors with the prediction from the Standard Model, which strongly suggests that the KM ansatz for CP violation is at least one of the sources of this interesting phenomenon.

• $\boldsymbol{B^0 \to \pi^+\pi^-}$

The tree and penguin terms appear at the same order in λ (see Eq. (22) and Table 2.) If penguin decays were negligible the asymmetry would directly measure $\sin(2\phi_2)$. Given the enhanced penguin contribution seen from comparing $\pi\pi$ and $K\pi$ decays, the penguins cannot be ignored, and a treatment that does not assume $|\rho(f)| = 1$ must be made.

If all six modes of $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and their charge conjugates can be measured with sufficient accuracy, ϕ_2 can be extracted using an isospin analysis [26], up to small corrections from electroweak penguins. However, the branching ratio for the charged modes is less than 10^{-5} [24], and that for the more difficult to measure $B^0 \to \pi^0\pi^0$ is expected to be even smaller. Therefore, further ingenuity is needed to get at this angle cleanly. A future possibility is to study the Dalitz plot of $B \to 3\pi$ decays [27].

To date only upper limits on CP-violating asymmetries in this mode have been reported [25].

Further Measurements

As Tables 1 and 2 suggest there are many more CP-eigenstate modes that are interesting to study, both for B_d and similarly for B_s decays. The latter states are not accessible for the B factories operating at the $\Upsilon(4S)$ resonance, but may be studied at hadronic colliders. The CDF result on the asymmetry in the $J/\psi K_S$ mode is an indication of the capabilities of such facilities for B physics [29]. Upgrades of the Fermilab detectors are in progress and proposals for new detectors with the capability to achieve fast triggers for a larger variety of purely hadronic modes are under development, promising some future improvement in this capability.

In addition to CP-eigenstate modes there are many additional modes for which particular studies have been proposed, in particular those focussed on extracting ϕ_3 (γ). Modes such as DK, DK^* and D^*K where the D mesons decay to CP eigenstates provide theoretically clean extraction of this parameter but have small branching ratios [30]. Other approaches involve the more copious $K\pi$ modes but rely on the use of isospin and SU(3) (U-spin) symmetries, so have larger theoretical uncertainties [31]. This is an active area of current theoretical work.

For a recent review of how predictions for CP-violating effects are affected by Beyond Standard Model effects see Ref. 28. There are also many ways to search for new physics effects in B decays that do not involve just the CP-violation effects. For example searches for isospin breaking effects in $K\pi$ modes have recently been suggested as a likely method to isolate such effects [32].

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Table 1: $B \to q\overline{q}s$ decay modes

Quark process	Leading term	Secondary term	Sample B_d modes	B_d angle	Sample B_s modes	B_s angle
$b \to c\overline{c}s$	$V_{cb}V_{cs}^* = A\lambda^2$ tree + penguin$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only$(u-t)$	$J/\psi\ K_S$	β	$J/\psi\eta$ $D_s\overline{D}_s$	0
$b \to s\overline{s}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ penguin only$(u-t)$	ϕK_S	β	$\phi\eta'$	0
$b \to u\overline{u}s$ $b \to d\overline{d}s$	$V_{cb}V_{cs}^* = A\lambda^2$ penguin only$(c-t)$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$ tree + penguin$(u-t)$	$\pi^0\ K_S$ ρK_S	competing terms	$\phi\pi^0$ $K_S\overline{K}_S$	competing terms

Table 2: $B \to q\overline{q}d$ decay modes

Quark process	Leading term	Secondary term	Sample B_d modes	B_d angle	Sample B_s modes	B_s angle
$b \to c\overline{c}d$	$V_{cb}V_{cd}^* = -A\lambda^3$ tree + penguin$(c-u)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-u)$	D^+D^-	$^*\beta$	$J/\psi K_S$	$^*\beta_s$
$b \to s\overline{s}d$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-u)$	$V_{cb}V_{cd}^* = A\lambda^3$ penguin only$(c-u)$	$\phi\pi$ $K_S\overline{K}_S$	competing terms	ϕK_S	competing terms
$b \to u\overline{u}d$ $b \to d\overline{d}d$	$V_{ub}V_{ud}^* = A\lambda^3(\rho - i\eta)$ tree + penguin$(u-c)$	$V_{tb}V_{td}^* = A\lambda^3(1-\rho+i\eta)$ penguin only$(t-c)$	$\pi\pi$; $\pi\rho$ πa_1	$^*\alpha$	$\pi^0 K_S$ $\rho^0 K_S$	competing terms
$b \to c\overline{u}d$	$V_{cb}V_{ud}^* = A\lambda^2$	0	$D^0\pi^0$, $D^0\rho^0$ ↳ CP eigenstate	β	D^0K_S ↳ CP eigenstate	0

*Leading terms only, large secondary terms shift asymmetry.